The possibility of synthesizing a doubly magic superheavy nucleus, $^{298}114_{184}$, is investigated on the basis of fluctuation-dissipation dynamics. In order to synthesize this nucleus, we must generate more neutron-rich compound nuclei because of the neutron emissions from excited compound nuclei. The compound nucleus $^{304}114$ has two advantages to achieving a high survival probability. First, because of low neutron separation energy and rapid cooling, the shell correction energy recovers quickly. Second, owing to neutron emissions, the neutron number in the nucleus approaches that of the double closed shell and the nucleus attains a large fission barrier. Because of these two effects, the survival probability of $^{304}114$ does not decrease until the excitation energy $E^* = 50$ MeV. These properties lead to a rather high evaporation residue cross section.

I. INTRODUCTION

The search for new elements is a long-standing important subject in nuclear physics [1,2]. Since 1966, according to macroscopic-microscopic calculations [3], a magic island of stability surrounding the doubly magic superheavy nucleus containing 114 protons and 184 neutrons has been predicted. The property and structure of nuclei in the superheavy mass region have been investigated, taking into account a large multidimensional deformation for the ground state [4–6].

Recently, within the relativistic mean-field model [7] and nonrelativistic Skyrme-Hartee-Fock approach [8], some other spherical magic numbers have been found, such as $Z = 120$ and $N = 172$ [9].

Attempts to synthesize heavy elements with atomic numbers beyond $Z \sim 100$ have been active since the 1970s, making use of various developments in experimental techniques [1,2,10,11]. For superheavy elements of around $Z \sim 114$ and $N \sim 184$, practical combinations of a target and projectile, such as $^{48}$Ca+$^{244}$Pu, have been used by the FLNR group [12]. In this case, the neutron number in the compound nucleus is less than $N = 184$.

Actually, if we plan to synthesize the doubly magic superheavy nucleus $^{298}114_{184}$, we must fabricate more neutron-rich compound nuclei because of the neutron emissions from excited compound nuclei. Since combinations of stable nuclei do not provide such neutron-rich nuclei, the reaction mechanism for nuclei with $Z = 114$, $N > 184$ has rarely been investigated until now. However, because of the characteristic properties of these nuclei, we find an unexpected reaction mechanism for enhancing the evaporation residue cross section. We report this mechanism in this paper.

As is well known, in heavy systems around $Z \sim 80$, the trajectory calculations with friction [13,14] were very useful for the explanation of the extra- or extra-extra-push energy. In superheavy mass region, however, the mean trajectory calculations are not suitable, because mean trajectories cannot reach the spherical shape region and around due to the strong dissipation [15]. However, the extremely small part of distribution can be found there due to fluctuation. Therefore, it is important to take into account the fluctuating part from the mean trajectory. It becomes necessary to solve a full dissipative dynamics, or a fluctuation-dissipation dynamics with the Kramers (Fokker-Planck) equation or with the Langevin equation [16–19].

In Sec. II, we explain our framework for the study and the model used. Results of the dynamical calculations are given in Sec. III. A summary is given in Sec. IV.

II. MODEL

Using the same procedure as described in Ref. [20], we apply the fluctuation-dissipation model and employ the Langevin equation for the fusion process. On the basis of our previous studies [21,22], to investigate the fission process, we employ the Smoluchowski equation, which is the strong friction limit of the Fokker-Planck equation. Here, we take into account the temperature-dependent shell correction energy.

The evaporation residue cross section $\sigma_{ER}$ is estimated as

$$\sigma_{ER} = \frac{\pi\hbar^2}{2\mu_0E_{c.m.}} \sum_{l=0}^{\infty} (2l + 1)T_l(E_{c.m.}, l)P_{CN}(E^*, l)W(E^*, l),$$

where $\mu_0$ denotes the reduced mass in the entrance channel. $E_{c.m.}$ and $E^*$ denote the incident energy in the center-of-mass frame and the excitation energy of the compound nucleus, respectively. $E^*$ is given as $E^* = E_{c.m.} - Q$ with $Q$ denoting the $Q$-value of the reaction. $T_l(E_{c.m.}, l)$ is the capture probability of the $l$th partial wave, which is calculated with the empirical coupled channel model [23]. $P_{CN}(E^*, l)$ is the probability of forming a compound nucleus in competition with quasi-fission events. $W(E^*, l)$ denotes the survival probability of compound nuclei during deexciting process.

To calculate $P_{CN}$, we employ the Langevin equation. We adopt the three-dimensional nuclear deformation space given by two-center parametrization [24,25]. The three collective parameters involved in the Langevin equation are as follows: $\delta_0$ (distance between two potential centers), $\delta$ (deformation of fragments), and $\alpha$ (mass asymmetry of the colliding nuclei); $\alpha = (A_1 - A_2)/(A_1 + A_2)$, where $A_1$ and $A_2$ denote the mass numbers of the target and projectile, respectively. The detail is explained in Ref. [20].
After the probabilities reaching the spherical shape and around, we must treat extremely small probabilities in the decay process of the compound nucleus. Therefore, we investigate the evolution of the probability distribution $P(q, l; t)$ in the collective coordinate space with the Smoluchowski equation [21,22]. We employ the one-dimensional Smoluchowski equation in the elongation degree of freedom $z_0$, which is expressed as follows:

$$
\frac{\partial}{\partial t} P(q, l; t) = \frac{1}{\mu \beta \partial q} \left\{ \frac{\partial V(q, l; t)}{\partial q} P(q, l; t) \right\} + \frac{T}{\mu \beta} \frac{\partial^2}{\partial q^2} P(q, l; t).
$$

$q$ denotes the coordinate specified by $z_0$. $V(q, l; t)$ is the potential energy, and the angular momentum of the system is expressed by $l$. $\mu$ and $\beta$ are the inertia mass and the reduced friction, respectively. For these quantities we use the same values as in Refs. [21,22]. $T$ is the temperature of the compound nucleus calculated from the excitation energy as $E^* = aT^2$ with $a$ denoting the level density parameter of Tóke and Swiatecki [26]. The temperature dependent shell correction energy is added to the macroscopic potential energy,

$$
V(q, l; t) = V_{\text{DM}}(q) + \frac{h^2l(l+1)}{2I(q)} + V_{\text{shell}}(q)\Phi(t),
$$

where $I(q)$ is the moment of inertia of rigid body at coordinate $q$. $V_{\text{DM}}$ is the potential energy of the finite range droplet model and $V_{\text{shell}}$ is the shell correction energy at $T = 0$ [20].

The temperature dependence of the shell correction energy is extracted from the free energy calculated with single particle energies [22,27]. The temperature-dependent factor $\Phi(t)$ in Eq. (3) is parametrized as

$$
\Phi(t) = \exp\left( -\frac{aT^2(t)}{E_d} \right),
$$

following the work by Ignatyuk et al. [28]. The shell-damping energy $E_d$ is chosen as 20 MeV. The cooling curve $T(t)$ is calculated by the statistical model code SIMDEC [22,27]. We assume that the particle emissions in the composite system are limited to neutron evaporation in the neutron-rich heavy nuclei. When the temperature decreases as a result of neutron evaporation, the potential energy $V(q, l; t)$ changes due to the restoration of shell correction energy.

The survival probability $W(E^*_0, l; t)$ is defined as the probability which is left inside the fission barrier in the decay process

$$
W(E^*_0, l; t) = \int_{\text{inside saddle}} P(q, l; t) dq.
$$

Here, $E^*_0$ is the initial excitation energy of the compound nucleus.

In the statistical code SIMDEC, we take into account the change of angular momentum during the deexcitation process. Roughly speaking, when one neutron is emitted from the compound nucleus, the angular momentum of the compound nucleus decreases $\hbar$, on average. Because of the decreasing angular momentum, the centrifugal part of the potential energy in Eq. (3) changes. The angular momentum dependence of the shell correction energy was reported in Ref. [29].

For the purpose of understanding well the characteristic enhancement in the excitation function, we first discuss the evaporation residue probability of one partial wave, i.e., of $l = 10\hbar$, which is one of the dominantly contributing partial waves [22,30]. In the present calculation, we do not take into account the variation of the potential energy that is due to the decrease of the angular momentum by neutron emission, because the initial angular momentum is small.

### III. RESULTS

In our previous studies [21,22], we showed the isotope dependence of the evaporation residue cross section for $Z = 114$. At that time, we investigated the isotope dependence with $N \leq 184$. However, in order to synthesize the doubly magic nucleus $^{298}114_{184}$ by hot fusion reactions, we must fabricate a more neutron-rich compound nucleus of $N > 184$ because of the neutron emissions from the excited compound nucleus.
Taking into account the neutron emissions, we investigate the possibility of synthesizing $^{298}_{114}184$.

The neutron separation energy depends on the neutron number. Figure 1(a) shows the neutron separation energies averaged over four successive neutron emissions ($B_n$) for the isotopes with $Z = 114$. We use the mass table in Ref. [31]. ($B_n$) values of $A = 292, 298,$ and $304$ are $6.43, 5.25,$ and $4.06$ MeV, respectively. With increasing neutron number of the nucleus, the neutron separation energy becomes low. Therefore many neutrons evaporate easily from the neutron-rich compound nuclei.

Because of rapid neutron emissions, the cooling speed of the compound nucleus is very high. Figure 1(b) shows the cooling curves of $A = 292, 298$ and $304$ at the initial excitation energy $E^*_0 = 40$ MeV, that were derived using the statistical code SIMDEC [22,27]. In the case of $A = 304$, the excited compound nucleus cools rapidly and the fission barrier recovers at a low excitation energy.

Moreover, owing to the neutron emissions, the neutron number of the deexciting nucleus with $A = 304$ approaches that of a nucleus with the double closed shell $^{304}_{114}184$. Figure 2(a) shows the shell correction energies $V_{\text{shell}}$ of isotopes with $Z = 114$ [31]. $V_{\text{shell}}$ of the $A = 304$ ($N = 184$) nucleus is smaller than that of the $A = 298$ ($N = 184$) nucleus. However, in the deexciting process of the nucleus with $A = 304$ ($N = 190$), the neutron number approaches $N = 184$ because of neutron emission. In Fig. 2(b), the time evolution of the neutron number for the compound nucleus $^{304}_{114}184$ is shown for eight different initial excitation energies, as calculated by SIMDEC [22,27]. At a high initial excitation energy, the neutron number of the compound nucleus quickly approaches $N \sim 184$, which is that of a neutron closed shell. This indicates the rapid appearance of a large fission barrier.

The compound nucleus with $^{304}_{114}$114 has two advantages for attaining a high survival probability. First, because of the low neutron separation energy and rapid cooling, the shell correction energy recovers quickly. Secondly, because of neutron emissions, the number of neutrons in the nucleus approaches that in the double closed shell, and a high shell correction energy is attained.

Generally, at a high excitation energy, the recovery of the shell correction energy is delayed. On the other hand, at a low excitation energy, the shell correction energy is established. Figure 3(a) shows the time evolution of the fission barrier height $B_f$ for $^{298}_{114}$114. We can see that the restoration of the shell correction energy is increasingly delayed with increasing excitation energy. Using the Smoluchowski equation, we calculate the survival probability, which is denoted by the red
FIG. 4. (Color online) Survival probabilities for $^{298}$114, $^{300}$114, and $^{304}$114, calculated using the one-dimensional Smoluchowski equation.

With increasing excitation energy, the survival probability decreases drastically. However, for $^{304}$114, the situation is opposite. At an excitation energy of 50 MeV, the fission barrier recovers faster than in the cases with lower excitation energies, as shown in Fig. 3(b). The reason is the double effects, that is to say, the rapid cooling and rapid approach to $N \sim 184$. The survival probability of $^{304}$114 is denoted by the blue line in Fig. 4. It is highly interesting that the excitation function of the survival probability has a flat region around $E^* = 20 \sim 50$ MeV. At $E^* = 50$ MeV, the survival probability of $^{304}$114 is three orders magnitude larger than that of $^{298}$114. For reference, the survival probability of $^{300}$114 is denoted by the green line in Fig. 4.

FIG. 5. (Color online) Fusion probabilities for each ideal system leading to the compound nucleus $^{304}$114, which were calculated using the three-dimensional Langevin equation. For the reaction $^{152}$La + $^{152}$La, it was calculated using the one-dimensional Smoluchowski equation. The arrows denote the corresponding Bass potential barriers [32].

For the ideal combinations for synthesizing the compound nucleus $^{304}$114, the fusion probabilities for each system are shown in Fig. 5, which were calculated using the Langevin equation, except for the reaction $^{152}$La + $^{152}$La. This symmetric reaction system with extremely low fusion probability is applied to the one-dimensional Smoluchowski equation. The combinations of the projectile and target are indicated in Fig. 5. The corresponding Bass potential barriers are denoted by the arrows [32]. We show the fusion probabilities above the barrier, because we use the classical models. To multiply the fusion probability with the survival probability of $^{304}$114 in Fig. 4, we obtain the evaporation residue cross section of superheavy elements, as shown in Fig. 6. The cross section is rather high. It is expected that neutron-rich isotopes are more favorable for the enhancement of the evaporation residue cross section. Thus, an investigation of the experimental feasibility of obtaining neutron-rich superheavy elements is an extremely urgent subject.

IV. SUMMARY

In summary, using the three-dimensional Langevin equation for the fusion process and the one-dimensional Smoluchowski equation for the survival process on the basis of our previous works [20–22], we investigated the possibility of synthesizing the doubly magic superheavy nucleus $^{298}$114. Because of the neutron emissions, we must generate more neutron-rich compound nuclei. The compound nucleus $^{304}$114 has two advantages to achieving a high survival probability. First, because of small neutron separation energy and rapid cooling, the shell correction energy recovers quickly. Secondly, owing to neutron emissions, the neutron number
of the nucleus approaches that of the double closed shell. Because of these two effects, the excitation function of the survival probability of $^{30}\text{N}^{14}$ has a flat region around $E^* = 20 \sim 50$ MeV. These properties lead to a rather high evaporation residue cross section. As a more realistic model, we plan to take into account the emission of the charged particles from the compound nucleus. Moreover, we treat the case of large angular momentum, and take into account the variation of the potential energy due to the decrease of angular momentum in the nucleus deexciting via neutron emissions.

Although the combinations of stable nuclei cannot yield such neutron-rich nuclei as $Z = 114$ and $N > 184$, we hope to make use of secondary beams in the future. We believe the mechanism that we discussed here can inspire new experimental studies on the synthesis of superheavy elements. Also, such a mechanism is very interesting and can be applied to any system that has the same properties, that is, low neutron separation energy and slightly larger neutron number than the closed shell.

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