Superheavy nuclei: Decay and Stability

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Abstract Decay properties of superheavy nuclei are required for exploring the nuclei from the upper part of the nuclear map. The stability of nuclei with $Z \leq 132$ is studied with respect to α-decay, β-decay and spontaneous fission. Performed calculations allow us to conclude that at existing experimental facilities the synthesis and detection of nuclei with $Z > 120$ produced in fusion reactions may be difficult due to their short half-lives (shorter than 1 μs). We found for the first time the region of β^+-decaying superheavy nuclei with $111 \leq Z \leq 115$ located to the "right" (more neutron-rich) to those synthesized recently in Dubna in ^{48}Ca-induced fusion reactions. This fact may significantly complicate their experimental identification. However it gives a chance to synthesize in fusion reactions the most stable superheavy nuclei located at the center of the island of stability. Our calculations yield that the β-stable isotopes ^{291}Cn and ^{293}Cn with a half-life of about 100 years are the longest-living superheavy nuclei located at the island of stability.

1 Motivation

More than 40 years passed from the first predictions that the region of rather stable superheavy (SH) nuclei should exist around $Z \sim 114$ and $N \sim 184$ [1, 2, 3]. Great success was achieved during the last twenty years in the experimental study of reac-

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tions leading to superheavy nuclei, their decay properties and structure. Up to now near-barrier fusion reactions have been used for the production of new SH elements in the "cold" [4, 5] and "hot" (using ^{48}Ca as a projectile) [6, 7] combinations of colliding nuclei. The heaviest yet discovered element is the 118 one, synthesized in "hot" fusion reaction of ^{48}Ca beam and ^{248}Cf target. However, californium is the heaviest available target which has been used in these experiments for the production of element 118 [8]. Thus, to get SH elements with $Z > 118$ in fusion reactions, one should proceed to heavier than ^{48}Ca projectiles (^{50}Ti, ^{54}Cr, etc.). The corresponding cross sections for the production of the elements 119 and 120 are predicted to be smaller by about two orders of magnitude [9] as compared with ^{48}Ca-induced fusion reactions leading to the formation of the elements 114-116. Another limitation of the fusion reactions (both "cold" and "hot") for producing superheavy elements consists in the fact that they lead to neutron-deficient isotopes having rather short life time.

The most stable SH nuclei are predicted to be located along the β-stability line in the region of more neutron-rich nuclei, which is unreachable directly by fusion reactions with stable beams. In fact, the predicted magic numbers, especially for protons, are quite different within different theoretical approaches. The magic number $Z = 114$ was predicted in earliest macro-microscopic calculations [1, 2, 10, 3] and confirmed later in Refs. [11, 12]. The fully microscopic approaches predict the proton shell closure at $Z = 120$ [13], $Z = 126$ [14], or $Z = 114, 120, 126$ [15] depending on the chosen nucleon-nucleon interaction in mean field theories. The neutron magic number $N = 184$ is almost firmly predicted by different theoretical models.

Nowadays the experimental study of heavy nuclei, in particular of superheavies, requires ideas, new theoretical predictions, and methods (reactions) that can be used for producing these nuclides. Knowledge of the decay modes and half-lives of nuclei in a very wide range of neutron and proton numbers (nuclear map) is necessary for such predictions and for the planning of the corresponding experiments. Moreover, the study of decay properties may help us to answer some principle but open questions: how far may we still move in synthesis of SH elements by the fusion reactions, where the island of stability is centered, what are the properties of the most stable SH nuclei, how to reach this region? Another field where the decay properties play a crucial role is the study of the r-process of nucleosynthesis in the superheavy mass region, and the related problem of a search of superheavy nuclei in nature.

2 Half-lives of heavy and SH nuclei

This work is aimed to the analysis of the decay properties of heavy and superheavy elements with respect to α-decay, β-decay, and spontaneous fission (SF) – the three main decay modes. All the calculations performed in this paper are based on the values of the ground-state masses obtained within the macro-microscopic approach. Here we use experimental masses for known nuclei and three sets of the ground-state masses for unknown ones, obtained by P. Möller et al. [16] (mainly these, as the most known ones), A. Sobiczewski et al. [17], and within the two-center shell-model potential [18, 19].

The α-decay is characterized by the energy release Q_α and the corresponding half-life T_α. The half-life for α-decay can be estimated quite accurately using the well-known Viola-Seaborg formula [20]

$$\log_{10} T_\alpha\,(\mathrm{sec}) = \frac{aZ+b}{\sqrt{Q_\alpha(MeV)}} + cZ + d + h_{\log}, \tag{1}$$

where a, b, c, d, and $h_{\log}$ are adjustable parameters. We use the values of these parameters obtained in [21] $a = 1.66175, b = -8.5166, c = -0.20228, d = -33.9069$. The quantity $h_{\log}$ takes into account hindrance of α-decay for nuclei with odd neutron and/or proton numbers [20]

$$h_{\log} = \begin{cases} 0, & \text{Z and N are even} \\ 0.772, & \text{Z is odd and N is even} \\ 1.066, & \text{Z is even and N is odd} \\ 1.114, & \text{Z and N are odd} \end{cases} \tag{2}$$

The phenomenological calculation of T_α is the most justified (as compared with T_β and T_{SF}) and the most accurate. The errors arising from uncertainty in Q_α are much larger than the one due to the inaccuracy of phenomenological Viola-Seaborg formula.

If one moves aside the stability line, the β-processes start to play an important role. Therefore, to estimate correctly the life time of such a nucleus we have to consider the competition of α-decay and spontaneous fission with $\beta^\pm$ decays and electron capture (EC). The decay properties of nuclei close to the β-stability line are mostly known (except for the region of superheavy nuclei). This means that we may restrict ourself to the case of nuclei far from the line of β-stability. It allows us to assume that the corresponding Q-values and the density of states are large enough to find in the daughter nucleus a level which is close to the ground state and which fulfils the conditions of allowed β-decays. Thus, the problem simplifies to the case of the ground-to-ground allowed β transitions. This assumption may be not accurate enough for some specific nuclei close to the β-stability line, but this can not alter the general trend in the decay modes, which we are interested in. We should mentioned here that previous systematic calculations of the half-lives with respect to β-decay (see, e.g., [22, 23]) were performed for allowed transitions as well. The half-life with respect to all kinds of β processes T_β is given by

$$1/T_\beta = 1/T_{\beta^-} + 1/T_{\beta^+} + 1/T_{EC}. \tag{3}$$

The half-life with respect to the allowed β-decay is defined by the following relation [24]:

$$\log_{10}\left[f_0^b T_b\,(\mathrm{sec})\right] = 5.7 \pm 1.1, \tag{4}$$

where f_0^b is the Fermi function (which is calculated using the standard relations, see e.g. [25]), $b = \beta^{\pm}$ or EC. Thus, the estimation of the β-decay half-lives is reduced to the calculation of the Fermi function f_0^b. We use in (4) the constant value 4.7, adjusted to the corresponding experimental data. The β-decay half-lives shorter than 1000 s should be addressed to the allowed decays. Our calculations agree with the experiment within two orders of magnitude for this case. This is sufficient to estimate the β-decay half-lives in competition with α-decay and spontaneous fission almost for all experimentally unknown nuclei.

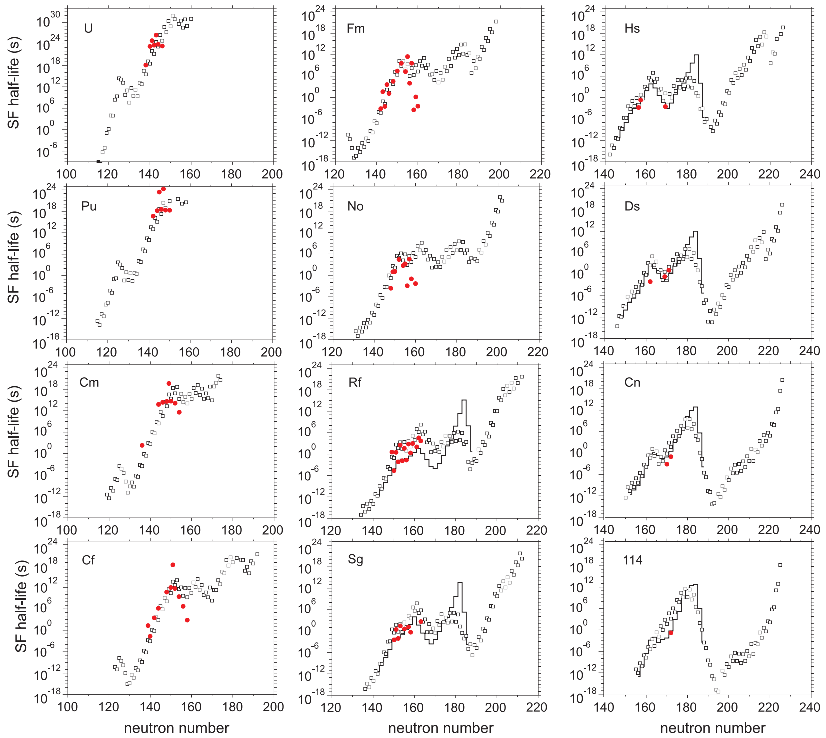

Fig. 1 Dependence of the SF half-lives on the neutron number for the isotopes of elements from U to 114. The open black squares are the estimation by the phenomenological formula (5), the full red circles are the experimental data [30, 32], and the full lines are the calculations of Ref. [26].

The spontaneous fission (SF) of nuclei is a very complicated process. Knowing the multidimensional potential energy surface only is not sufficient for the accurate determination of the corresponding decay time. The most realistic calculations of the SF half-life are based on the search for the least action path in the multidimensional deformation space. Only few examples of such calculations are known [10, 26, 27], that were performed in a rather restricted area of the nuclear map due to long calculation times. In Ref. [28] we propose the systematics based on idea of W.J. Swiatecki [29] that the SF half-lives are mainly determined by the height of the fission barrier. To determine the coefficients of the systematics we include in the fitting procedure not only the experimental data [30] but also the realistic theoretical predictions [26, 27] for the region $100 \leq Z \leq 120$ and $140 \leq N \leq 190$

$$\begin{aligned} \log_{10} T_{SF}\,(\text{sec}) &= 1146.44 - 75.3153 Z^2/A + \\ &+ 1.63792\left(Z^2/A\right)^2 - 0.0119827\left(Z^2/A\right)^3 + \\ &+ B_f\left(7.23613 - 0.0947022 Z^2/A\right) + \\ &+ \begin{cases} 0, & \text{Z and N are even} \\ 1.53897, & \text{A is odd} \\ 0.80822, & \text{Z and N are odd} \end{cases} \end{aligned} \tag{5}$$

Here B_f is the fission barrier, which is calculated as a sum of the liquid-drop barrier $B_f(LDM)$ [31] and the ground-state shell correction $\delta U(g.s.)$ [16], i.e. $B_f = B_f(LDM) + \delta U(g.s.)$. Figure 1 shows the dependence of the SF half-life on the neutron numbers for nuclei with even atomic numbers from Uranium to $Z = 114$ element. Obviously Eq. (5) qualitatively reproduces the behavior of the half-lives in the experimentally known region. However the proposed relation substantially underestimates the abrupt decrease of the half-life for Cf, Fm, and No around $N = 160$. In the region of superheavy nuclei we get reasonable agreement with the data. The reason for larger deviation from the experimental SF half-lives for neutron-rich isotopes of Cf, Fm, and No is the influence of exit channel, caused by clusterization with two nearly double-magic tin fragments, which is a special case of this region of nuclei. This effect is not included in the relation (5), but is accounted for within the dynamical approach mentioned above. However, even in such advanced calculations, this steep decrease of the SF half-live is underestimated (see Fig. 4 in Ref. [27]). In Fig. 1 we also show the calculations of Ref. [26] for the isotopes of $Z = 104 - 114$. One may see that in this region both models give similar results for those nuclei, for which experimental data exist. However, the model of Ref. [26] predicts for some nuclei a too steep decrease of the half-lives around $N \simeq 170$ and much longer times around the closed shell numbers $N = 184$.

3 Analysis of the nuclear map

Figure 2 shows upper part of the nuclear map for the total half-lives and decay modes of the nuclei with $Z \leq 132$ obtained with the ground-state masses from Ref. [16]. The known nuclei are situated along the β-stability line with a shift to the proton-rich region especially for heavy and superheavy nuclei. Almost all proton-rich nuclei with $Z \leq 118$ having half-lives sufficiently long for their experimental identification are already synthesized. The red circles in Figs. 2 and 3 (a) correspond to the nuclei with $Z = 119 - 124$, which may be obtained in the $3n$ channel of the fusion reactions: ^{50}Ti$+^{249}$Bk, ^{50}Ti$+^{249}$Cf, ^{54}Cr$+^{248}$Cm, ^{54}Cr$+^{249}$Bk, ^{54}Cr$+^{249}$Cf, ^{58}Fe$+^{248}$Cm, ^{58}Fe$+^{249}$Bk, and ^{58}Fe$+^{249}$Cf. The synthesis cross section of these new superheavy

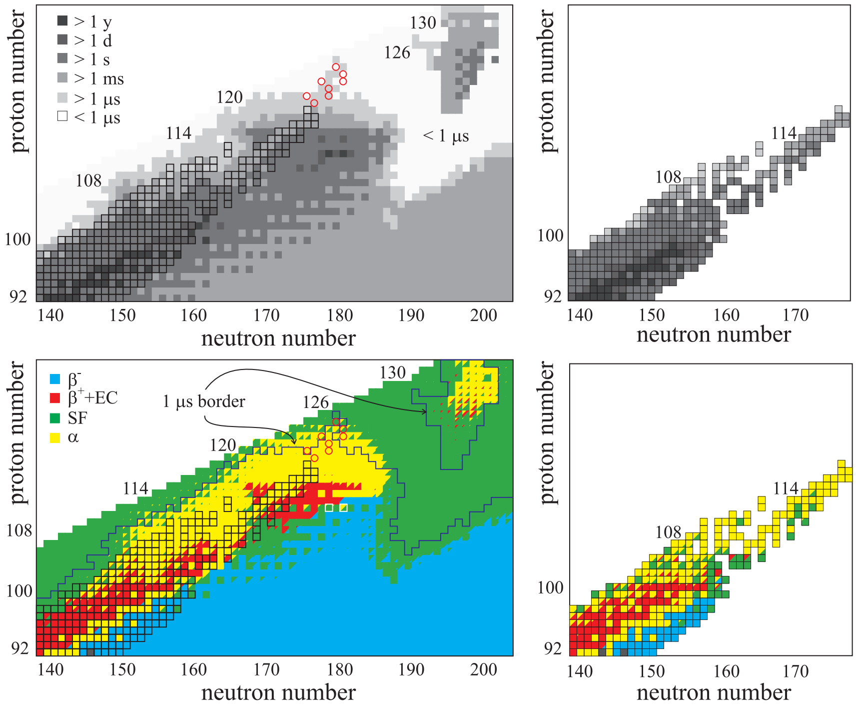

Fig. 2 The total half-lives (top) and the decay modes (bottom) of nuclei in the upper part of the nuclear map. The left panels are calculations (performed with the ground-state masses from Ref. [16]) and the right panels are the experimental data taken from [32]. The contour lines on the left bottom panel correspond to the border of 1 μs half-life. The circles show the nuclei with $Z = 119 - 124$, which may be synthesized in 3n channel of fusion reactions ^{50}Ti$+^{249}$Bk,^{249}Cf and ^{54}Cr,^{58}Fe$+^{248}$Cm,^{249}Bk,^{249}Cf (see the text). The bounded cells correspond to the experimentally known nuclei. The bounded nuclei with the white color border are the most stable Copernicium isotopes ^{291}Cn and ^{293}Cn.

nuclei with $Z > 118$ in fusion reactions is predicted to decrease substantially due to the change of the projectile from ^{48}Ca to a heavier one [9]. Moreover, as can be seen from Figs. 2 and 3 (a) these nuclei are very short-living. They are located at the border of 1 μs area – the critical time required to pass separator to be detected. It means that the nuclei heavier than the 120 element – even if they will be synthesized – could be hardly detected because of their very short half-lives. This conclusion is nearly model independent. Both models [see Figs. 2 and 3 (a)] give quite similar predictions of the half-lives for the nuclei which could be synthesized in the above mentioned projectile-target combinations. However the borders of 1 μs area on the neutron-rich side differ substantially for these two models. This discrepancy appears due to the extrapolation of the model parameters to the unknown region, while the results for experimentally studied nuclei are quite similar.

The discovery of new elements mentioned above (even proton-rich isotopes) is certainly of interest. However, in our opinion, the most challenging region for fu-

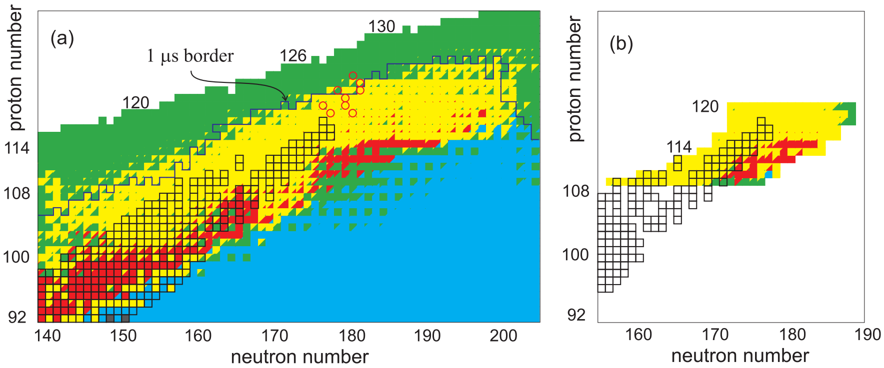

Fig. 3 The decay modes calculated using the ground-state masses from the two-center shell model [18, 19] (a) and those obtained in Ref. [17] (b). The SH half-lives for the panel (b) are taken from Ref. [26] (with the hindrance factor 100 for odd and odd-odd nuclei). Other notations are the same as in Fig. 2.

ture studies is the region of more heavy and more neutron-rich nuclei. This is especially the island of stability of superheavy nuclei centered at $Z \sim 114$ and $N \sim 184$ (remember, however, that the microscopic meson filed theory also predicts nuclei around $Z \sim 120$ and $N \sim 184$ as a candidates for a stability island). According to our predictions (made with the masses [16]) the most long-living nuclei in the $Z \sim 114$ and $N \sim 184$ area are the β-stable isotopes of Copernicium ^{291}Cn and ^{293}Cn with the half-lives of about 100 years shown in Fig. 2 by the white-border squares. The main decay mode of ^{291}Cn is predicted to be SF and ^{293}Cn is decaying by α-decay and SF with nearly equal probability. Because of their relatively long half-lives these isotopes – if synthesized – could be accumulated. Unfortunately these two isotopes are unreachable directly by any fusion reaction with stable ion beams. In principle, there is a chance to produce these nuclei in multi-nucleon transfer reactions [33] or by multiple neutron capture processes [34]. However the corresponding cross sections are very low. A new way for the synthesis of neutron-enriched superheavy nuclei and, in particular, those from the center of the stability island may be found basing on the found area of β^+-decaying nuclei in the vicinity of the island of stability.

We found (see Figs. 2 and 3) that some isotopes of superheavy elements with $111 \leq Z \leq 115$, more neutron-rich than those synthesized recently in Dubna in the ^{48}Ca-induced fusion reactions, also may undergo β^+-decay. Note, that such an area of β^+-decaying nuclei appears independently of the model used for the nuclear masses calculation. However, the size of this region is sensitive to the underlying shell model. The appearance of such an area of β^+-decaying nuclei in the vicinity of the island of stability becomes quite evident from the schematic Fig. 4. In this figure we consider the situation where the neutron closure $N = 184$ coincides with the region of β-stable nuclei (which is expected close by the proton number $Z = 114$). The left panel of Fig. 4 shows the typical behavior of the characteristic energies of EC, α-decay, and SF playing the role in this region (Q_{β^-} is negative here and not

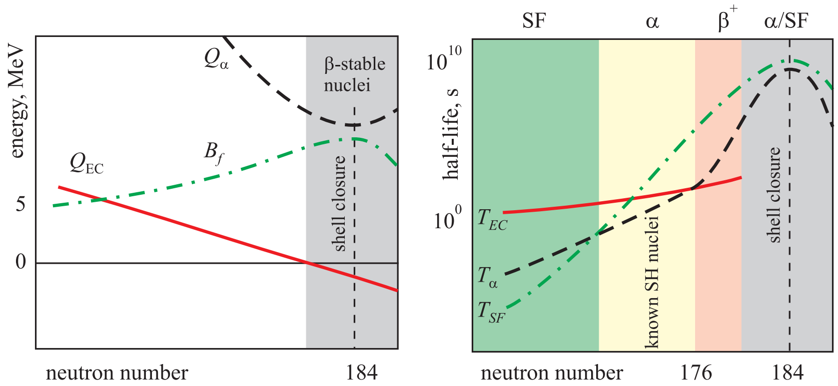

Fig. 4 Schematic picture explaining the existence of the region of β^+-decaying nuclei in the vicinity of the stability island. (left panel) Dependence on the neutron number of the characteristic energies of β^+-decay (Q_{EC}, solid curve), alpha-decay (Q_α, dashed curve), and spontaneous fission (B_f, dash-dotted curve). The region of β-stable nuclei and the position of the neutron shell closure ($N = 184$) are shown. (right panel) Expected behavior of the half-lives T_{EC} (solid curve), T_α (dashed curve), and T_{SF} (dash-dotted curve) from the proton-rich side up to the center of the stability island. The dominating modes of decay and the position of known SH nuclei in the vicinity of $Z = 114$ are shown.

shown). In this case one may expect the following order of decay modes starting from the proton drip line up to the top of the stability island (see the right panel in Fig. 4). Due to the strong Coulomb field, the most proton-rich nuclei should undergo SF with rather short half-lives. Moving to the “right” the fission barriers increase because of increase of the neutron number (and, therefore, decrease of the Coulomb forces) as well of the stabilizing effect of the neutron shell $N = 184$. Then α-decay starts to play a main role. Note, that most nuclei known at the moment close to $Z = 114$ (both synthesized in “cold” and “hot” fusion reactions) experience α-decay. Approaching the island of stability the half-lives of α-decay as well as those of SF increase by many orders of magnitude due to influence of the neutron shell $N = 184$. When these half-lives are longer than minutes and days (the typical half-lives with respect to EC of nuclei in the vicinity of the β-stability region), the EC process may dominate. Finally, the most stable nuclei (which should be β-stable) again undergo α-decay or/and SF. This consideration of the decay modes sequence is rather natural and model independent. It explains an appearance of the area of β^+-decay found here. However, the size of this area depends on the nuclear masses and nuclear structure. It should be stressed ones more, that our calculations of β-decay half-lives are based on the assumption of allowed β-transitions. As was said above, β-decay can be substantially suppressed, especially for nuclei close to the β-stability line (i.e. having small Q-values of β-decays). This means that some of the nuclei found here to have the β^+-decay as the main mode, may have much longer β-decay time, whereas the main decay mode could be α-decay or SF. However, the gross decay-mode structure of the nuclear map (i.e. existence of the region of β^+-decaying superheavy nuclei) should remain.

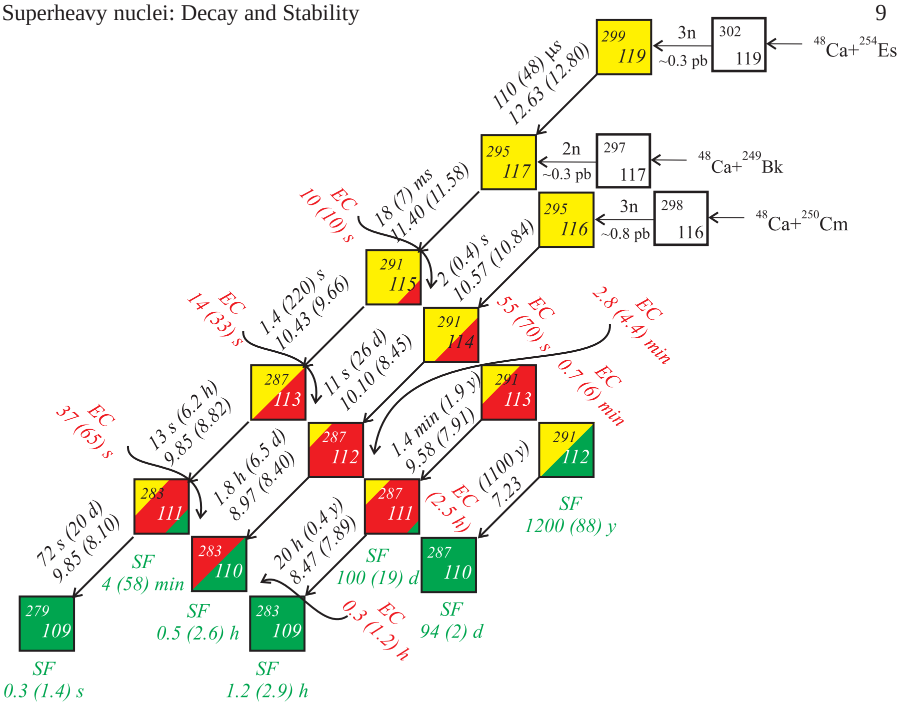

Fig. 5 The pathway to the middle of the island of stability via a possible β^+-decay of the isotopes ^{291}115 and ^{291}114. Decay half-lives and Q_α values (in MeV) calculated with nuclear masses [17] and [16] (in brackets) are shown.

Our finding indicates that the experimental identification of the nuclei to the "right" of already discovered ones may meet significant difficulties. However, the existence of the area of β^+-decay gives us the hypothetic way to reach the middle of the island of stability just in fusion processes of "stable" nuclei. In Fig. 5 several possible decay chains of the isotopes ^{291}115 and ^{291}114 are shown along with the corresponding values of Q_α and half-lives calculated with the use of nuclear masses predicted in Ref. [17] and in Ref. [16]. The SF half-lives are taken from Ref. [26] (with the hindrance factor 100 for odd and odd-odd nuclei), while the values in brackets are calculated by phenomenological relations (5). The isotope ^{291}115 may be formed after α-decay of ^{295}117 (the 2n evaporation channel of the ^{48}Ca + ^{249}Bk fusion reaction, cross section is 0.3 pb [9]) or after two α-decays of ^{299}119 (the 3n evaporation channel of the ^{48}Ca + ^{254}Es fusion reaction, cross section is 0.3 pb [35]). The second one, ^{291}114, is formed after α-decay of ^{295}116 in the 3n evaporation channel of the ^{48}Ca + ^{250}Cm fusion reaction with cross section of about 0.8 pb [35]. These isotopes should have rather long half-lives and, thus, they could be located already in the "red" area of the nuclear map; that is, they may be β^+-decaying nuclei. In accordance with our calculations of decay properties of SH nuclei [28], the isotopes ^{291}115 and ^{291}114 may experience not only α-decay but also EC. This prediction opens a narrow pathway to the middle of the island of stability of SH nuclei by sequence of β^+ decays ending at the ^{291}Cn nucleus. Note that, for the moment, the proposed method is the highest in cross section method for production of the nuclei located in the middle of the first island of stability. Hopefully it may be realized in future with the progress in experimental techniques.

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