STATISTICAL MODEL OF DECAY OF EXCITED NUCLEI

Level density

The level density of the atomic nucleus consisting of Z protons and N neutrons with the total angular momentum J and the excitation energy U has the following form:

$$\rho(U,Z,N,J)=K_{coll}(U,Z,N)\rho_0(U,Z,N,J),$$

where $\rho_0(U,Z,N,J)$ is the single-particle level density, $K_{coll}(U,Z,N)$ is the enhancement factor taking into account collective states of the nucleus.

The single-particle level density is given by [Ignatyuk83]:

$$\rho_0(U,Z,N,J)=\frac{(2J+1)\sqrt{a}}{24\left(U-\Delta-\frac{\hbar^2 J(J+1)}{2\Im_\perp}\right)^2}\left(\frac{\hbar^2}{\Im_\perp}\right)^{3/2}\exp\left(2\sqrt{a\left(U-\Delta-\frac{\hbar^2 J(J+1)}{2\Im_\perp}\right)}\right),$$

where a is the level density parameter, $\Im_\perp$ is the moment of inertia of the nucleus about the axis perpendicular to its axis of symmetry, Δ is the pairing energy:

$$\Delta=\chi\frac{12}{\sqrt{A}},$$

where A is the mass number of the nucleus, and χ is the parameter equal to 0, 1 or 2 for odd-odd, odd and even-even nuclei, respectively.

The level-density parameter is given by [Ignatyuk83]

$$a=\tilde{a}\left[1+\delta U\frac{1-\exp(-\gamma U)}{U}\right],$$

$$\tilde{a}=\alpha A+\beta A^{2/3}B_s,$$

where δU is the shell correction; α,β and γ are the coefficients with the default values 0.073 MeV^{-1}, 0.095 MeV^{-1} and 0.061 MeV^{-1}, respectively [Ignatyuk75]. The functional $B_s(\beta_2)$ is the dimensionless quantity equal to the ratio of the surface area of the deformed nucleus with the quadrupole deformation parameter β_2 to the area of the spherical nucleus with the equal volume. The quantity $B_s(\beta_2)$ is calculated for the deformation of the ground state (widths of particle emission) and for the saddle point (fission width). The values of the deformation of the nucleus at the saddle point are calculated according to [Cohen63]. The ground-state properties of nuclei (masses, shell corrections, deformations) are taken from [Moller16]. The ground-state masses for known nuclei are from [Wang12].

The moment of inertia of the nucleus is calculated by the formula:

$$\Im_\perp=k_\perp\Im_{r.b.}(1+\beta_2/3),$$

where the coefficient $k_\perp$ is by default equal to 0.5, $\Im_{r.b.}$ is the moment of inertia of a rigid body.

The deformation-dependent collective enhancement factor $K_{coll}\left(U,Z,N\right)$ is determined according to [Zagrebaev01]:

$$K_{coll}=K_{rot}\varphi\left(\beta_2\right)+K_{vib}\left(1-\varphi\left(\beta_2\right)\right), \qquad \varphi\left(\beta_2\right)=\left[1+\exp\left(\frac{\beta_2^0-\left|\beta_2\right|}{\Delta\beta_2}\right)\right]^{-1},$$

where K_{rot} is the rotational enhancement factor, K_{vib} is the vibrational enhancement factor. The default values of parameters are $\beta_2^0=0.15$, $\Delta\beta_2=0.04$ [Zagrebaev01]. The deformation dependence provides a smooth transition from the vibrational enhancement of the level density for spherical nuclei to the rotational enhancement for well-deformed ones.

By default, the enhancement factor K_{vib} is determined by the expression [Ignatyuk83]:

$$K_{vib}=\exp\left(0.0555A^{2/3}T^{4/3}\right),$$

where A is the mass number, $T=\sqrt{U/a}$ is the nuclear temperature.

Alternatively, K_{vib} can be determined by the expression [Junghans98]:

$$K_{vib}=25\frac{\Im_{\perp}T}{\hbar^2}\left(0.022+0.003\Delta N+0.005\Delta Z\right)^2,$$

where ΔN (ΔZ) are the absolute values of the number of neutrons (protons) above or below the nearest shell closure.

The enhancement factor K_{rot} is determined by the following formula:

$$K_{rot}=k\frac{\Im_{\perp}T}{\hbar^2},$$

where the parameter k has the default value 1.

The energy dependence of K_{rot} and K_{vib} is taken as:

$$K_{rot(vib)}\left(E\right)=\frac{K_{rot(vib)}-1}{1+\exp\left[\left(E-E_{cr}\right)/\Delta E_{cr}\right]}+1,$$

where E_{cr} and ΔE_{cr} are the parameters with the default values $E_{cr}=40$ MeV, $\Delta E_{cr}=10$ MeV [Junghans98].

Decay widths

It is assumed that the excited rotating nucleus may decay through the following channels: emission of light particles (neutrons, protons or alpha particles), emission of gamma rays and by fission into heavy fragments.

The expression for calculation of the particle emission widths has the form:

$$\Gamma_{C\to B+b}\left(E^*,J\right)=\frac{2s_b+1}{2\pi\rho_C\left(E^*,J\right)}\int_0^{E^*-B_b}\sum_l T_l\left(e_b\right)\cdot\sum_{I=\left|J-l\right|}^{I=J+l}\rho_B\left(E^*-B_b-e_b,I\right)de_b\ .$$

Here s_b is the spin of emitted particle b and B_b is its binding energy, where $b = n, p, \alpha$; $T_l(e)$ is the probability that the particle will pass through the potential barrier, e_b is the kinetic energy of the emitting particle.

For charged particles the following expression for $T_l(e)$ is used:

$$T_l(e) = \frac{P(e)}{\left[1 + \exp\left(\frac{2\pi}{\hbar\omega}\left(V_c + \frac{\hbar^2 l(l+1)}{2\mu R^2} - e\right)\right)\right]},$$

where $P(e) = \frac{4\sqrt{\frac{e}{e+40}}}{\left(1+\sqrt{\frac{e}{e+40}}\right)^2}$, $R = r_0 A^{\frac{1}{3}} + 2$ fm and μ is the reduced mass of the particle.

The height of the Coulomb barrier and its width are given by

$V_c = 0.106Z + 0.9 + 0.02\left(Z + N^* - A\right)$ MeV, $\hbar\omega = 6.2 + 0.04\left(Z - 20\right)$ MeV for protons,

$V_\alpha = 0.195(Z-2) + 4.5 + 0.02(Z + N^* - A)$ MeV, $\hbar\omega = 6.03 + 0.016\left(Z - 100\right)$ MeV for α-particles.

Here $N^* = 0.2638 + 1.0676Z + 0.0055Z^2$ is the number of neutrons for the stable nucleus with the atomic number *Z*.

For neutrons the probability is calculated using the formula:

$$T_l(e) = P(e)\exp\left(-2\sqrt{\frac{2\mu e}{\hbar^2}}\left(\rho^2 - R^2 + \rho\log\left(\frac{\rho + \sqrt{\rho^2 - R^2}}{R}\right)\right)\right), \qquad \rho = \sqrt{\frac{\hbar^2 l(l+1)}{2\mu e}}.$$

The gamma-ray emission width is

$$\Gamma_\gamma^L\left(E^*, J\right) = \frac{1+\kappa}{\rho_C\left(E^*, J\right)}\int_0^{E^*} f_L\left(e_\gamma\right)\sum_{I=|J-L|}^{I=J+L} e_\gamma^{2L+1}\rho_C\left(E^* - e_\gamma, I\right)de_\gamma,$$

where $\kappa = 0.75$ [Nedoresov85] the force function for dipole radiation ($L = 1$) has the form:

$$f_1\left(e_\gamma\right) = 3.31\cdot 10^{-6}\left(MeV^{-1}\right)\frac{Z(A-Z)}{A}\frac{e_\gamma\Gamma_0}{\left(E_0^2 - e_\gamma^2\right)^2 + \left(e_\gamma\Gamma_0\right)^2},$$

where E_0 and Γ_0 are, respectively, the energy and the width of the giant dipole resonance, calculated as

$$E_0 = \frac{167.23}{\sqrt{1.959A^{2/3} + 14.074A^{1/3}}}\text{ MeV}, \quad \Gamma_0 = 5\text{ MeV [Schmidt91]}.$$

The fission width is calculated from the expression:

$$\Gamma_{fiss}\left(E^*, J\right) = \frac{K_{Kramers}}{2\pi\rho_C\left(E^*, J\right)}\int_0^{E^*} T_{fiss}\left(e, J\right)\rho_C^{s.p.}\left(E^* - e, J\right)de, \tag{1}$$

where $T_{fiss}\left(e,J\right)=\dfrac{1}{1+\exp\left[-\dfrac{2\pi}{\hbar\omega_{s.p.}}\left(e-B_{fiss}\left(E^*\right)\right)\right]}$ is the fission barrier penetrability, $K_{Kramers}=\dfrac{\hbar\omega_{g.s.}}{T\omega_{s.p.}}\left(\sqrt{\omega_{s.p.}^2+\eta^2/4}-\eta/2\right)$ is the Kramers factor [Kramers40], η is the viscosity parameter, $\omega_{g.s.}$ and $\omega_{s.p.}$ are the curvatures of the potential at the ground state and saddle point, respectively. The fission barrier is calculated by the formula:

$$B_{fiss}\left(U\right)=B_{LDM}-\delta U,$$

where B_{LDM} is the liquid-drop fission barrier [Sierk86], δU is the shell correction to the ground state [Moller16]. The subscript $s.p.$ indicates the calculation of the corresponding quantity at the saddle point.

Survival probability

There are two methods applied to calculate the survival probability. The Monte Carlo method allows one to obtain survival probability for all possible reaction channels, while the direct integration gives the survival probability for *xn* channels only (up to 6*n*). The Monte Carlo method has additional possibilities of calculating, for example, the fission fragment properties (the mass-charge yields, etc.). These possibilities are absent in the case of the direct integration method. While the Monte Carlo approach is more general, it has a limitation associated with computational time required to accumulate the necessary statistics. For some rare processes (e.g., surviving in *xn* channels in the superheavy mass region) the required large statistics is nearly unreachable, and the direct integration is the only method working for such the cases.

A. The Monte Carlo method

At each stage of the decay of the nucleus one of the possible events is randomly selected – the emission of neutrons, protons, alpha particles, gamma-rays or fission. The probabilities of each event are

$$P_b=\frac{\Gamma_b}{\Gamma_{tot}},\quad b=n,p,\alpha,\gamma,\text{fission},$$
$$\Gamma_{tot}=\Gamma_n+\Gamma_p+\Gamma_\alpha+\Gamma_\gamma+\Gamma_{fiss}.$$

The energy of the emitted particles is simulated determined according to the following spectrum:

$$W_b(e)=C_b\cdot\sqrt{e}\cdot\exp\left[-e/T\right],\quad b=n,p,\alpha,$$
$$W_\gamma(e)=C_\gamma\cdot f_L(e)e^{2L+1}\cdot\exp\left[-e/T\right],\quad L=1,$$

where C_i are the normalization constants.

During evaporation cascade the nucleus either undergoes fission at some stage or loses its excitation energy below the fission barrier and neutron binding energy by the emission of particles and cools forming the evaporation residue.

B. Direct integration

Subsequent estimation of the total probability for the formation of a cold residual nucleus after the emission of *x* neutrons $C\rightarrow B+xn+N\gamma$ is usually performed within numerical calculations based on the analysis of the multistep decay cascade. The expression for such the probability accounting for the Maxwell-Boltzmann energy distribution of evaporated neutrons reads

$$P_{xn} = \int_0^{E_0^*-B_n(1)} \frac{\Gamma_n}{\Gamma_{tot}}(E_0^*,I_0)\cdot W_n(E_0^*,e_1)de_1 \int_0^{E_1^*-B_n(2)} \frac{\Gamma_n}{\Gamma_{tot}}(E_1^*,I_1)\cdot W_n(E_1^*,e_2)de_2 \ldots$$
$$\int_0^{E_{x-1}^*-B_n(x)} \frac{\Gamma_n}{\Gamma_{tot}}(E_{x-1}^*,I_{x-1})\cdot W_n(E_{x-1}^*,e_x)\cdot \prod_{i=1}^{N} \frac{\Gamma_\gamma}{\Gamma_{tot}}(E_i^*,I_i)\cdot de_x.$$

Here $B_n(x)$ and e_i are the binding and kinetic energies of the *i*th evaporated neutron, E_i^* is the excitation energy of the residual nucleus after the emission of i neutrons. $W_n(E^*,e) = C_n\sqrt{e}\exp\left[-e/T(E^*)\right]$ is the probability for the evaporated neutron to have energy e, and the normalization coefficient C_n is determined from the condition $\int_0^{E^*-B_n} W_n(E^*,e)de = 1$.

Fusion-fission and fusion-survival cross sections

The cross section of the formation of evaporation residue in $xn,yp,z\alpha$ channel in collisions of heavy nuclei can be calculated as follows

$$\sigma_{EvR}^{xn,yp,z\alpha} = \frac{\pi\hbar^2}{2\mu E}\sum_{l=0}^{\infty}(2l+1)T_l(E)\cdot P_{CN}(E,l)\cdot P_{xn,yp,z\alpha}(E,l),$$

than the total survival cross section is

$$\sigma_{surv} = \frac{\pi\hbar^2}{2\mu E}\sum_{l=0}^{\infty}(2l+1)T_l(E)\cdot P_{CN}(E,l)\cdot \sum_{x,y,z} P_{xn,yp,z\alpha}(E,l),$$

and, finally, the fusion-fission cross section is defined as

$$\sigma_{fiss} = \sigma_{fus} - \sigma_{surv},$$

where the fusion cross section is

$$\sigma_{fus} = \frac{\pi\hbar^2}{2\mu E}\sum_{l=0}^{\infty}(2l+1)T_l(E)\cdot P_{CN}(E,l).$$

Here P_{CN} is the probability for the compound nucleus (*CN*) formation by two nuclei coming in contact. For specific nuclear combinations leading to so called “cold” synthesis a simple parameterization of $P_{CN}(E,l)$ was proposed in [Zagrebaev08]

$$P_{CN}(E,l) = \frac{P_{CN}^0(Z_1,Z_2)}{1+\exp\left(\frac{E_B^* - E_{\text{int}}^*(l)}{\Delta}\right)}.$$

Here E_B^* is the excitation energy of *CN* at the Bass barrier and $E_{\text{int}}^*(l) = E + Q - E_{rot}(l)$, where Q is the fusion Q value, $E_{rot}(l) = \frac{\hbar^2}{2\Im_{g.s.}} l(l+1)$ is the rotational energy and

$$P_{CN}^0 = \frac{1}{1 + \exp\left(\frac{Z_1 Z_2 - \varsigma}{\tau}\right)},$$

where $\varsigma \approx 1760$ and $\tau \approx 45$.

The models allows one to take into account the dynamical delay of the fission process in the Monte-Carlo simulation of the decay of excited nucleus that is extremely important for analysis of the pre-scission and post-scission particle multiplicities. We use a simplified approximation of realistic dynamical calculations of the fission width (see Fig.1). The fission width is parametrized by a step function with only one parameter – delay time τ_d. When $t < \tau_d$ only the evaporation process is considered and the fission probability is neglected, while for $t > \tau_d$, all the decay channels are treated as in the standard statistical model (see above). Here t is the time of the decay process (passed since the CN formation) which is estimated as $\hbar/\Gamma_{tot}$.

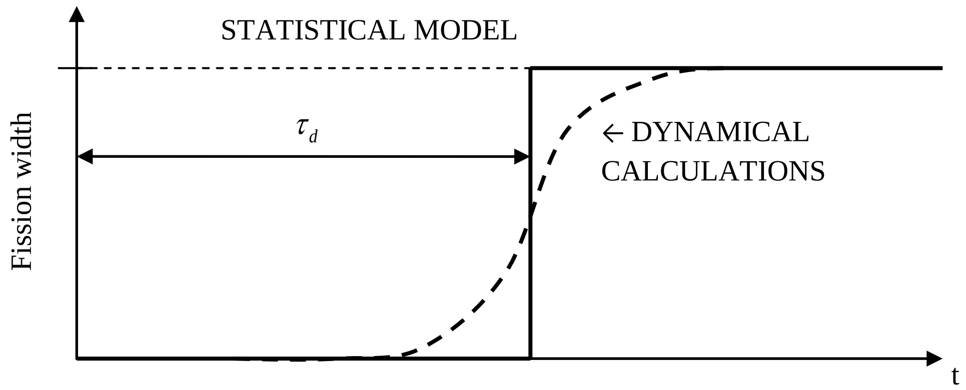

Fig. 1. Schematic time dependence of fission width

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