# Few-Nucleon Transfers and Weakly Dissipative Processes in the GRAZING Model of Low-Energy Nuclear Reactions

V. V. Samarin

Joint Institute for Nuclear Research, Dubna, Moscow oblast, 141980 Russia e-mail: samarin@jinr.ru

**Abstract**—The applicability of Winther's semi-classical model in the GRAZING program to a representative set of low-energy nuclear reactions is examined. It is demonstrated that the model describes the processes of few-nucleon transfers with low dissipation of energy for spherical nuclei <sup>40</sup>Ca, <sup>96</sup>Zr, <sup>208</sup>Pb, and others. Satisfactory agreement with experimental data is observed for transfers of no more than one proton and approximately 6–8 neutrons. The reactions in which the agreement with experimental data is poor at a standard set of parameters are indicated.

**DOI:** 10.3103/S1062873813070198

### **INTRODUCTION**

Grazing inelastic collisions of atomic nuclei are accompanied by the transfer of nucleons and deformation of nuclei surfaces, i.e., the excitation of collective degree of freedom. At the same time, the relative motion of heavy nuclei can be considered on the basis of classical mechanics using terms of classic trajectories and scattering angles, depending on impact collision parameters. Internal degrees of freedomsingle-particle states, low-lying oscillations of surface nuclei (quadrupole, octupole, etc.) and highlying oscillations (gigantic resonances)—must be described using quantum theory. A typical parameter here would be the minimum distance  $s_{\min}$  between nucleus surfaces (even in an approximation of their unchanging spherical shape). Deeply inelastic reactions and substantial transfers of nucleons correspond to low values of  $s_{\min}$  and grazing and the intersection of nuclei surfaces. The low probabilities of nucleon transfer and collective excitation at quite large  $s_{\min}$  allow the use of approximated approaches (perturbation theory, linear parts of expansions into series, empirical models, etc.). The semi-classical model of few-nucleon transfers with low energy dissipation developed by A. Winther [1, 2] and used in the GRAZING program [3] corresponds to conditions of high and intermediate values of distance  $s_{\min}$ . Universal computation formulas of the model are based on the following physical assumptions: single-particle transfers to inelastic scattering with the generation of elementary excitations are independent; collective elementary excitations of nuclei are phonons whose perturbations are linear in terms of deformation parameters; the probabilities of independent single-particle transfers are less than 1 (which is typical for single-par-

ticle transfers); and nuclei in the ground states (before collision) are counted as spherical. The model uses tabular data on the bond energy of nuclei (mass defects), energies of quadrupole and octupole excitations, and probabilities of electromagnetic transfers from states with excitation of a single oscillating quantum (B(E2)), B(E3)). Other parameters (characteristics of gigantic resonances, densities of neutron and proton states, form factors of nucleon transfers, etc.) are determined from universal empirical or model formulas. For each classic trajectory (weakly depending on dissipation and fluctuation energy), the model includes the approximated computation of nucleon transfer probabilities, excitations of low-lying and high-lying oscillations in the neighborhood of the point of the minimum closing distance of nuclei  $r_0$ , and a procedure to average the probabilities of these processes by the impact parameters of nuclei collisions b (or moments of relative motion *l*). The practical independence of neutron and proton transfers with respect to each other and to inelastic processes in the given model provides a convenient opportunity to compare computation results on mass and charge distributions with corresponding experimental data. Detailed analysis of inelastic processes is more complicated because of the superimposition of dissipation energy conditioned by different collective excitations and nucleon transfers. Computations of angular distribution are sensitive to dissipation and energy fluctuations, along with possible deviations of nucleus shape from spherical. The GRAZING program is widely used for analyzing experimental data [4]. The launching of the program on Internet server NRV [5] provides a convenient opportunity for computing mass and charge distributions and presenting the results in vivid graphic form.

This work shows that computations performed in GRAZING program describe processes of fewnucleon transfers with low energy dissipation for spherical nuclei <sup>40</sup>Ca, <sup>90</sup>Zr, <sup>208</sup>Pb and others. Satisfactory agreement with the experimental data is observed for transfers of no more than one proton and approximately 6-8 neutrons. The reactions in which the agreement with experimental data is poor at a standard set of parameters are indicated.

# THEORETICAL

The assumption of independent transfer for collisions with defined impact parameters (defined moments of the relative motion of nuclei) leads to a Bessel distribution according to the transfer numbers of neutrons  $\Delta N$  and protons  $\Delta Z$ 

$$P(\Delta N, \Delta Z) = P(\Delta N)P(\Delta Z), \tag{1}$$

where

$$P(\Delta N) = \left(\frac{q^{\nu P}}{q^{\nu S}}\right)^{\frac{\Delta N}{2}}$$

$$\times I_{|\Delta N|} \left(2\sqrt{q^{\nu P}q^{\nu S}}\right) \exp\left(-q^{\nu P} - q^{\nu S}\right),$$
(2)

$$P(\Delta Z) = \left(\frac{q^{\pi P}}{q^{\pi S}}\right)^{\frac{\Delta Z}{2}} I_{|\Delta Z|} \left(2\sqrt{q^{\pi P}q^{\pi S}}\right) \exp\left(-q^{\pi P} - q^{\pi S}\right), (3)$$

 $q^{\nu P}, q^{\pi P}$  are average numbers of picked-up neutrons (v) and protons ( $\pi$ ),  $q^{vS}$ ,  $q^{\pi S}$  are average numbers of stripped neutrons and protons. With the standard set of parameters in the GRAZING program, the average number of pickup and/or stripped nucleons turn out to be of the order of unity. The distribution diagram for such a case is shown in Fig. 1. Reducing the probability by three orders of magnitude corresponds to a sixfold increase in the average numbers of nucleon transfers. The resulting distribution is obtained by averaging these probabilities over impact parameters (moments) of collision while taking inelastic processes of colliding nuclei into account. The value of parameters  $q^{\vee P}, q^{\pi P}$ .  $q^{vS}, q^{\pi S}$  in the semi-classical Winther model depend on the properties of single-particle states in colliding nuclei, mainly on the energy (Q value) released (Q > 0) or absorbed (Q < 0) during the pickup or stripping of a nucleon, and on densities of nucleon levels g near Fermi level

$$g^{\nu} = \frac{3N}{2\epsilon^{\nu}} \frac{15}{\delta^{\nu}}, \quad g^{\pi} = \frac{3Z}{2\epsilon^{\pi}} \frac{15}{\delta^{\pi}}, \tag{4}$$



**Fig. 1.** Diagrams of probabilistic distribution (2) for average numbers of pickup and stripped neutrons  $q^{\nu P} = 1$ ,  $q^{\nu S} = 1$  (dots) and  $q^{\nu P} = 2$ ,  $q^{\nu S} = 2$  (circles).

where  $\delta^{\nu}$ ,  $\delta^{\pi}$  are variable parameters with standard value  $\delta^{\nu} = \delta^{\pi} = 8$  and

$$\varepsilon^{\nu} = 52 - 22 \frac{N - Z}{A} - 8,$$
(5)
$$\varepsilon^{\pi} = 52 + 22 \frac{N - Z}{A} - \frac{Ze^2}{1.2A^{1/3}} - 8.$$

Parameters  $\delta^{\nu}$ ,  $\delta^{\pi}$  can be associated with so called Fermi-gas parameter of level density [6]

$$a = \frac{\pi^2 g(\varepsilon_F)}{6} = \frac{\pi^2}{6} \left( g^{\nu} + g^{\pi} \right).$$
 (6)

Figure 2a shows values of the Fermi gas parameter of level density *a* obtained from analyzing the experimental data according to the average distance between neutron resonances and the dependences of parameter *a* determined from formula (6) for four values of parameter  $\delta^{v} = \delta^{\pi} = 5, 8, 10, 20$ . Figure 2b shows the dependency of reaction <sup>40</sup>Ca + <sup>96</sup>Zr of average numbers of pickup and stripped neutrons (v) and protons ( $\pi$ ), and the cross sections for parameters  $\delta^{v}$ ,  $\delta^{\pi}$  determining the densities of nucleons near the Fermi levels in colliding nuclei. Complete cross sections of the formation of nuclei with different mass numbers from the initial nucleus of <sup>40</sup>Ca upon colliding with a nucleus of <sup>96</sup>Zr  $E_{lab} = 152$  MeV are shown in Fig. 3a.

Intervals of the energy loss for a projectile nucleus in the GRAZING program are restricted by several low-lying oscillation quanta (quadrupole and octupole) and the energy of gigantic dipole resonance,  $\sim$ 20 MeV.



**Fig. 2.** (a) Values of the Fermi gas parameter of level density *a* obtained in [6] by analyzing the experimental data on average distance between neutron resonances (( $\bigcirc$ ) even–even nuclei; ( $\bigcirc$ ) even–odd nuclei; ( $\triangle$ ) odd–even nuclei; ( $\bigcirc$ ) odd–odd nuclei) and dependences of parameter *a* determined using formula (6) for four values of parameter  $\delta = \delta^{v} = \delta^{\pi} = 5$ , 8, 10, and 20 (dashed, solid, dashed-and-dotted, and dotted curves, respectively); (b) dependency for reaction  ${}^{40}\text{Ca} + {}^{96}\text{Zr}$  of average numbers of pickup and stripped protons (solid and dashed curves, respectively) on parameters  $\delta^{v}$ ,  $\delta^{\pi}$ ; the vertical dashed line indicates the standard value  $\delta^{v} = \delta^{\pi} = 8$  for the GRAZING program.

#### **RESULTS AND DISCUSSION**

Experimental data on few-nucleon transfer in reactions  ${}^{40}Ca + {}^{96}Zr$ ,  ${}^{90}Zr + {}^{208}Pb$ ,  ${}^{40}Ca + {}^{208}Pb$  from [4, 7–9] were compared to the computational results from the GRAZING program. Figures 3a, b show good agreement between the computational results and experimental data that almost does not violate the accompanying transfer of single proton; Fig. 3c shows that computed cross-sections with two protons underestimates the experimental data. This can be seen in detail from angular distributions (differential cross-sections) of transfer reaction  ${}^{40}Ca + {}^{208}Pb$  shown in Fig. 4. A similar situation is observed for few-nucleon



**Fig. 3.** Complete cross-sections for pure neutron pickup (circles) and pickup with the stripping of one (triangles) and two (squares) protons in reactions (a)  ${}^{40}\text{Ca} + {}^{96}\text{Zr}$  with  $E_{\text{lab}} = 152 \text{ MeV}$ , (b)  ${}^{90}\text{Zr} + {}^{208}\text{Pb}$  with  $E_{\text{lab}} = 560 \text{ MeV}$ , and (c)  ${}^{40}\text{Ca} + {}^{208}\text{Pb}$  with  $E_{\text{lab}} = 249 \text{ MeV}$ . Black symbols represent experimental data from (a, b) [4] and (c) [12, 13, 14]; white symbols, computations performed using the GRAZING program with (a)  $\delta^{\nu} = \delta^{\pi} = 6$ , (b)  $\delta^{\nu} = \delta^{\pi} = 8.5$ , and (c)  $\delta^{\nu} = \delta^{\pi} = 8$ .

transfers in reactions  ${}^{40}Ca + {}^{124}Sn [10-12], {}^{32}S + {}^{208}Pb [13], {}^{48}Ti + {}^{208}Pb [14], {}^{58}Ni + {}^{208}Pb [15].$ 

In reaction  ${}^{40}Ca + {}^{96}Zr$  (Fig. 3a), the transfer of up to six neutrons to the  ${}^{40}Ca$  nucleus and up to two protons to the  ${}^{96}Zr$  nucleus indicate that the average number of pickup nuclei at closest tangent collision are on the order of unity (Fig. 1); the average number of stripped protons at the same time is reduced by about two-thirds. Computations using the GRAZING program satisfactorily reproduce such values. At the stan-



**Fig. 4.** Computed angular distributions in a center of mass system for pure neutron pickup (solid curve) and neutron pickup with the stripping of two protons (dashed line) at  $\delta^{\nu} = \delta^{\pi} = 8$  in reaction  ${}^{40}\text{Ca} + {}^{208}\text{Pb}$  with  $E_{\text{lab}} = 249$  MeV. Dots represent the experimental data [13].

dard set of parameters, however, the computational results obtained from the GRAZING program differ slightly from the experimental data (at large numbers of transferred neutrons up to one order of magnitude). Consideration of evaporation and some variation in parameters  $\delta^{v}$ ,  $\delta^{\pi}$  near standard value  $\delta^{v} = \delta^{\pi} = 8$ enhances the agreement between theory and experiment. According to Fig. 2a, for light nuclei more precise consideration of nucleon levels near Fermi level requires that we lower the values of parameters  $\delta^{v}$ ,  $\delta^{\pi}$ (particularly for <sup>40</sup>Ca + <sup>96</sup>Zr, down to 6). For heavy nuclei near lead, we must slightly raise the values of  $\delta^{v}$ ,  $\delta^{\pi}$  (particularly for <sup>90</sup>Zr + <sup>208</sup>Pb, up to 8.5).

Multi-nucleon transfers and deeply inelastic reactions are not described by this model, as can be seen from reaction <sup>56</sup>Fe + <sup>165</sup>Ho [16] (Fig. 5). Experimental data on multiproton transfers and computational results from the GRAZING program are on the same order of magnitude within a narrow interval that corresponds to the stripping and pickup of a single proton (Fig. 5a). The difference outside this interval is several orders of magnitude. The GRAZING program also allows us to describe the energy distributions in a narrow region (~20 MeV wide) of the initial energy (Fig. 5b). A similar situation is observed place for multi-nucleon transfers and large energy transfers in reactions <sup>86</sup>Kr + <sup>166</sup>Er [17–19], <sup>84</sup>Kr + <sup>209</sup>Bi [20], <sup>136</sup>Xe + <sup>209</sup>Bi [21–24].

The semi-classical Winther model and its computer form (the GRAZING program) thus describe neutron transfers of one to several (6–8) neutrons for nuclei close to magic nuclei <sup>40</sup>Ca, <sup>90</sup>Zr, <sup>208</sup>Pb, which



**Fig. 5.** Charge (a) and energy (b) distributions of products of reaction <sup>56</sup>Fe + <sup>165</sup>Ho with  $E_{lab} = 462$  MeV. Dots represent experimental data [16]; computation results with parameters  $\delta^{\nu} = \delta^{\pi} = 8$  are shown by circles and solid lines; those with  $\delta^{\nu} = \delta^{\pi} = 3$ , by triangles and the dashed line.

are characterized by relatively low density of their neutron levels near the Fermi level. The transfers of large number of protons in the considered model are underestimated. The slight energy losses within 20 MeV in the model are described satisfactorily.

## CONCLUSIONS

The performed computations and comparisons with experimental data provide a better understanding of the possibilities and applicability ranges of the GRAZING program. This is also useful for analyzing experimental data, for enhancing theoretical models of grazing nucleus-nucleus collisions, and for planning new experiments to obtain, e.g., new atomic nuclei near the boundaries of proton and neutron stability.

## ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, grant no. 11-07-00583-a. The author thanks Prof. V.I. Zagrebaev for his helpful comments and advice, along with A.V. Karpov, A.S. Denikin, Yu.A. Muzychka, and M.A. Naumenko.

## REFERENCES

- 1. Winther, A., Nucl. Phys. A, 1995, vol. 594, p. 203.
- 2. Winther, A., Nucl. Phys. A, 1994, vol. 572, p. 191.
- 3. http://personalpages.to.infn.it/~nanni/grazing/
- 4. Szilner, S., et al., Phys. Rev. C, 2007, vol. 76, p. 024604.
- Data Base on Low-Energy Nuclear Reactions Nuclear Reaction Video. http://nrv.jinr.ru/nrv/
- Ignatyuk, A.V., Statisticheskie svoistva vozbuzhdennykh atomnykh yader (Statistical Properties of Excited Atomic Nuclei), Moscow: Energoatomizdat, 1983.
- 7. Szilner, S., et al., Phys. Rev. C, 2005, vol. 71, p. 044610.
- Pollarolo, G., *AIP Conf. Proc.*, 2006, vol. 853, no. 29, p. 29.
- 9. Corradi, L., Pollarolo, G., and Szilner, S., J. Phys. G: Nucl. Part. Phys., 2009, vol. 36, p. 113101.

- 10. Corradi, L., et al., Phys. Rev. C, 1996, vol. 54, p. 201.
- 11. Corradi, L., et al., *Phys. Rev. C*, 2000, vol. 61, p. 024609.
- 12. Corradi, L., Nucl. Phys. A, 2001, vol. 685, p. 37.
- 13. Corradi, L., et al., Phys. Rev. C, 1994, vol. 49, p. 2875.
- 14. Rehm, K.E., et al., Phys. Rev. C, 1988, vol. 37, p. 2629.
- 15. Corradi, L., Pollarolo, G., and Winther, A., *Phys. Rev. C*, 2002, vol. 66, p. 024606.
- 16. Hoover, A.D., et al., Phys. Rev. C, 1982, vol. 25, p. 256.
- 17. Eyal, Y., et al., Phys. Rev. C, 1980, vol. 21, p. 2509.
- 18. Tserruya, I., et al., *Phys. Rev. Lett. C*, 1981, vol. 47, p. 16.
- 19. Tserruya, I., et al., Phys. Rev. C, 1982, vol. 26, p. 2509.
- 20. Birkelund, J.R., et al., *Phys. Rev. C*, 1982, vol. 26, p. 1984.
- 21. Schroder, W.U., et al., Phys. Rep., 1978, vol. 45, p. 301.
- 22. Wilcke, W.W., et al., Phys. Rev. C, 1980, vol. 22, p. 128.
- 23. Wollersheim, H.J., et al., *Phys. Rev. C*, 1981, vol. 24, p. 2114.
- 24. Bondorf, J.P., et al., Phys. Rev. C, 1993, vol. 48, p. 459.

Translated by K. Gumerov