Nucleon Transfer within Distorted Wave Born Approximation

N$_R$V Project‡

Flerov Laboratory of Nuclear Reactions, 141980, Dubna, Russian Federation

Abstract. The finite range Distorted Wave Born Approximation (DWBA) is described in the application to nucleon transfer reactions at low collision energies. The DWUCK5 computer code [1] is adapted in the N$_R$V . The detailed description of the DWBA can be found in the books by G. R. Satchler [2] and P. Fröbrich [3].

‡ The N$_R$V team: V.I. Zagrebaev, A.S Denikin (denikin@jinr.ru), A.V. Karpov (karpov@jinr.ru), A.P. Alekseev, V.A. Rachkov, M.A. Naumenko, V.V. Samarin (samarin@jinr.ru)

1. DWBA transfer amplitude and cross section

The transfer reaction section of the NRV is based on the computer code DWUCK5 [1]. It calculates the scattering observables for binary nuclear reactions using the finite range distorted wave Born approximation (DWBA). We consider the $A(a,b)B$ reaction where the projectile a is treated as the composite system $a = b + x$ (see Fig. 1). The particle x could be either a nucleon or a few-nucleon cluster, for example the α-particle. The transfer of the particle x to the target leads to the formation of the composite target-like fragment in the exit channel $B = A + x$. We also assume that all particles may have non-zero spin denoted below as J_A, J_B, j_a, j_b and j_x.
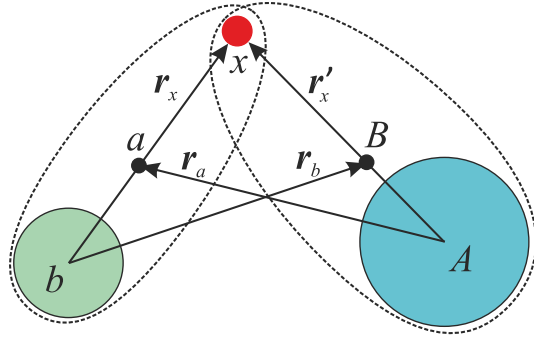
Figure 1. The definition of coordinates for the nucleon transfer reaction $A(a,b)B$.

The computer code DWUCK5 calculates the transition amplitude for the reaction $A(a,b)B$ within the post (β) or the prior (α) form

$$T_{fi}^{\alpha(\beta)} = \mathcal{J} \int \int d^3\mathbf{r}_b \, d^3\mathbf{r}_a \chi_{\mathbf{k}_f}^{(-)*}(\mathbf{r}_b) \, \langle bB| \, \Delta V_{\alpha(\beta)} \, |aA\rangle \, \chi_{\mathbf{k}_i}^{(+)}(\mathbf{r}_a), \tag{1}$$

where $\chi^{(-)}$ and $\chi^{(+)}$ are the distorted waves, $\mathbf{r}_a$ and $\mathbf{r}_b$ are the relative coordinates for the systems (a,A) and (b,B) (see Fig. 1), respectively, and $\mathcal{J}$ is the Jacobian for the transformation to these coordinates

$$\mathcal{J} = \left(\frac{m_a m_B}{m_a m_B - m_b m_A} \right)^3.$$

The quantity $\langle bB| \, \Delta V \, |aA\rangle$ discussed below is the form factor for the transfer reaction that couples the bound states in the entrance and exit channels.

The asymptotics of the distorted waves $\chi_{\mathbf{k}}^{(\pm)}(\mathbf{r})$ is the combination of the plane wave with the momentum $\mathbf{k}$ and the outgoing (or incoming) spherical scattered wave. Without the Coulomb potential

$$\chi_{\mathbf{k}}^{(\pm)}(\mathbf{r} \to \infty) \to e^{i\mathbf{k}\cdot\mathbf{r}} + f(\theta) \frac{e^{\pm ikr}}{r}. \tag{2}$$

The final distorted wave which has an incoming scattered wave condition is related to the solution with the outgoing waves by

$$\chi^{(-)*}(\mathbf{k}, \mathbf{r}) = \chi^{(+)}(-\mathbf{k}, \mathbf{r}). \tag{3}$$

It is convenient to decompose the distorted wave functions into the partial waves. As shown in detail in [2, 3] the radial partial wave functions satisfy the equation

$$\left(\frac{d^2}{dr^2} + k^2 - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2}U_{\alpha(\beta)}(r)\right)\chi_{Jls}(r) = 0. \tag{4}$$

Here $U_{\alpha(\beta)}(r) = U(r) + U_C(r) + (\mathbf{l}\cdot\mathbf{s})U_{ls}(r)$ is the optical potential describing the relative motion of the projectile and the target in the entrance (α) or exit (β) channels. U is the central potential with real and imaginary parts, U_C is the Coulomb potential for the uniform charge distribution with the radius R_C and U_{ls} is the spin-orbit potential. We take into account the total spin s coupled with the orbital momentum l into the total angular momentum J. The radial functions $\chi_{Jls}(k, r)$ satisfy the boundary conditions $\chi_{Jls}(k, 0) = 0$ at the origin and

$$\chi_{Jls}(k, r) \rightarrow \frac{i}{2}e^{i\sigma_l}\left[H_l^-(kr) - S_{Jl}H_l^+(kr)\right] \tag{5}$$

for $r \rightarrow \infty$ where the nuclear part of the potential may be neglected. The functions $H_l^{\pm}(kr) = G_l(kr) \pm iF_l(kr)$ are the outgoing ($+$) and the incoming ($-$) Coulomb waves, S_{Jl} is the elastic scattering S matrix element and σ_l is the Coulomb phase shift.

The form factor $\langle bB|\Delta V_{\alpha(\beta)}|aA\rangle$ contains information about the nuclear structure. The potential $\Delta V_{\alpha(\beta)}$ usually reads

$$\Delta V_{\nu} = \sum_i V_{i,\nu}(r_i) - U_{\nu}(r),$$

where $V_{i,\nu}$ are the interaction potentials between the target (A) and the projectile constituents ($i = b$ or x) in case of the prior form ($\nu = \alpha$), and between the projectile-like fragment (b) and the components of the target-like fragments ($i = A$ or x) in case of the post form ($\nu = \beta$). The prior-form transition potential is expressed by

$$\Delta V_{\alpha} = V_{xA} + V_{bA} - V_{aB} \approx V_{xA}, \tag{6}$$

where $U_{\alpha} = V_{aB}$. It is valid when the core b and the target A are heavier than the transferred particle x and the term ($V_{bA} - V_{aB}$) may be omitted.

Within the post-form DWBA the transition potential is treated as

$$\Delta V_{\beta} = V_{bx} + V_{bA} - V_{bB} \approx V_{bx}, \tag{7}$$

where $U_{\beta} = V_{bB}$. The approximation in (7) is valid when the target A and the target-like fragment B are similar, thus the contribution of the term ($V_{bA} - V_{bB}$) is negligible.

Eqs. (6) and (7) show that the coupling potentials are the binding potentials in the entrance and the exit channels, respectively. These potentials are used in order to obtain the bound state wave functions for the projectile ($a = b + x$) in the entrance channel and for the target-like fragment ($B = A + x$) in the exit channel

$$\langle\,\mathbf{r}_x\,|aA\rangle = \sqrt{\mathcal{S}_a}\,\varphi_a(\mathbf{r}_x), \,\langle\,\mathbf{r}_x'\,|bB\rangle = \sqrt{\mathcal{S}_B}\,\varphi_B(\mathbf{r}_x'), \tag{8}$$

where $\mathcal{S}$ is the spectroscopic coefficient carrying information about the structure of the composite system, and coordinates $\mathbf{r}_x$ and $\mathbf{r}_x'$ are defined as in Fig. 1.

Decomposing the distorted wave functions into the partial waves and taking into account the spin coupling of the particles the cross section for the reaction $A(a,b)B$ may now be expressed in terms of the transition amplitude

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} \frac{k_\beta}{k_\alpha} \frac{1}{(2J_A + 1)(2j_a - 1)} \sum_{M_A M_B m_a m_b} |T_{M_A M_B, m_a m_b}|^2 , \qquad (9)$$

where $\mu_{\alpha(\beta)}$ and $k_{\alpha(\beta)}$ are the reduced mass and the wave number in the corresponding channel.

2. Input parameters

The N$_R$V knowledge base provides the web service (dialog) to set the input parameters for calculation of nucleon transfer cross sections within the DWBA. The screenshot of the web dialog is shown in Fig. 2.
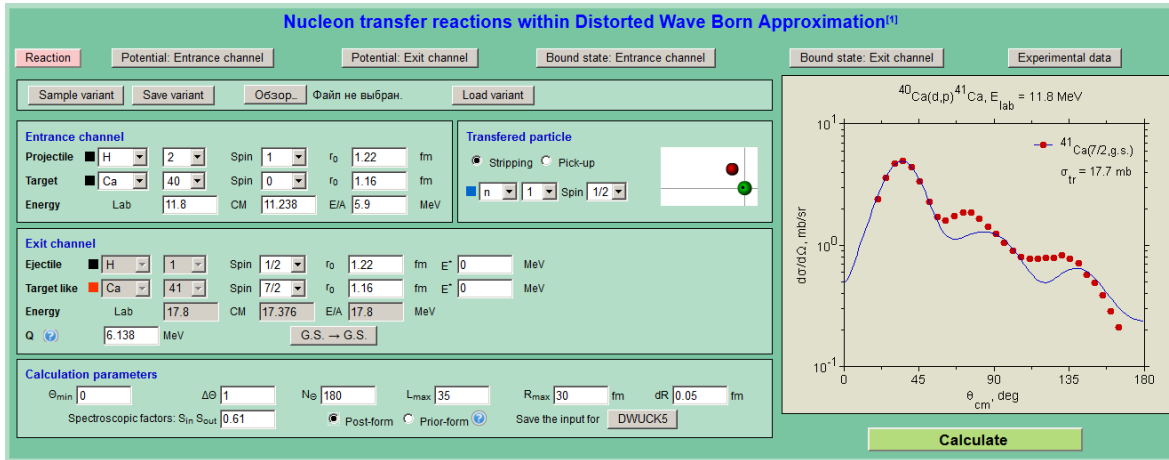
Figure 2. N$_R$V web dialog allowing to set the input parameters for calculation of nucleon transfer reaction cross sections within the DWBA.

In the upper row of buttons the user may choose the specific sections (*Reaction, Potentials, Bound states* or *Experimental data*) in order to modify the corresponding parameters.

2.1. Reaction

In the *Reaction* section the user may choose the masses and charges of interacting nuclei, their spins and radii (radii are used in calculation of the proximity potential) both for the entrance and exit channels as well as the transferred particle and the type of the reaction (*pick-up* or *stripping*).

The excitation energies E^* of the fragments and the Q-value of the transfer process for the exit channel may also be set. The energy of the relative motion in the exit channel is set automatically as $E'_{cm} = E_{cm} - Q$.

The button G.S.→G.S. allows to set the Q-value and binding energies (see the *Bound states* section) corresponding to the transition from the ground state to the ground state (for this the database of nuclear masses is used).

The calculation parameters include the center-of-mass angular range for the cross section: the starting angle θ_{min}, the angular mesh step $\Delta\theta$ and the number of steps N_θ. The value L_{max} is the maximum value of the orbital momentum taken into account. The quantities R_{max} and dR define the radial mesh.

The combined spectroscopic factor $\mathcal{S}_{in}\mathcal{S}_{out}$ is used to normalize the DWBA cross section. The user may also choose either the prior or the post form of the DWBA.

The button *DWUCK5* provides the input data in the proper format suitable for the DWUCK5 code.
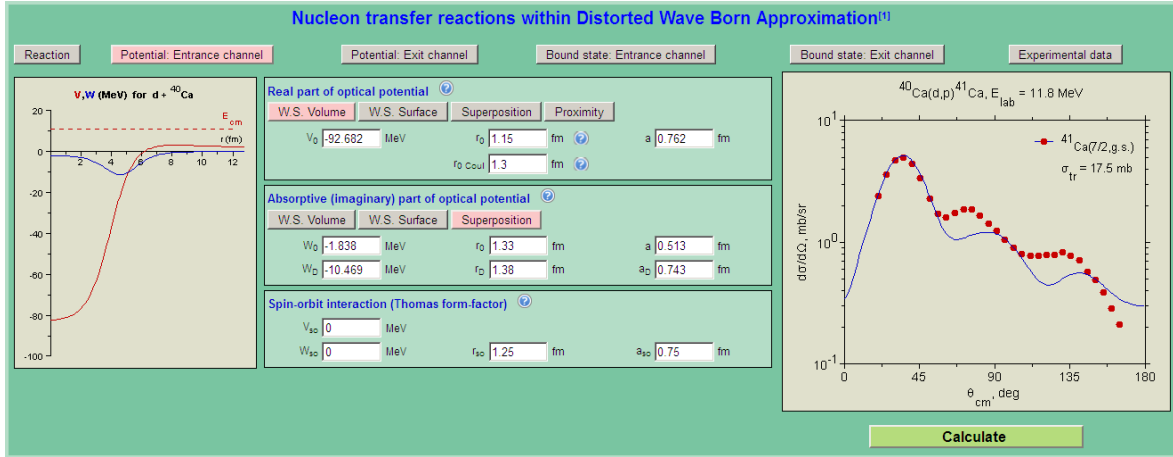
Figure 3. N_RV web dialog allowing to set the optical potential for the entrance channel.

2.2. Potential: Entrance channel

This section is used to set the parameters of the optical potential applied to the distorted wave calculation. The standard form of the point-sphere Coulomb potential is used

$$
V_C(r) = Z_1 Z_2 e^2 \begin{cases} \dfrac{1}{r}, & r > R_C, \\ \dfrac{1}{2R_C}\left(3 - \dfrac{r^2}{R_C^2}\right), & r \le R_C, \end{cases} \tag{10}
$$

where the Coulomb radius $R_C = r_{0,Coul} A_T^{1/3}$ may be specified by the user. The user may also choose one of the four different types of the real part of the nuclear potential: the volume and the surface *Woods-Saxon* types, their *Superposition*

$$
V_N(r) = \frac{V_0}{1 + \exp\left(\frac{r - r_0 A_T^{1/3}}{a}\right)} - 4a_D V_D \frac{d}{dr} \frac{1}{1 + \exp\left(\frac{r - r_D A_T^{1/3}}{a_D}\right)},
$$

or the *Proximity* potential (see Ref. [4]). Note that the user may modify the reduced radii, e.g., r_0 or r_D, while the potential includes the radii depending only on the target mass $R_i = r_{0,i} A_T^{1/3}$ in all the cases. The radii of nuclei required for the calculation of the proximity potential are set in the section *Reaction* next to the specific nucleus.

For the imaginary part of the optical potential everything is the same except for the proximity potential which is not provided.

The spin-orbit interaction may only be chosen in the Thomas form

$$V_{SO}(r) = -(\mathbf{L} \cdot \mathbf{s})(V_{SO} + iW_{SO})\frac{1}{r}\frac{d}{dr}\left[1 + \exp\left(\frac{r - r_{SO}A_T^{1/3}}{a_{SO}}\right)\right]^{-1}.$$

For convenience the applet on the left panel shows the potential plot with the chosen parameters.

The optical potential in the exit channel may be specified in the *Potential: Exit channel* section in a similar way.
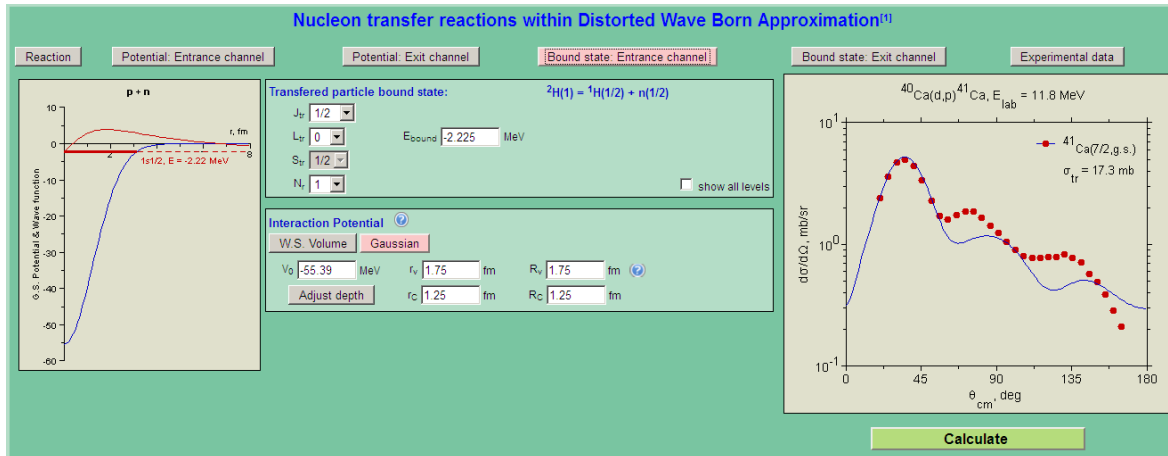

Figure 4. $N_R V$ web dialog allowing to set the bound state parameters and the interaction potential for the system $a = (b + x)$ in the entrance channel.

2.3. Bound state: Entrance channel

This section (see Fig. 4) is used to specify the quantum state parameters and the interaction potential for the composite system $a = (b + x)$ in the entrance channel. The quantity E_{bound} is the binding energy in MeV. The quantity N_r is the main quantum number, J_{tr}, L_{tr} and S_{tr} are the total angular momentum, orbital momentum and spin of the transferred particles.

The spin S_{tr} in this section is fixed since it is the same as the spin of the transferred particle in the *Reaction* section. The total angular momentum $\mathbf{J}_{tr}$ is the result of the $\mathbf{L}_{tr} + \mathbf{S}_{tr}$ coupling. Note also that the total spin of the composite particle in the entrance channel is the sum of the transferred particle momentum $\mathbf{J}_{tr}$ and the spin of the core
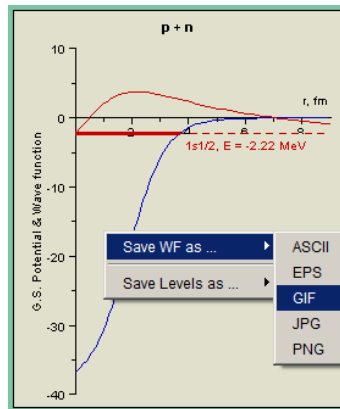
Figure 5. The additional features provided by the bound state Java applet. Activation by the mouse button click.

particle. In the case shown in Fig. 4 these particles are the neutron in the $1s_{1/2}$ state (the transferred particle) and the proton (the core with the spin $\frac{1}{2}$). They form the deuteron projectile with the total momentum 1.

The interaction potential is the potential binding the cluster of the composite system. The user may choose one of the two types of the potential: the Woods-Saxon volume potential

$$V(r) = \frac{V_0}{1 + \exp\left(\frac{r - R_v}{a_v}\right)},$$

or the Gaussian potential

$$V(r) = V_0 \exp\left(-\frac{r^2}{R_v^2}\right).$$

The Coulomb interaction is chosen in the same form as in the optical potential case.

The plot (Java applet) on the left panel demonstrates the interaction potential and the bound state wave function for the entrance channel. Note that the Java applet provides additional features, see Fig. 5. In particular, the wave function and the list of the levels found in the potential may be *saved* by the user in the ASCII text or graphics formats.

The Java applet also automatically *adjusts* the potential depth V_0 in case of any parameter change. In some cases the search routine fails to find the appropriate potential depth. In this case the user has to manually set the initial value of the V_0 parameter, then the search routine starts again.

The bound state in the exit channel may be specified in the same way as in the *Bound state: Exit channel* section.

2.4. Experimental data

In this section (see Fig. 6) the user may enter the experimental data specifying the entrance channel, the transferred particle and its final state in the exit channel. The
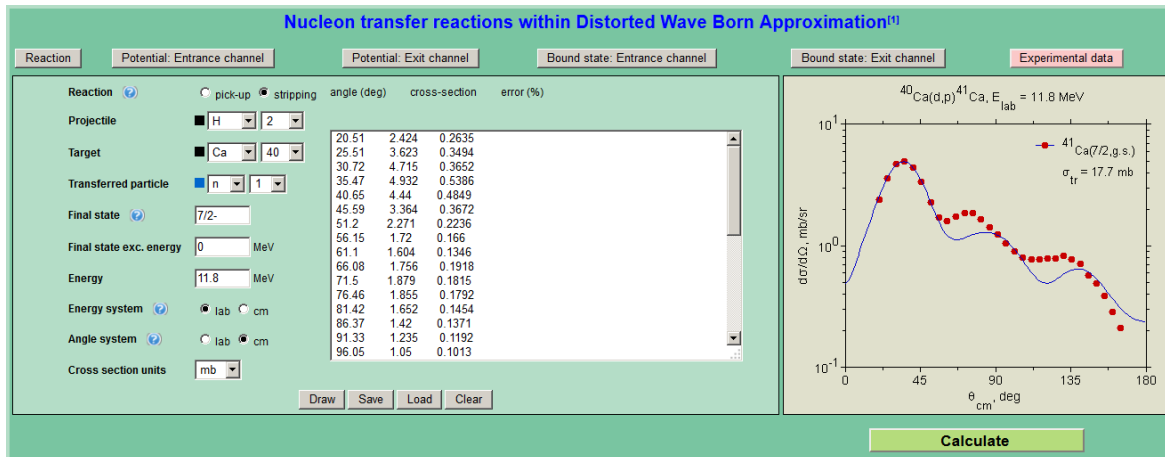
Figure 6. N$_R$V web dialog allowing to enter the experimental data for the transfer reaction.

collision energy and the scattering angle may be entered either in lab. or c.m. system.

The experimental data must be entered in the three-column form: the scattering angle θ in degrees, the differential cross section $d\sigma/d\Omega$ in the chosen units and the error value in % of the cross section value.

The experimental data are optional. The DWUCK5 code does not require any experimental data to perform the calculation since it does not fit the theoretical curve to the data.

2.5. Results

If all parameters are set, the cross section calculation may be started by pressing the *Calculate* button. In this case the input data as well as the entered experimental data are sent to the N$_R$V server and stored in the user folder. On the basis of these parameters the N$_R$V server prepares the input file and runs the DWUCK5 code on the server side. After the calculation the server passes the results to the Java applet located on the left side of the web dialog. The results are shown by the solid curve together with the experimental data. The integrated transfer reaction cross section is also shown in the plot.

The user may open an additional Java window with the differential cross section by clicking the corresponding Java applet. This window is shown in Fig. 7. The plot in this window is the same as the plot in the web dialog, but provides additional options allowing to process the data using the menu items. For example, the user may save the data in the text or graphics formats (menu *File/Save as ...*), plot the cross section vs. scattering angle in c.m. or lab. system, etc.

Note also that the next session will start with the last calculated variant if the same computer and the same Internet browser are used.
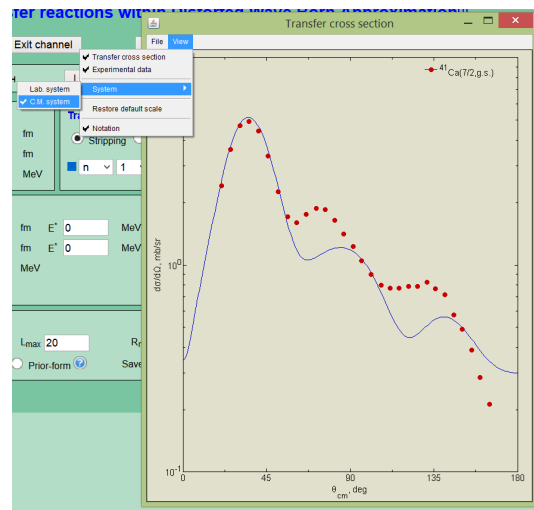
Figure 7. The additional Java window allowing to process the obtained data.

References

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