GENERALIZED OPTICAL POTENTIAL OF LIGHT WEAKLY BOUND CLUSTER NUCLEI

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A generalized optical potential for elastic scattering induced by light weakly bound nuclei is calculated within the Feshbach projection operator method. The model explicitly takes into account the contribution of the projectile break-up continuum treated within a microscopic cluster model. The model is tested on deuteron and \( ^6\)Li elastic scattering by different targets at intermediate energies. The optical potentials are then calculated for the \(^6\)He(230 MeV)\(^{12}\)C reaction treating the projectile nuclei within two (\( \alpha + 2n \)) and three (\( \alpha + n + n \)) cluster models. The differences are analyzed.

1. Introduction

During the last years, reactions with light weakly bound nuclei have been of increased interest from experimental and theoretical points of view and the progress in the investigation of these nuclei has been impressive.\(^1\)\(^-\)\(^3\) This progress is conditioned by the important efforts devoted to investigate reaction mechanisms and by new techniques enabling the production of exotic nuclei. In spite of that we are still far from a clear understanding of the unusual structure of exotic nuclei, as well as of the reaction mechanisms taking place under conditions of a strong coupling of all the reaction channels with the break-up channel of weakly bound projectile.

Generally it is assumed that in light exotic nuclei the nucleons tend to group into clusters, whose relative motion mainly defines the properties of these nuclei. This assumption leads to great advantages for models employing the cluster concept both for the structure and reactions involving light exotic nuclei (see references in Refs. 1–3). In particular, the interplay between elastic and break-up channels in reactions involving two-cluster projectiles can be effectively studied within the continuum-discretized coupled channel method\(^4\)\(^,\)\(^5\) (CDCC). However, the application of the CDCC approach to reactions with few-body projectiles (such as \(^6\)He = (\( \alpha + n + n \)), for example) meets significant computational difficulties if a realistic few-body wave function is used. Some recent publications on this topic should be mentioned.\(^6\)
The generalized optical model (GOM) of H. Feshbach is an alternative approach to the problem. Models based on the Feshbach theory are extensively used for the study of coupling effects on different reaction channels. Nevertheless, the studies of the projectile elastic and break-up channels coupling within the Feshbach method were done in the past with many simplifications because of significant computational burden. Instead the CDCC approach is applied usually. However, the application of the CDCC method becomes very problematic in the case of a few-body projectile, while modern computer technics makes the GOM more feasible.

The main goal of this work is to apply the GOM to study the elastic scattering of weakly bound light nuclei using a realistic few-body model description of their internal structure.

2. Model and Applications

The method proposed earlier is extended here in order to avoid the simplifying assumptions. In Refs. 8 and 9 the authors (i) neglected the spins of the particles, (ii) considered coupling with s-wave continuum only, (iii) neglected or treated the Coulomb force in an approximate way, and (iv) used the free-particle Green function. Our approach goes beyond these assumptions. Within the method a structureless target nucleus interacts with a projectile treated as a system of few bound clusters. The Hamiltonian of the problem reads

\[ \hat{H}_{R,\xi} = \hat{T}_R + \hat{H}_\xi + \hat{V}_{R,\xi}. \]

It includes the kinetic energy operator of the projectile-target relative motion \( \hat{T}_R \) and the Hamiltonian \( \hat{H}_\xi = \hat{t}_\xi + \hat{v}_\xi \) describing the projectile internal structure. Here \( \xi \) denotes an appropriate set of the projectile internal coordinates. The interaction potential \( \hat{V}_{R,\xi} \) is considered as a sum of effective complex cluster-target potentials.

The Feshbach operators \( \hat{P} \) and \( \hat{Q} \) are constructed on the basis functions of the Hamiltonian \( \hat{H}_\xi \). The operator \( \hat{Q} \) includes just the continuum part of the projectile spectrum, since a weakly bound nucleus is supposed to have only one bound state, i.e. the ground one. Operator \( \hat{P} \) extracts the elastic component of the total wave function which satisfies the Schrödinger equation with the optical potential

\[ \hat{U}^{(OM)} = \hat{P}\hat{V}_{\xi,\xi}\hat{P} + \hat{P}\hat{V}_{\xi,\xi}\hat{Q}\frac{1}{E - \hat{Q}\hat{H}_{\xi,\xi}\hat{Q} + i\hbar} \hat{Q}\hat{V}_{\xi,\xi}\hat{P}, \]

where the first term \( \hat{U}_R^{(1)} = \hat{P}\hat{V}_{\xi,\xi}\hat{P} \) is a sum of the cluster-target interactions folded over the projectile ground state, while the second one (we will refer it as \( \hat{U}_R^{(2)} \)) is a non-local polarization potential, which describes the coupling with break-up channels.

In order to obtain an explicit expression for the optical potential, (i) we use the Fourier–Bessel expansion of the interaction potential \( \hat{V}_{\xi,\xi} \), and (ii), following the usual technics, we decompose the total wave function over the spin-angular functions \( \Theta_{L(\gamma)}^{JM}(\xi, \Omega_R) = \left[ \phi_{\gamma}(\xi) \otimes Y_{\xi}(\Omega_R) \right]^{JM} \), which results from the orbital \( L \)
and spin $j$ vector coupling. The wave functions $\phi_{jm}(\xi)$ describe either discrete or continuum states of the projectile.

The calculation of the dynamical polarization potential requires also the definition of matrix elements of the few-body Green operator $\hat{G}(z) = (z - \hat{H}_{R,\xi})^{-1}$. The factorization of the Green operator in this form is impeded since interaction $\hat{V}_{R,\xi}$ is a nonseparable function of the relative $R$ and internal $\xi$ coordinates. Instead we express the $\hat{G}$ operator in the form of the Born series

$$\hat{G} = \hat{G} + \hat{G}(\hat{V}_{R,\xi}^{(1)} \hat{G} + \cdots),$$  

where $\hat{G} = (z - \hat{T}_R - \hat{H}_\xi - \hat{U}_R^{(1)})^{-1}$ is the Green operator, whose matrix elements can be calculated explicitly. In this paper the series (3) is truncated at the second term. By this approximation we neglect the rearrangements and multi-step excitation processes. The formers, however, have been included implicitly through the effective cluster-target interactions. Multi-step processes (like continuum-to-continuum coupling) become important at low energies, i.e. at large collision time $t_{\text{coll}}$. Thus, to validate the approximation we use a criteria $\tau = t_{\text{coll}}/t_{\text{int}} \leq 1$, where $t_{\text{int}}$ is a characteristic time of the projectile internal motion.

The decompositions of the total wave function and of the Green function allow to decouple the radial and angular coordinates in Eq. (2). Performing integration over the internal coordinates $\xi$ and over relative angular coordinates $\Omega_R$, we obtain the local central cluster-folding interaction $U^{(1)}(R)$ and the non-local $L$-dependent polarization potential $U^{(2)}(R, R')$ (see Refs. 12 for details). The set of the Schrödinger equations with non-local optical potential is solved numerically with appropriate boundary conditions in order to calculate elastic scattering cross sections.

We test the model on deuteron and $^6\text{Li}$ elastic scattering reactions. In this case, the deuteron ($p+n$) and $^6\text{Li}(p+d)$ projectiles are treated within a two-cluster model with $l$-dependent interaction$^{11}$ reproducing the projectile’s ground state energies, r.m.s. radii and low energy phase-shifts. The cluster-target interactions are extracted from the analysis of experimental data within the phenomenological optical model. Details may be found in Ref. 12.

The obtained cluster-folding and polarization potentials are used to calculated the cross sections of elastic scattering on different targets at energies of 30–50 MeV/u. These results are shown in Fig. 1 and indicate a good agreement with experimental data. This justifies the efficiency of the model.

To apply the model to the $^6\text{He}$ scattering, we start with a comparison of the two-cluster ($\alpha + ^2n$) and three-cluster ($\alpha + n + n$) approximations to the description of the $^6\text{He}$ internal structure. The $^6\text{He}$ two-cluster approximation is often used in reaction calculations, and its validity is an important question.

It is convenient to express the cluster-folding and polarization potential using elastic and transition form-factor components, which depend on the internal projectile properties only. These quantities are calculated separately within a two-cluster or three-cluster model and can be compared directly. The three-cluster structure
Fig. 1. Comparison of the calculations and experimental data\textsuperscript{14-18} on the deuteron and $^6$Li elastic scattering.

Fig. 2. (a) The $\alpha$-cluster elastic form-factor in the $^6$He calculated within two-body (dashed line) and three-body (solid line) models. (b) In the same notation the $\alpha$-cluster transition form-factors are compared for the $0^+,1^-$ and $2^+$ states of the $^6$He. (c) $^6$He($230$ MeV) + $^{12}$C elastic scattering cross section calculated within the GOM using two-body (dashed lines) and tree-body (solid lines) model of $^6$He. The thick and thin curves show calculations with and without coupling, correspondingly. Experimental data are from Ref. 19.

of the $^6$He is described within the hyperspherical harmonics method,\textsuperscript{13} which was found very effective both in structure and reaction calculations. In Figs. 2(a) and 2(b) the form-factors are shown for the $\alpha$-cluster in $^6$He. One may see that the $\alpha$-cluster form-factors are noticeably narrower within two-body model than within three-body one. The narrower the form-factor, the wider the spatial distribution. This discrepancy is due to the presence of a cigar-like component in three-body $^6$He...
The \( ^6 \text{He} \) wave function. The \( \alpha \)-cluster in three-body system moves closer to the \( ^6 \text{He} \) center of mass in comparison with the two-body case, where only di-neutron component is taken into account (see the insets in Fig. 2(a)).

3. Conclusions

A wider spatial cluster distributions lead to a wider optical potential. In Fig. 2(c) the elastic scattering cross sections calculated for the \( ^6 \text{He} + ^{12} \text{C} \) reaction are shown. The calculations are performed using the GOP obtained within two-cluster and three-cluster projectile models. It is clear that the two-cluster model significantly overestimates absorptive part of the optical potential and, consequently, gives suppressed angular distribution.

In the recent paper\(^{20} \) the analysis of the same reaction has been done within the CDCC method using a two-body \( ^6 \text{He} \) model. The obtained cross section considerably underestimates data. The authors found also that the additional contribution from the transfer channels are not able to cover the observed difference. It allows us to conclude that (i) this discrepancy is a demonstration of the \( ^6 \text{He} \) few-body nature, (ii) the elastic scattering is quite sensitive to the \( ^6 \text{He} \) internal structure, and (iii) the two-cluster approximation in the \( ^6 \text{He} \) case leads to inadequate results.

References