Examining the enhancement of sub-barrier fusion cross sections by neutron transfer with positive Q values

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Background: Significant enhancement of sub-barrier fusion cross sections owing to neutron rearrangement with positive Q values were found for many combinations of colliding nuclei. However, several experimental results on fusion reactions were reported recently in which such enhancement has not been observed in spite of a possibility for neutron rearrangement with positive Q values. Purpose: We aim to clarify much better the mechanism of neutron rearrangement in sub-barrier fusion reactions to find the other requirements (beside positive Q values) which favor (or prevent) sub-barrier fusion enhancement.

Methods: A channel coupling approach along with the semiclassical model for neutron transfer has been used for analysis of available experimental data on sub-barrier fusion of heavy ions.

Results: The role and interplay of different factors determining the enhancement of sub-barrier fusion (such as Q values for neutron rearrangement, properties of collective excitations, and neutron binding energies) have been studied and clarified.

Conclusions: (1) Only 1n and 2n transfers with positive Q values have a noticeable impact on sub-barrier fusion. A positive Q value for neutron rearrangement is a necessary but not sufficient requirement for additional sub-barrier fusion enhancement to take place. (2) The “rigidity” of colliding nuclei with respect to collective excitations is important for sub-barrier fusion enhancement due to neutron rearrangement with positive Q values to be clearly visible. (3) The neutron binding energy of the “donor” nucleus has a strong impact only in the case of fusion of light weakly bound nuclei.

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I. MOTIVATION

Fusion enhancement below the Coulomb barrier has become an intensively studied phenomena owing to progress in experimental techniques. It is by now well established that to describe the fusion cross sections one needs to include coupling of relative motion to other degrees of freedom such as rotation of deformed nuclei and their surface vibrations. The sub-barrier fusion enhancement induced by surface deformations or rotation of heavy deformed nuclei is well understood and properly described within the quantum coupled channel (QCC) approach [1–6] and within the empirical coupled channel (ECC) model [7].

At the same time, there is much experimental evidence testifying to additional enhancement of the sub-barrier fusion cross section due to neutron rearrangement with positive Q values. This effect can be easily observed if one compares the sub-barrier fusion cross sections for two close projectile-target combinations in which neutron rearrangement with positive Q values is possible in one whereas all neutron transfers have negative Q values in the other. The combinations (40Ca + 96Zr, 40Ca + 90Zr) [8] and (16O + 60Ni, 18O + 58Ni) [9] are good examples of such systems. Experimental and theoretical cross sections for these fusion reactions are shown in Fig. 1 (taken from Ref. [10]). The coupling of relative motion to the surface vibrations of target nuclei describes quite well the fusion cross sections for the 40Ca + 90Zr and 16O + 60Ni reactions, but it is insufficient to describe additional sub-barrier fusion enhancement for the 40Ca + 96Zr and 18O + 58Ni reactions caused by the intermediate neutron rearrangements with positive Q values.

Another example comes from a comparison of the fusion cross sections for the 40Ca + 48Ca and 48Ca + 48Ca reactions [3]. In this case, quite unexpectedly sub-barrier fusion was found to be less probable for the more neutron rich system. This is explained again by the possibility for neutron rearrangement with positive Q values in the case of 40Ca + 48Ca but not in the case of 48Ca + 48Ca. The mechanism of sequential fusion was proposed in [10], in which an additional sub-barrier fusion enhancement owing to neutron rearrangement with positive Q values at the approaching stage was described for the first time. The corresponding semiclassical model of this process was developed and successfully used for the predictions [10,11] and description of several fusion reactions [12–14].

Microscopic consideration of the problem performed within the time-dependent Schrödinger equation [15] and in the time-dependent Hartree-Fock (TDHF) approach [16] clearly demonstrates that spreading of the valence neutron’s wave function into the volume of the other nucleus takes place before the colliding nuclei overcome the Coulomb barrier. Thus, neutron rearrangement at the approaching stage may really influence the sub-barrier fusion dynamics, increasing the kinetic energy of colliding nuclei if it occurs with positive Q value.

In the past few years the study of fusion reactions involving light weakly bound nuclei has been of increasing interest [17–27]. For these nuclei, coupling to surface vibrations and rotation of heavy target are less important because of their smaller size. However, just the rearrangement of nucleons at the approaching stage may lead to significant sub-barrier...
It turns out that the role of neutron transfer in sub-barrier fusion reactions is not completely clear. Recently, several projectile-target combinations were reported (for example, $^{58.6}\text{Ni} + ^{130}\text{Te}$ [28], $^{16.18}\text{O} + ^{76.74}\text{Ge}$ [29], and $^{60.64}\text{Ni} + ^{160}\text{Mo}$ [30]) for which the measured fusion cross sections do not demonstrate noticeable enhancement at sub-barrier energies in spite of positive $Q$ values for neutron rearrangement. It is important to note that the semiclassical model [10], which takes into account neutron rearrangement, describes perfectly all these new data and also does not predict any fusion enhancement for these combinations (see below). This means that positive $Q$ values for neutron rearrangement is a necessary but not sufficient requirement for sub-barrier fusion enhancement to takes place.

The main purpose of this paper is to find the other factors (conditions) of neutron rearrangement with positive $Q$ values which favor (or prevent) sub-barrier fusion enhancement. To find these conditions we clarified more deeply the mechanism of neutron rearrangement in fusion reactions. Our findings can be formulated briefly as follows: (1) Only $\text{In}$ and $\text{2n}$ transfer with positive $Q$ values play a noticeable role in sub-barrier fusion of heavy ions. (2) The “rigidity” of colliding nuclei with respect to collective excitations is important for sub-barrier fusion enhancement due to neutron rearrangement with positive $Q$ values to be observed. (3) The neutron binding energy of the “donor” nucleus has a strong impact on the transfer probability for light weakly bound nuclei.

II. THE MODEL

It is rather difficult to include the nucleon transfer channels in a rigorous quantum channel-coupled approach. The problem appears when, following the standard channel-coupled method, the total wave function is decomposed over collective (rotational and/or vibrational) states and simultaneously over neutron transfer states. In such a decomposition, overcomplete and nonorthogonal basis functions are used, which requires a special complicated technique or some simplifying assumptions. Moreover, in medium mass and heavy nuclei single-particle states are spread over numerous exited states of these nuclei (with appropriate spectroscopic factors), which hardly can be included in any microscopic coupled-channel scheme.

At the same time, neutron rearrangement was quite consistently incorporated into the ECC approach [7] using a semiclassical approximation for the transfer probability [10]. This method is not fully microscopic, but it takes into account approximately the main effects of neutron rearrangement with positive $Q$ values. The ECC model with neutron rearrangement has been already successfully used in several papers [10–14] to reproduce and predict cross sections for sub-barrier fusion reactions of stable nuclei.

In fusion reactions of light and medium mass nuclei it was found that compound nucleus (CN) is formed right away colliding nuclei come into contact. However, with increasing masses of fusing nuclei (for example, in reactions leading to formation of superheavy nuclei) the quasi-fission process starts to play a more significant role. For such reactions the ECC and QCC models give the so-called capture cross section,

![Diagram](image-url)

**FIG. 1.** (Color online) (Upper panel) Fusion excitation functions for $^{40}\text{Ca} + ^{96}\text{Zr}$ (open circles) and $^{40}\text{Ca} + ^{90}\text{Zr}$ (filled circles) [8]. The no-coupling limits are shown by the dotted curves. The dashed curves show the calculations with coupling to surface vibrations and without neutron transfer, whereas the solid line was obtained by accounting for neutron rearrangement in the entrance channel of the $^{40}\text{Ca} + ^{90}\text{Zr}$ reaction. (Bottom panel) Fusion excitation functions for $^{18}\text{O} + ^{58}\text{Ni}$ (open circles) and $^{16}\text{O} + ^{60}\text{Ni}$ (filled circles) [9]. The no-coupling limit is shown by a dotted curve (which is practically indistinguishable for the two reactions). Other notations are the same as in the upper panel.

fusion enhancement owing to the large positive $Q$ value. For example, the deep sub-barrier fusion cross section of $^6\text{He}$ with $^{206}\text{Pb}$ was predicted to be several orders of magnitude larger as compared with fusion of $^4\text{He} + ^{208}\text{Pb}$ [10], which was confirmed later in experiments [22,27]. However, there is a quantitative inconsistency between the results of the two measurements as well as with the calculated cross section [10] at sub-barrier region. In spite of the fact that a large enhancement of sub-barrier fusion follows from all the experiments [22,27], further study of this reaction is desirable. Moreover, deep sub-barrier fusion of light nuclei (including exotic ones) may also have an impact on astrophysical nucleosynthesis [15].
which is larger than the fusion cross section (formation of CN) by the quasi-fission cross section. In this paper we consider fusion reactions of light and medium mass nuclei, for which the impact of the quasi-fission process is expected to be small. Therefore, one may use the traditional notion “fusion cross section” for the cross section calculated within the empirical (or quantum) coupled-channel model.

The collision dynamics of two nuclei is regulated mostly by the nucleus-nucleus potential, consisting of the Coulomb, nuclear, and centrifugal terms. The contact point, \( R_{\text{cont}} = R_1 + R_2 \), is located at a shorter distance than the position of the Coulomb barrier, \( R_{\text{cont}} < R_B \). Thus, the fusion probability (or cross section) is determined by the probability of passing through the potential barrier. The fusion cross section is usually decomposed over the partial waves as

\[
\sigma_{\text{fus}}(E) = \frac{\pi \hbar^2}{2\mu E} \sum_{l=0}^{l_{\text{cr}}} (2l + 1) T_l(E),
\]

(1)

where \( E \) is the center-of-mass energy, \( \mu \) is the reduced mass of the system, \( l \) is the orbital angular momentum, \( l_{\text{cr}} \) is the lowest angular momentum at which a pocket in the interaction potential disappears, and \( T_l(E) \) is the barrier penetration probability. Approximating the radial dependence of the barrier by a parabola one can use the simple Hill-Wheeler formula \([31]\) for the penetration probability:

\[
T_l^{\text{HW}}(B; E) = \left[ 1 + \exp \left( \frac{2\pi}{\hbar \omega_B(l)} \left( B + \frac{\hbar^2(l(l+1)}{2\mu R_B^2(l)} - E \right) \right) \right]^{-1},
\]

(2)

where \( B \) and \( R_B(l) \) are the height and position of the potential barrier, respectively, and \( \hbar \omega_B(l) \) is the width of the parabolic barrier.

Generally, the nucleus-nucleus interaction potential depends not only on the relative distance between the colliding nuclei but also on their deformations \( \beta_{\ell_i}, i = 1, 2 \) and mutual orientations, \( \theta_i \). Thus, the interaction potential is characterized by the multidimensional Coulomb barrier, the height of which is a function of orientations and deformations: \( B(\beta_{\ell_1}, \beta_{\ell_2}; \theta_1, \theta_2) \).

Within the ECC model \([7]\) the coupling of the radial motion to the collective degrees of freedom is taken into account by averaging the transmission coefficient over the dynamic surface deformations and/or orientations of deformed colliding nuclei.

The collision dynamics and, consequently, the fusion cross section for spherical nuclei depends mainly on the coupling to their surface vibration degrees of freedom. Therefore, the partial penetration probability should be averaged over the deformation-dependent barrier height,

\[
T_l(E) = \int f(B) T_l^{\text{HW}}(B; E) dB,
\]

(3)

where dynamic deformations are assumed to take place along the internuclear axis, and \( f(B) \) is the empirical dynamic barrier distribution function \([7]\) normalized to unity: \( \int f(B) dB = 1 \). It is not the same as the (conventional) “experimental” barrier distribution function \( D(B) = \frac{d^2\sigma_{\text{fus}}}{d\Omega dB} \) \([32]\). For a one-dimensional (no-coupling) barrier model, \( f(B) = \delta(B - B_0) \), whereas \( D(B) \) in this case is still a smooth function with one peak at \( B = B_0 \) and with a width of about 0.56\( \hbar \omega_B \) (see, for example, \([4]\)). For a realistic multidimensional barrier (simulating channel coupling) we use the Gaussian approximation for \( f(B) \):

\[
f(B) = N_B \exp \left( -\frac{(B - B_0)^2}{\Delta B} \right),
\]

(4)

where \( B_0 = (B_1 + B_2)/2 \). Here \( B_1 \) is the height of the barrier at zero dynamic deformation of colliding nuclei, \( B_2 \) is the height of the saddle point calculated with realistic vibrational properties of nuclei, i.e., with the surface stiffness parameters obtained from the experimental values of the excited vibrational states, \( \Delta B = (B_1 - B_2)/2 \), and \( N_B \) is the normalization coefficient.

In the case of collisions of statically deformed nuclei \( \beta_{\ell_i} \neq 0 \) the Coulomb barrier height and its position depend on the mutual orientation of nuclei, so averaging over the orientations of both nuclei is required and

\[
T_l(E) = \frac{1}{4} \int_0^{\pi} \int_0^{\pi} T_l^{\text{HW}}(B(\beta_{\ell_1}, \beta_{\ell_2}; \theta_1, \theta_2); E) \times \sin \theta_1 \sin \theta_2 d\theta_1 d\theta_2.
\]

(5)

Note that in Eq. (5) we have assumed a uniform distribution over the initial orientations (i.e., the corresponding dynamic barrier distribution function is unity in the region of integration).

The neutron rearrangement channels can be easily included \([10]\) in this ECC model of fusion reactions. The total penetration probability (which takes into account the rearrangement of neutrons) can be estimated again by formulas (3) or (5) in which \( T_l^{\text{HW}} \) is replaced by the following expression:

\[
\tilde{T}_l^{\text{HW}}(B; E) = \frac{1}{N_{\ell r}} \int_{-E}^{Q_{\alpha}} \int_{-E}^{Q_{\beta}} \alpha_k(E, l, Q) \times T_l^{\text{HW}}(B; E + Q) dQ,
\]

(6)

where \( Q_{\alpha} \) is the \( Q \) value of the ground-to-ground transfer of \( x \) neutrons. The probability of the transfer of \( x \) neutrons with a given \( Q \) value (less than \( Q_{\alpha} \)) is calculated as follows:

\[
\alpha_k(E, l, Q) = N_k \exp(-CQ^2) \exp(-2k[D(E, I) - D_0]),
\]

(7)

where \( k = k(\ell_1) + k(\ell_2) + \cdots + k(\ell_k) \) for sequential transfer of \( k \) neutrons, \( k(\ell_i) = \sqrt{2\mu \epsilon_i / \hbar^2} \), \( \epsilon_i \) is the binding energy of the \( i \)th transferred neutron, \( D(E, I) \) is the distance of the closest approach along the Coulomb trajectory with angular momentum \( l \), \( D_0 = R_1^{(n)} + R_2^{(n)} + d_0 \), \( R_i^{(n)} = r_i^{(n)} A_i^{1/3} \) are the orbit radii of the valence (transferred) neutrons of colliding nuclei (with \( r_i^{(n)} \) and \( d_0 \) being adjustable parameters), \( N_{\ell r} \) is the normalization constant, \( \alpha_0 = \delta(Q), C = R_B \mu_{12}/4k \hbar^2 B \), and \( \mu_{12} \) is the reduced mass of the two nuclei in the entrance channel.

The values of \( r_i^{(n)} \) are 1.25 fm and \( d_0 = 2.5 \text{ fm} \) were fixed to reproduce the experimental data for the fusion reactions such as \(^{40,48}\text{Ca} + ^{48}\text{Ca} \) \([3]\), \(^{40}\text{Ca} + ^{90,92}\text{Zr} \) \([8]\), \(^{16,18}\text{O} + ^{60,58}\text{Ni} \) \([9]\), etc. Note that the values \( r_i^{(n)} = 1.4 \text{ and } d_0 = 0 \text{ are usually extracted} \([33,34]\) from the analysis of the data on transfer...
reactions. This leads to smaller values of $D_0$ (and smaller values of $\alpha_k$) for the transfer reactions compared to those required for the fusion reactions. This difference can be understood because, for the fusion reactions, the effect of neutron rearrangement depends on how strongly the wave function of the valent neutron is spread over the two-center molecular states at the moment of closest approach, whereas the final (measured) neutron transfer probability is given by the situation after the re-separation of the colliding nuclei at infinite distance between them.

As can be seen from (6), enhancement of the fusion probability may appear at sub-barrier energies if rearrangement of neutrons leads to a gain in energy (positive $Q$ values). In reactions with negative $Q$ values, neutron rearrangement in the entrance channel does not influence the total fusion cross section because the penetration probability $T^B_{1 Haw}(B; E + Q)$ becomes smaller for negative $Q$. In this case $\alpha_0$ is the only nonvanishing term in sum (6). Note that the probability of neutron rearrangement depends not only on the $Q$ value but also on the binding energy of the transferred neutron [see Eq. (7)]. The coefficients $\alpha_k$ decrease rapidly with increasing binding energies in the “donor” nucleus. However, this effect is usually ignored while discussing the influence of neutron rearrangement on the fusion process.

In our calculations, up to four neutron transfer channels are taken into account. However, owing to the rapid decrease of $\alpha_k$ with increasing number of transferred neutrons, $k$, only $1n$ and $2n$ transfer channels with positive $Q$ values were found to play a significant role (see the discussion below). The experiments indicate (see, for example, [34,35]) that simultaneous transfer of two neutrons might be enhanced by a factor of $N_{2n} \sim 3$ compared to independent (subsequent) transfer of these neutrons.

### III. WHY ENHANCEMENT FROM NEUTRON TRANSFER WITH POSITIVE $Q$ VALUES IS NOT ALWAYS VISIBLE

All calculations presented below have been performed (and can be easily repeated) with the Nuclear Reactions Video (NRV) fusion code available at the NRV website with free access [6]. The Woods-Saxon potential with the parameters listed in Table I is used as the nuclear part of the nucleus-nucleus interaction. The choice of the potential parameters is rather uncertain. Moreover, often the same behavior of fusion cross sections can be obtained with a few different sets of parameters. Therefore, to make the predictions for a chosen reaction reliable, first we fit the potential to reproduce the available experimental data for the nearest projectile-target combination and then we use the same parameters for the studied reaction.

The coupling to target and projectile collective states was taken into account for each studied system. The parameters of the vibrational excitations for the QCC calculations are taken from the NRV experimental databases. For the stiffness parameters required for the ECC calculations we use the liquid-drop values [36] as an initial approximation. To treat the excitation of rotational states for deformed nuclei we use the corresponding experimental data for the energy of the first rotational $2^+$ state and the ground-state deformation parameters according to Ref. [37].

In each case we consider a few close projectile-target combinations, one of which reveals the influence of neutron rearrangement and the other does not. As a rule the chosen combinations lead to the same compound nuclei in order to have the same decay properties that would simplify the experiment itself and the analysis of the corresponding experimental data. This is a reasonable approach in the current studies when the collective properties of the fusing nuclei are similar. However, if the collective properties are different (first of all those of the targets), this may lead to a large difference between sub-barrier fusion cross sections besides the one caused by neutron rearrangement.

First, we show that the ECC and QCC models give quite similar results for the systems where only the coupling to collective states play a role and not the neutron transfer. Figure 2 shows fusion cross sections in the reaction $^{16}O + ^{154}Sm$ [38]. Coupling to rotational states of $^{154}Sm$ ($E_{2+} = 82$ keV, $\beta_2 = 0.3$, $\beta_8 = 0.11$) was included in the QCC and ECC calculations. The projectile is treated as a structureless nucleus. All the $Q$ values for neutron rearrangement are negative in this reaction, and the neutron transfer channels do not influence the fusion cross sections. As can be seen, the QCC and ECC approaches give very similar results. All other combinations of fusing nuclei yield similar results. We use the ECC model below just because it allows us to include the coupling to neutron rearrangement channels by Eqs. (6) and (7).

It is known (see the discussion above) that the sub-barrier fusion of weakly bound nuclei is a “classical” example of the reaction revealing a strong enhancement due to neutron rearrangement. Figure 3 shows the fusion cross sections for two systems $^4He + ^{64}Zn$ (with no neutron transfer, since all $Q_{xn}$ values are negative) and $^6He + ^{64}Zn$ having large...
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FIG. 2. (Color online) Fusion cross section for $^{16}\text{O} + ^{154}\text{Sm}$. The solid and dashed curves correspond to ECC and QCC calculations, respectively. Experimental data are from [38]. The dotted curve shows the result for one-dimensional barrier penetrability. Arrows indicate the barriers for two limit orientations of the deformed $^{154}\text{Sm}$ nucleus ($B_1 = 53.6$ MeV and $B_2 = 60.7$ MeV) and for its spherical shape with the same volume, $B_{sp}$. The bottom panel shows the corresponding “experimental” barrier distribution functions, $D(B)$.

FIG. 3. (Color online) Fusion cross section for $^6\text{He} + ^{64}\text{Zn}$. The solid and dashed curves correspond to ECC calculations accounting for and without accounting for the neutron transfer, respectively. The dash-dotted curve shows the $^6\text{He} + ^{64}\text{Zn}$ fusion cross section multiplied by a factor of 0.8. Experimental data are from [25].

$Q_{in}$ values ($Q_{in} = 6.11$ MeV, $Q_{2n} = 18.06$ MeV, and $Q_{3n} = 4.54$ MeV). One may notice a good agreement with recent experimental data [25] for both systems. The ECC calculations without neutron rearrangement (dashed curves) are very close for the two reactions. The enhancement owing to neutron rearrangement already reaches about one order of magnitude at 1 MeV below the Coulomb barrier and it is even stronger at deeper sub-barrier energies.

In the literature there are discussions of two counteractive factors influencing the fusion of weakly bound nuclei (such as $^6\text{He}$). The first factor results from the neutron rearrangement with positive $Q$ values studied here. The second one is caused by the breakup of a weakly bound projectile, which is not included in either the QCC or the ECC calculations (but the problem is studied in many papers; see, e.g., Refs. [39–41]). Coupling to the breakup and nucleon transfer channels reveals itself in the nucleus-nucleus potential as polarization terms having different signs. The neutron rearrangement with positive $Q$ values leads to the enhancement of sub-barrier fusion; i.e., it provides attractive polarization potential. The breakup processes lead to a repulsive term being added to the potential and, therefore, suppress the fusion cross section (see, for example, Refs. [42,43]). In order to roughly estimate these two factors let us consider the fusion cross section at above-barrier energies. It will be shown in Sec. IV that the influence of the neutron rearrangement processes become weak with increasing energies. Nevertheless, the calculated cross section in Fig. 3 overestimates the experimental data. In the case of fusion reactions with the participation of light weakly bound nuclei such as $^6\text{He}$ this damping of the cross section of fusion reactions with the participation of light weakly bound nuclei (such as $^6\text{He}$) is usually attributed to the influence of the breakup channels. It plays a noticeable role at near- and above-barrier energies and results in about a 20%–30% reduction of the magnitude of the fusion cross section [39,44]. Thus the calculated cross section can be easily fitted to the data after being multiplied by an empirical coefficient of $\sim 0.7–0.8$ (dash-dotted curve in Fig. 3). At the deep sub-barrier energies which are the focus of our study the role of the breakup channels is much less while the influence of the transfer and inelastic channels dominate the growth of the cross section by several orders of magnitude. Therefore we do not use any additional coefficients in this paper.

One may also see some overestimation of the fusion cross section at above-barrier energies for the $^6\text{He} + ^{64}\text{Zn}$ reaction for which the breakup is not expected at all. It is well known that the behavior of the fusion cross section in this energy range is completely determined by the potential parameters. They are chosen here to be identical for both reactions in order to avoid additional factors influencing the calculated cross sections. This leads to close values of the cross sections at higher energies, where they can be approximated by the geometrical value $\pi R_r^2$. This overestimation is not important for our consideration since it is rather small and the aim of this paper is the analysis of sub-barrier fusion. Thus we did not alter the potential parameters.

In this connection, we also should mention the case of $^6\text{He} + ^{209}\text{Bi}$ fusion reactions, in which enhancement is clearly visible and properly explained within the ECC model with neutron rearrangement (see Fig. 4). Good agreement of the
FIG. 4. (Color online) $^6$He + $^{209}$Bi: The solid and dashed curves correspond to ECC calculations of the fusion cross section accounting for and without accounting for the neutron transfer, respectively. The experimental data for the fusion cross sections (filled circles) are taken from [18]. The long-dashed curve gives the fusion-fission cross section, and the corresponding experimental data [45] are shown by filled stars. $^4$He + $^{209}$Bi: The dashed curve is the fusion cross section calculated by using the ECC model without neutron transfer. The dotted curves show the cross sections for 1n and 2n evaporation channels. The respective experimental data are given by open circles [47] (1n channel) and open squares [48] (2n channel).

calculations with the available experimental data on the fusion-fission cross sections [45] is also obtained for the $^6$He + $^{209}$Bi reaction using the NRV statistical model code [46].

As already mentioned there is much evidence for additional sub-barrier fusion enhancement owing to neutron rearrangement with positive $Q$ values for both stable and weakly bound nuclei. However, several projectile-target combinations were reported recently ($^{58,64}$Ni + $^{130}$Te [28], $^{16,18}$O + $^{76,74}$Ge [29], and $^{60,64}$Ni + $^{100}$Mo [30]) for which no noticeable enhancement of the sub-barrier fusion cross sections was observed in spite of positive $Q$ values for neutron transfer. Here we performed an analysis of these fusion reactions within the model formulated above. We found that the model describes well all the experimental data and it also does not predict any significant fusion enhancement for these specific combinations having positive $Q$ values for neutron rearrangement. This motivates us to study more deeply the mechanism of intermediate neutron rearrangement in sub-barrier fusion reactions.

The corresponding fusion cross sections and the results of our analysis are shown in Figs. 5, 6, and 7. The theoretical calculations agree well with the experimental data. In all cases neutron rearrangement with positive $Q$ values is taken into account. However, some excess in the sub-barrier fusion cross section is visible only in the case of the $^{58}$Ni + $^{130}$Te reaction with positive $Q_{1n} = +0.58$ MeV and $Q_{2n} = +5.89$ MeV (see Table II) compared with the more neutron rich system $^{64}$Ni + $^{130}$Te having all negative $Q$ values for neutron transfers besides $Q_{2n} = +0.55$ MeV (see Fig. 5).

The data do not show any significant effect of neutron rearrangement for the $^{60}$Ni + $^{100}$Mo fusion reaction (having
positive $Q$ value neutron transfer from target to projectile; see Table II) compared with the $^{64}\text{Ni} + ^{100}\text{Mo}$ reaction (with the only positive value of $Q_{2n} = +0.83$ MeV). The same result is seen for the $^{18}\text{O} + ^{76}\text{Ge}$ fusion reaction (having positive $Q_{2n} = +3.75$ MeV) compared with $^{16}\text{O} + ^{76}\text{Ge}$ (all $Q_{in} < 0$) shown in Fig. 6.

Note that the model used takes into account neutron rearrangement, reproduces quite well the experimental data, and also does not predict any sub-barrier fusion enhancement for these specific reactions with positive $Q$ values of neutron transfers. Thus, we have to understand what features of these reactions (properties of colliding nuclei) suppress a gain coming from positive $Q$ value neutron rearrangement (which is clearly visible in many other cases).

**IV. INTERPLAY OF NUCLEAR PROPERTIES AND SUB-BARRIER FUSION ENHANCEMENT**

In this section different factors influencing the enhancement of the sub-barrier fusion due to neutron rearrangement are discussed. We use here the term “enhancement factor” to characterize the effect of coupling with neutron-transfer channels. A standard way to measure the enhancement factor consists in studying two close projectile-target combinations (to ensure similar fusion barriers and properties of the collective excitations): one with negative $Q$ values (no neutron rearrangement effect) and another one with positive $Q$ values. The difference of the cross sections in the sub-barrier region (if it can be observed) must be attributed to the additional coupling with neutron transfer channels. This method is not straightforward because a difference of the fusion cross sections may still appear owing to a difference of fusion barriers or the collective excitation properties. However, in many cases this method gives a good approximation to the “real” enhancement factor, which we define as the ratio of the fusion cross sections obtained by accounting for and without accounting for the coupling to the neutron rearrangement channels. It is clear that such a quantity can be obtained in theoretical calculations only.

### A. $Q$ values

The $Q$ values of neutron transfer as the factor determining the enhancement of the sub-barrier fusion cross section have been discussed many times (see, e.g., [3,10]). We repeat here the main points. If a system of two colliding nuclei has positive $Q$ values of neutron transfer then one may expect that the sub-barrier fusion cross section will demonstrate enhance-

<table>
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<th>$1n$</th>
<th>$2n$</th>
<th>$3n$</th>
<th>$4n$</th>
<th>$E_{2n}$</th>
<th>$(\beta^2_E)$</th>
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<td>$^{40}\text{Ca} + ^{96}\text{Zr}$</td>
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<td>+5.24</td>
<td>+9.64</td>
<td>1.75</td>
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<td>+5.89</td>
<td>+4.92</td>
<td>+9.23</td>
<td>0.84</td>
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<td>+4.20</td>
<td>+2.39</td>
<td>+5.23</td>
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<tr>
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</tbody>
</table>

**TABLE II. $Q_{in}$ values (in MeV) of neutron transfers and vibrational properties of the targets. The energies of the first vibrational state are given in MeV.**

FIG. 8. (Color online) Fusion excitation functions for $^{40}\text{Ca} + ^{96}\text{Zr}$. The curves show the ECC calculations with and without (dotted curve) neutron rearrangement channels being accounted for. The solid curve is obtained with real $Q_{in}$ values; the dashed and dash-dotted curves are the model calculations with $Q_{in}$ values halved and doubled, respectively.

FIG. 9. (Color online) Fusion excitation functions for $^{40}\text{Ca} + ^{96}\text{Zr}$ system. The $Q_{in}$ values for this system are listed in Table II. The experimental fusion cross sections [8] are well reproduced within the model. However, if one assumes that the $Q_{in}$ values are halved (dashed curve) then the neutron rearrangement enhancement is drastically reduced, and the calculated fusion cross section becomes much closer to the ECC calculations without neutron rearrangement (dotted curve). If the $Q_{in}$ values are doubled (dash-dotted curve) the effect is much stronger.

The last but not least point to make is that only the rearrangement of the outermost neutrons (normally $1n$ and $2n$ transfer channels) may enhance the sub-barrier fusion significantly. This occurs because the coupling with neutron rearrangement channels influences the fusion probability only if such rearrangement takes place *before* the Coulomb barrier is overcome. Therefore, only a few valent neutrons having the largest radii of their wave functions should be taken into account in the analysis of sub-barrier fusion. Figure 9 shows the fusion cross section for the $^{40}\text{Ca} + ^{96}\text{Zr}$ system calculated within the ECC model with different numbers of neutron rearrangement channels taken into account. The $Q_{in}$ values of neutron transfer are given in Table II up to $4n$; the rest values are $Q_{5n} = +8.42$ MeV and $Q_{6n} = +11.62$ MeV. One may see that the main effect comes from $1n + 2n$ channels. A much smaller but still visible effect is due to rearrangement of the third and fourth neutrons, whereas transfer of more neutrons does not influence the sub-barrier fusion probability.

### B. Properties of collective excitations

The $^{40}\text{Ca} + ^{96}\text{Zr}$ combination is a typical example of a reaction revealing strong fusion enhancement at sub-barrier energies due to neutron rearrangement. In contrast, some of the above-mentioned projectile-target combinations already studied experimentally ($^{60}\text{Ni} + ^{100}\text{Mo}$ [30] and $^{58}\text{Ni} + ^{130}\text{Te}$ [28])...
The studied enhancement due to neutron rearrangement is more pronounced for these reactions in the case of positive fusion. The enhancement due to neutron rearrangement should be large influence that neutron rearrangement has on sub-barrier fusion for these systems. The collective states of a heavy target has a rather low impact on barrier penetrability. Additionally, we consider only the first term \( l = 0 \) in the partial-wave decomposition of the fusion cross section. In this case the “enhancement factor” can be approximated by

\[
F = \frac{\sum_i (2l + 1) \tilde{T}_i}{\sum_i (2l + 1) \tilde{T}_i} \approx 1 + \text{const.} \frac{Q_{1n}}{\Delta B} \exp \left( -\frac{E - B_0}{\Delta B} \right) \frac{1}{1 + \text{erf} \left( \frac{E - B_0}{\Delta B} \right)}
\]

This expression is obtained for \( Q_{1n} > 0 \) and under the assumption that \( C Q_{1n} \ll 1 \), which is quite reasonable for \( Q_{1n} \leq 5 \text{ MeV} \). The constant value in (8) consists of all normalization factors as well as nuclear properties playing a second-order role (e.g., the neutron binding energies discussed below). This approximation of the enhancement factor allows us to conclude the following:

(i) The enhancement factor increases with increase of the \( Q_{1n} \) value.
(ii) The enhancement is larger for smaller \( \Delta B \) values, i.e., for more rigid nuclei.
(iii) The enhancement factor increases when the energy goes deeper into the sub-barrier region. For above-barrier energies \( F \) tends to unity.

The influence of the collective properties of nuclei on the enhancement of the fusion cross section due to neutron rearrangement is illustrated in Fig. 10. Figures 10(a) and 10(b) show the fusion cross sections and the barrier distributions for the \( ^{40}\text{Ca} + ^{96}\text{Zr} \) system. Fusion due to neutron rearrangement is enhanced by two orders of magnitude at energies 12 MeV below the barrier [compare the solid and dashed curves in Fig. 10(a)]. This enhancement is also well seen in the barrier distributions, when one compares the distributions calculated with and without neutron rearrangement in the low-energy region (shadowed area). Both reaction partners are magic spherical nuclei, and, therefore, they are hardly deformed nuclei, since their first excited states (see Table II) are rather high. According to our conclusion, this leads to the large influence that neutron rearrangement has on sub-barrier fusion.

If now we assume that these nuclei are softer with respect to their deformation than they actually are, and we replace the vibrational properties of \( ^{40}\text{Ca} \) and \( ^{96}\text{Zr} \) by those of \( ^{60}\text{Ni} \) and \( ^{100}\text{Mo} \) (softer nuclei), the influence of the collective excitations increases [see Fig. 10(c)]. As a result, accounting for neutron transfer channels gives now an additional enhancement factor of 5 instead of 100. The effect is well seen in the behavior of the barrier distributions shown in Fig. 10(d). The barrier distribution after changing the vibrational properties of colliding nuclei (the dashed curve) shifts to lower energies and becomes wider (smaller value of \( B_0 \) and larger value of \( \Delta B \)). As a result, the low-energy tails of the distributions with and without neutron rearrangement taken into account get very close to each other. The fusion enhancement due to the coupling to the neutron channels is, therefore, much smaller than for the original \( ^{40}\text{Ca} + ^{96}\text{Zr} \) system. Note that in these calculations we alter only the vibrational properties of nuclei and do not seem to be similar (having similar \( Q \) values of neutron transfers) to the \( ^{40}\text{Ca} + ^{96}\text{Zr} \) case but without any significant effect from neutron rearrangement.

The reason why one observes a very different influence of the neutron rearrangement on the sub-barrier fusion for these “similar” systems can be attributed to their different vibrational properties. Our conclusion is that the fusion enhancement due to neutron rearrangement is larger for systems having smaller fusion enhancement due to the coupling to collective states.

In the case of fusion reactions of light nuclei the coupling to the collective states of a heavy target has a rather low impact on the barrier penetrability. This means that the sub-barrier fusion enhancement due to neutron rearrangement should be more pronounced for these reactions in the case of positive \( Q \) value neutron transfers.

For collisions of medium and heavy statically deformed nuclei the coupling to the rotational states always plays a significant role. Thus, for such combinations the effect of neutron rearrangement is expected to be small. For spherical nuclei the sub-barrier fusion enhancement owing to the coupling to the vibrational degrees of freedom can be very different in magnitude depending of the vibrational properties of the reaction partners. Below we will focus just on the analysis of the fusion of spherical nuclei.

The influence of the vibrational properties of nuclei (their softness) on barrier penetrability can be estimated quantitatively. In the ECC model the coupling to the deformation degrees of freedom is determined by the shape of the empirical dynamic barrier distribution function \( f(B) \) [see Eqs. (1)–(4)]. In Eq. (4) the effective fusion barrier \( B_0 \) and the width of the distribution, \( \Delta B \), are determined by the energy of the first vibrational state, \( E_2 \), and the rms value of zero vibrations, \( \langle \beta_0^2 \rangle \). The smaller is \( \langle \beta_0^2 \rangle \) and the larger is \( E_2 \) (“rigid” nuclei) the closer is \( B_0 \) to the barrier for spherical nuclei and the smaller is \( \Delta B \), i.e., the narrower is the barrier distribution. The studied enhancement due to neutron rearrangement is determined by the ratio of the transmission probabilities obtained by accounting for and without accounting for the neutron channels. For simplicity, but without lose of generality, one may assume that only the \( 1n \) rearrangement channel plays a role. Additionally, we consider only the first term \( l = 0 \) in the partial-wave decomposition of the fusion excitation functions for \( ^{40}\text{Ca} + ^{96}\text{Zr} \). The curves show the ECC calculations accounting for different number of neutron rearrangement channels (from 0 to 6n).

FIG. 9. (Color online) Fusion excitation functions for \( ^{40}\text{Ca} + ^{96}\text{Zr} \). The curves show the ECC calculations accounting for different number of neutron rearrangement channels (from 0n to 6n).
change the rest of the model parameters (potential, charges, masses, binding energies, $Q$ values, etc.).

The analysis performed clearly explains why the effect of neutron rearrangement is negligible in the above-mentioned systems (Figs. 5, 6, and 7) while it is well pronounced for the “similar” $^{40}\text{Ca} + ^{96}\text{Zr}$ combination. It is just because of the different properties of their collective excitations (shown in Table II). The correlation between these properties and the observed fusion enhancement is clearly seen. The smaller the rms deformation parameter is (and the higher the energy of the vibrational state), the smaller is the effect expected from the coupling with collective states, and, hence, the larger is the influence of neutron rearrangement on sub-barrier fusion. The weak neutron-channel-caused enhancement for $^{18}\text{O} + ^{74}\text{Ge}$ is additionally because the only positive and rather moderate $Q_{xn}$ value corresponds to the $2n$ channel, while the $Q$ value for the $1n$ channel is negative. Note that in Table II only the target collective properties are shown, because for the studied systems the coupling to the collective states of the targets is more important and has the largest effect. However, in calculations the collective properties of both targets and projectiles are included.

C. Neutron binding energies

In order to clarify the role of neutron binding energies we performed the following calculations. The binding energies of two valent neutrons were varied simultaneously in the target and projectile while preserving all the other properties ($Q$ values, potentials, etc.). In what follows we will discuss the neutron binding energies in the “donor” nucleus only. The influence of neutron binding energies on the sub-barrier fusion of light weakly bound nuclei is shown in Fig. 11. One may see that at energies $\sim 3$ MeV below the barrier the total effect of neutron rearrangement amounts to about two orders of magnitude (the solid curve as compared to the dotted one).

FIG. 10. (Color online) The fusion cross sections (a) and (c) and the experimental barrier distribution functions (b) and (d). The solid and dashed curves show the results of the ECC calculations accounting for and without accounting for the neutron rearrangement channels, respectively. Panels (a) and (b) correspond to the $^{40}\text{Ca} + ^{96}\text{Zr}$ system (with the symbols denoting the experimental data), while panels (c) and (d) are made for the same system but the vibrational properties of $^{40}\text{Ca}$ and $^{96}\text{Zr}$ are substituted by those of $^{60}\text{Ni}$ and $^{100}\text{Mo}$, respectively. The dash-dotted curve on panel (d) is the same as the dashed one on panel (b).

FIG. 11. (Color online) Fusion cross sections for the $^6\text{He} + ^{64}\text{Zn}$ reaction. The data (symbols) are taken from Ref. [25]. The curves are the ECC calculations with (solid and dashed) and without (dotted) neutron rearrangement. The solid curve corresponds to real neutron binding energies in $^6\text{He}$. The dashed curve shows the model calculations in which larger neutron binding energies in $^6\text{He}$ (the same as in $^7\text{Li}$) are assumed. The binding energies are shown in MeV.
If in the calculations one assumes more bound neutrons in $^6\text{He}$ (the same values of $\varepsilon_{\text{n}}$ as in $^9\text{Li}$) while keeping all other properties (potential, $Q$ values, etc.) unchanged, then the effect is reduced to one order of magnitude.

However, in sub-barrier fusion of heavy nuclei the binding energy of the transferred neutron has almost no impact on fusion enhancement! One may see in Fig. 12 that, in the case of $^{40}\text{Ca} + ^{90}\text{Zr}$, the sub-barrier fusion cross sections shift only a little if one assumes neutrons in $^{90}\text{Zr}$ are less bound (as in $^6\text{He}$) or are bound twice as strongly (as in $^{16}\text{O}$).

The reason why the neutron binding energy plays a more important role in the fusion of light nuclei becomes clear from Fig. 13, where two nucleus-nucleus potentials are shown for the $^6\text{He} + ^{64}\text{Zn}$ and $^{40}\text{Ca} + ^{90}\text{Zr}$ systems. For the lighter projectile the Coulomb barrier is lower but wider and the classical turning point corresponds to a larger distance between nuclear surfaces than for the heavier one due to the smaller $Z_1 Z_2$ Coulomb factor. This discrepancy increases with decreasing energy below the Coulomb barrier. The neutron binding energy determines the “compactness” of its wave function and hence the neutron transfer probability (7), which is also dependent on the position of the turning point [the “$D - D_0$” factor in (7)]. Thus, the neutron transfer probability decreases much faster with increasing binding energy in the case of lighter nuclei.

On the other hand, for the lighter systems the same gain in energy (determined by the $Q$ value) has a larger relative influence on each item in the sum for the transmission coefficient (6) than for the heavier one because of the lower Coulomb factor. This explains why the largest enhancement of the sub-barrier fusion cross sections owing to neutron rearrangement is expected (and observed experimentally) for the fusion of light weakly bound nuclei having large positive $Q$ values for neutron transfer.

V. CONCLUSIONS

The role of neutron rearrangement channels in near-barrier fusion reactions is studied within the ECC model. It is shown that the model reproduces the available experimental data quite well. It also gives results close to those of the QCC model when neutron rearrangement does not play a role. In contrast with the generally accepted opinion, we found that having a positive $Q$ value for neutron transfer is not the only factor determining enhancement of the sub-barrier fusion probability.

A noticeable additional enhancement of the sub-barrier fusion cross section (besides that caused by coupling to collective degrees of freedom) can be expected in the following cases: (i) when the system has large positive $Q$ values for transfer of one or/and two neutrons (with the role of $\gamma n$ channels being negligible for $x > 4$) and (ii) when the coupling to the collective states is not so important at sub-barrier energies. The latter is always the case when one of the reaction partners is a light nucleus. In all other cases this is realized for the fusion of spherical nuclei having high energies of the first vibrational state and small values of the rms dynamical deformation. The largest effect from neutron rearrangement is expected for spherical nuclei having magic or nearly magic numbers of protons or/and neutrons.

Additional enhancement of the sub-barrier fusion cross sections takes place for light neutron-rich, weakly bound nuclei due to the smaller binding energies of valent neutrons.

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