Cross sections for the production of superheavy nuclei

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Abstract

Production and studying properties of superheavy (SH) nuclei meet significant experimental difficulties owing to extremely low cross sections of their formation in heavy ion fusion or multinucleon transfer reactions. Accurate predictions of these cross sections along with the corresponding excitation functions are quite desirable for planning and performing experiments of such kind. Study of these cross sections (their dependence on projectile–target combination and energy dependence) gives us an opportunity to investigate complicated dynamics of low-energy heavy ion collisions as well as decay properties of excited SH nuclei.

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1. Introduction

There are three methods for the production of transuranium elements, namely, a sequence of neutron capture and $\beta^-$ decay processes, fusion of heavy nuclei, and multinucleon transfer reactions. The neutron capture process is an oldest (and natural) method for the production of new heavy elements. Strong neutron fluxes might be provided by nuclear reactors and nuclear explosions under laboratory conditions and by supernova explosions in nature. The so-called “fermium gap”, consisting of the short-living fermium isotopes $^{258-260}$Fm located at the beta stability line and having very short half-lives for spontaneous fission, impedes the formation of...
nuclei with $Z > 100$ by the weak neutron fluxes realized in existing nuclear reactors. In nuclear and supernova explosions (fast neutron capture) this gap may be bypassed, if the total neutron flux is high enough. Theoretical models predict also another region of short-living nuclei located at $Z = 106–108$ and $A \sim 270$. A macroscopic amount of long-living SH nuclei located at the island of stability might really be produced in the pulsed nuclear reactors of the next generation, if their neutron fluence per pulse is increased by about 3 orders of magnitude (i.e. up to $10^{20}$ neutrons/cm$^2$), that is sufficient to bypass the both gaps of instability. Detailed consideration of the neutron capture process (including a possibility for the production of SH elements in nature) can be found in [1]. Here we restrict ourselves by analysis of the fusion and multinucleon transfer reactions for the production of SH nuclei.

The corresponding cross sections are extremely low (around one picobarn, i.e., $10^{-36}$ cm$^2$). Moreover, energy dependence of the cross sections for the formation of SH nuclei in fusion reactions is rather narrow (few MeV, see below). This significantly complicates planning and conducting experiments on production and studying properties of SH nuclei. In experiments based on heavy ion fusion reactions target material is irradiated by heavy ions during several weeks (sometime, several months). Incorrect choice of the beam energy (deviation by a few percents from the optimal value) may lead to a total loss of required events. Accurate predictions of the corresponding cross sections (or knowledge them from the previous experiments) are very desirable to perform experiments of such kind.

Cross sections for the production of SH nuclei both in fusion and transfer reactions are themselves of great interest. Their measurements allow one to study complicated dynamics of low-energy heavy ion collisions and decay properties of excited heavy nuclei formed in such processes (competition of light particle evaporation, $\gamma$ emission, and fission).

2. Formation of superheavy nuclei in fusion reactions

Formation process of SH residual nucleus $B$(g.s.), which is the product of light particle evaporation and $\gamma$ emission from an excited compound nucleus $C$, formed in fusion reaction of two heavy nuclei $A_1 + A_2 \rightarrow C \rightarrow B + n, p, \alpha, \gamma$, can be decomposed in the 3 reaction stages shown schematically in Fig. 1.

In the first reaction stage colliding nuclei overcome the Coulomb barrier and come in close contact with overlapped nuclear surfaces. This process competes with elastic and quasi-elastic scattering (including few nucleon transfers) with formation of projectile-like and target-like nuclei. This competition strongly depends on collision energy and impact parameter (angular momentum of relative motion). At sub-barrier energies the probability for nuclei to form the contact configuration is small even at zero angular momentum.
At the second reaction stage the contact configuration of two touching nuclei is transformed into more or less spherical configuration of compound nucleus (CN). In the course of this evolution the heavy nuclear system may, in principle, split again into two fragments \((f_1'\) and \(f_2'\)) before CN is formed. This process is named “quasi-fission”. There are many experimental evidences (see below) pointing out on a strong competition between CN formation and quasi-fission for heavy nuclear systems.

If, nevertheless, CN is formed it has some excitation energy \(E^*\) and angular momentum \(l\) equal to the angular momentum in the entrance channel. Excitation energy of CN depends on collision energy and on the projectile–target combination:

\[
E^*(C) = E + Q_{\text{fus}}^{\text{fus}} = E + [B(C) - B(A_1) - B(A_2)].
\] (1)

Here \(E\) is the beam energy in the center-of-mass system, and \(B(A)\) is the binding energy of nucleus \(A\).

Fission barriers of SH nuclei are rather low and, thus, fission is the dominated decay process of excited CN. To survive in competition with fission, excited SH nucleus \(C\) has to evaporate light particles (mostly neutrons) and emit several \(\gamma\) quanta which remove the excitation energy and angular momentum.

Cross section for the formation of a given residual nucleus \(B(\text{g.s.})\) in fusion reaction can be decomposed over partial waves and written in the form reflecting the three-step formation mechanism (that is a standard method used in the majority of existing models) shown in Fig. 1

\[
\sigma_{\text{EvR}}(E) \approx \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l + 1) P_{\text{cont}}(E, l) P_{\text{CN}}(A_1 + A_2 \rightarrow C; E, l) P_{\text{EvR}}(C \rightarrow B; E^*, l). \] (2)

Here \(P_{\text{cont}}(E, l)\) is the probability for the colliding nuclei to overcome the potential barrier in the entrance channel and form a contact configuration at which the distance between nuclear centers is less than sum of their radii, \(R \leq R_{\text{cont}} = R_1 + R_2\), which is smaller than the radius of the Coulomb barrier \(R_B\) by 2 or 3 fm (see below Figs. 3 and 4).

\(P_{\text{CN}}\) is the probability that the nuclear system will evolve from a configuration of two touching nuclei into a spherical or nearly spherical form of the compound mono-nucleus. The last term in (2), \(P_{\text{EvR}}(C \rightarrow B)\), defines the probability of producing the residual nucleus \(B(\text{g.s.})\) in its ground state in the “cooling” process of CN (evaporation of light particles and \(\gamma\) emission in competition with fission).

Eq. (2) is written in an approximate form because the whole process of CN formation and decay is divided here into 3 individual reaction stages even if connected with each other but treated and calculated separately: (1) overcoming the Coulomb barrier and formation of the contact configuration, (2) formation of the compound mono-nucleus \(A_1 + A_2 \rightarrow C^*\) in competition with quasi-fission, and (3) “cooling” of excited CN (by evaporation of light particles and gamma emission) in competition with fission \(C \rightarrow B(\text{g.s.})\).

A possibility of such a division is justified first of all by different time scales of all the three reaction stages. The time required to overcome the Coulomb barrier and pass the distance \(R_B - R_{\text{cont}}\) does not exceed several units of \(10^{-21}\) s, whereas the characteristic time of neutron emission from a weakly excited CN is at least by two orders of magnitude longer. The intermediate stage of the CN formation is not an entirely independent process: it is closely connected with the initial as well as with the final reaction stages. In particular, pre-compound light particles could be emitted at this stage (though highly improbable) further complicating the whole process. Nevertheless, this reaction stage, namely, its beginning and end, are also well defined in the configuration space of parameters with the help of which the entire process is described,
and hence the use of the separate factor $P_{CN}$ for the modeling of that stage is justified in the calculation of the EvR cross section.

2.1. Experimental study of reaction dynamics of SH element formation

Experimental discrimination and observation of all the reaction stages shown in Fig. 1 is very desirable but not always possible.

To measure the cross section (2) itself one need to separate and count a number of residual nuclei in their ground state. Usually residual nucleus in its ground state (named evaporation residue, EvR) is distinguished from other reaction products by detecting correlated signals from recoil nucleus C (moving in forward direction with well defined velocity $v_{CN} = v_1 \cdot \frac{A_1}{A_1+A_2}$) and from subsequent (delayed) signals from decay products of nucleus $B$(g.s.) ($\alpha$ particles and/or fission fragments).

The number of such events is very small in the case of SH element production ($\sigma_{EvR} \sim 1$ pb), but it brings only technical problems in experimental measurement of this cross section.

If one detects (and sum up) all the fragments $f'_1, f'_2, f_1, f_2$ and EvR then we get the contact (sticking) cross section

$$\sigma_{cont}(E) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l + 1) P_{cont}(E, l).$$  \hspace{1cm} (3)

Very often it is named also “capture” or “sticking” cross section. We prefer to use the term “contact” because “capture” has less defined meaning both experimentally and theoretically (see below boundary conditions for the contact process).

If we detect only EvR and real fission fragments of CN, $f_1$ and $f_2$, then we get the fusion (CN formation) cross section

$$\sigma_{CN}(E) = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l + 1) P_{cont}(E, l) \cdot P_{CN}(E, l).$$  \hspace{1cm} (4)

Unfortunately in collisions of heavy nuclei it is very difficult (in many cases absolutely impossible) to distinguish real fission fragments of CN, $f_1$ and $f_2$ from the quasi-fission fragments $f'_1, f'_2$). For more or less mass symmetric combinations of colliding nuclei these fragments are also quite similar to the projectile-like and target-like fragments $A'_1, A'_2$ formed in deep inelastic scattering. This means that experimental extraction of the fusion and contact cross sections is rather difficult, and one should be careful when experimental data are compared with theoretical calculations.

For light and medium mass systems different reaction channels are distinguished rather easily. Numerous experimental data on fusion reactions of medium mass nuclei or light ions with heavy targets testify that there is no competition between CN formation and quasi-fission at the second reaction stage, i.e., the probability of the CN formation after overcoming the Coulomb barrier and reaching the contact configuration by these nuclei is very close to unity, $P_{CN} \approx 1$ and $\sigma_{cont} = \sigma_{CN}$.

In Fig. 2 the mass-energy distributions of reaction fragments are shown for low-energy collisions of $^{16}$O and $^{40}$Ca with $^{238}$U target [2]. In the case of light projectile $^{16}$O the two dominating reaction mechanisms are clearly visible: (1) Quasi-elastic scattering leading to formation of the projectile-like and target-like fragments, (2) fusion reaction leading to formation of CN (here it is $^{254}$Fm) and its subsequent fission into two fission fragments with more or less equal masses.
\( (A_1 + A_2)/2 \). Their total kinetic energy corresponds exactly to the standard value of kinetic energy released in fission of heavy nuclei.

In collisions of heavier projectile \( ^{40}\text{Ca} \) with \( ^{238}\text{U} \) target the yield of reaction fragments with more or less equal masses \( (A_1 + A_2)/2 \) (which could be interpreted as fission fragments of formed CN) is much less that testifies about much lower fusion probability in this reaction. At the same time, there are many reaction fragments with intermediate masses, in particular, in the region of \( A_1 \sim 80, A_2 \sim 200 \) (these are just quasi-fission fragments \( f_1' \) and \( f_2' \) shown in Fig. 1). These experimental data clearly indicate that the quasi-fission process plays a significant role in low-energy collisions of heavy nuclei significantly decreasing the fusion (CN formation) probability.

Partial wave analysis of reaction mechanisms in collisions of light and medium mass nuclei allows one to conclude that for such systems low partial waves (low impact parameters \( b = l/k \)) contribute completely to the fusion reaction channel, whereas intermediate ones contribute to deep inelastic scattering, and peripheral collisions with largest impact parameters lead to quasi-elastic and pure elastic scattering.

For heavier colliding systems the situation is not so simple. Different reactions mechanisms are strongly overlapped here. It is especially clear for heavy mass symmetric combinations (such as \( \text{Xe} + \text{Xe} \), for example). Even in head-on collisions of these nuclei at above-barrier energy all the reaction mechanisms discussed above take place, and experimentally it is impossible to distinguish deep inelastic scattering of these nuclei from quasi-fission and fusion–fission processes.

3. Contact (capture) cross sections

The first reaction stage and the contact (sticking) cross section are studied very well both experimentally and theoretically. As already mentioned, for medium mass nuclei and for the cases when one nucleus is rather light (roughly \( A_1 < 20 \)) there is no quasi-fission process at all (or its contribution is negligibly small), and after overcoming the Coulomb barrier and sticking two nuclei fuse, that is they form CN which then undergoes to fission or survives depending on its charge and mass and excitation energy. For such combinations of nuclei \( \sigma_{\text{cont}} = \sigma_{\text{CN}} \equiv \sigma_{\text{fus}} \) and the term “fusion” is normally used in the literature for this process. Many hundreds experiments have been performed aimed on study of near-barrier fusion of light and medium mass nuclei, and very precision theoretical models have been developed for description of this process. Today we may not only describe available experimental data but also predict quite accurately the contact cross section for any combination of colliding nuclei if properties of these nuclei are known.
At above-barrier collision energies the capture cross section is close to the geometrical one (slightly squeezed by the repulsive Coulomb field). In Fig. 3 effective potential energy, \( V(r) + \frac{\hbar^2 l(l + 1)}{2\mu r^2} \), fusion cross section and the field of classical trajectories are shown for the case of \(^{12}\text{C} + ^{27}\text{Al}\) collisions. At a given above-barrier energy \( E \) two nuclei come in contact (reach the distance \( r = R_1 + R_2 \) and fuse without tunneling at all partial waves \( l \leq l_{\text{fus}}(E) \) (or at all impact parameters less than \( b_{\text{fus}} = l_{\text{fus}}/k, k = \sqrt{2\mu E/\hbar^2} \)).

The value of \( l_{\text{fus}}(E) \) is easily defined from equality of the effective Coulomb barrier to the center-of-mass energy, \( \frac{\hbar^2 l_{\text{fus}}(l_{\text{fus}} + 1)}{2\mu R_B^2} = E, \) or \( l_{\text{fus}}^2(E) \approx \frac{2\mu R_B^2}{\hbar^2} (E - V^B) \).

For heavy nuclei the probability to overcome the effective potential barrier \( P_{\text{cont}}(l > l_{\text{fus}}) \ll 1 \) and \( P_{\text{cont}}(l < l_{\text{fus}}) \approx 1 \). Thus, the contact cross section (3) can be approximately written as

\[
\sigma_{\text{cont}}(E) = \frac{\pi l_{\text{fus}}}{k^2} \sum_{l=0}^{l_{\text{fus}}} (2l + 1) \approx \pi l_{\text{fus}}^2(E)/k^2 = \pi b_{\text{fus}}^2(E) = \pi R_B^2(1 - V^B/E). \]

It is really close to the geometrical one with accounting of non–straight line trajectories in the Coulomb field. This formula gives the straight line in the \( 1/E \) scale shown in Fig. 3(c).

3.0.1. One-dimensional barrier penetration model

To decrease excitation energy of the SH compound nucleus one should choose the collision energy as low as possible. However at deep sub-barrier energies both the contact cross section and fusion (CN formation) probability sharply decrease. Optimal energies for the production of SH nuclei in fusion reactions were found to be close to the Coulomb barrier. Thus, the contact cross section at near barrier energies should be considered more carefully taking into account quantum tunneling over the barrier.

In Fig. 4 near-barrier experimental [4] and theoretical fusion cross section is shown for \(^{36}\text{S} + ^{90}\text{Zr}\). Theoretical calculation is performed here within the so-called one-dimensional barrier penetration model, in which radial Schrödinger equation is solved for each partial wave with “absorptive” boundary condition at \( r = R_{\text{cont}} \). This means that the flux, which penetrates the Coulomb barrier (shown in the right panel of Fig. 4), is absorbed completely (forming CN) and not reflected from the inner region, i.e. the radial wave function at \( r < R_{\text{cont}} \approx R_1 + R_2 \) is an incoming wave and has not outgoing component reflected from the region \( 0 < r < R_{\text{cont}} \). At large distances \( (r \to \infty) \) this wave function has an ordinary behavior of scattering wave: incoming and outgoing waves. The partial penetration probability \( P(E,l) \) is defined as the ratio of absorptive
and incoming fluxes (see appropriate formula in Section 3.2). Found in such a way contact cross section is shown in Fig. 4(a) by the dashed curve.

As can be seen the one-dimensional barrier penetration model describes properly experimental data at above-barrier energies but strongly underestimate fusion probability at sub-barrier region. The reason for that (ignoring coupling to other degrees of freedom of colliding nuclei) has been understood many years ago. Before describing more sophisticated channel coupling model of sub-barrier fusion (see the next section) note that the penetration probability of one-dimensional barrier can be calculated even more easily (without solving Schrödinger equation) by the use of the Hill–Wheeler formula [5] for the penetration probability of parabolic potential.

Radial dependence of interaction potential of two heavy nuclei can be approximated in the vicinity of the Coulomb barrier by inverted parabola (see the right panel of Fig. 4). Such a parabola is characterized by the oscillator frequency \( \hbar \omega = \sqrt{\frac{\hbar^2 |V''(R_B)|}{\mu}} \) dependent on the width of the barrier. Penetration probability of the parabolic barrier can be estimated by the Hill–Wheeler formula [5]

\[
P^{\text{HW}}(E, l) = \left[ 1 + \exp \left( \frac{2\pi}{\hbar \omega} \left( B(l) - E \right) \right) \right].
\]  

(5)

In this formula the dependence of the effective barrier height (owing to centrifugal potential) is taken into account, \( B(l) = V_B + \frac{\hbar^2 l(l+1)}{2\mu R_B^2} \). For deep sub-barrier energies it is rather easy to take into account some broadening of the barrier and decreasing the value of \( \hbar \omega(E) \) with decreasing energy. For heavy ions the penetration probability calculated by the Hill–Wheeler formula is very close to the explicit value obtained by numerical solution of the Schrödinger equation.

3.1. Interaction potential of two deformable rotating nuclei

Interaction potential of two colliding nuclei depends on the distance between surfaces, \( \xi \), which, in its turn, depends not only on relative distance between nuclear centers but also on dynamic deformations of nuclei and on their orientation (see Fig. 5).

Potential energy of two (deformable and rotating) nuclei is a sum of Coulomb, nuclear and deformation energies
Fig. 5. Schematic picture of two deformed nuclei rotating in the reaction plane.

Fig. 6. Interaction potentials of $^{16}$O + $^{154}$Sm (left) depending on orientation of statically deformed $^{154}$Sm nucleus ($\beta_{0.8}^{u.s.} = 0.3$, $\beta_{2}^{u.s.} = 0.1$) and of $^{40}$Ca + $^{90}$Zr (right) depending on quadrupole dynamic deformations of two nuclei. For the second case the minimal value of the fusion barrier (saddle point) is marked by the open circle.

$$V_{12}(r, \beta_1, \theta_1; \beta_2, \theta_2) = \frac{Z_1Z_2e^2}{r} \left[ 1 + \frac{3R_1^2}{5r^2} \beta_1 Y_{20}(\theta_1) + \frac{3R_2^2}{5r^2} \beta_2 Y_{20}(\theta_2) \right]$$

$$+ \frac{1}{2} \left[ C_1 \beta_1^2 + C_2 \beta_2^2 \right].$$

Formula (6) is valid for $r > R_1 + R_2$ and for rather small quadrupole deformations $\beta_1$ and $\beta_2$ of both nuclei (with the corresponding rigidities $C_1$ and $C_2$). More detailed expression for interaction potential can be found in [6].

From Eq. (6) it is clear that the one-dimensional barrier penetration model is too simplified to describe properly the motion of nuclei in the entrance channel of fusion reaction. In Fig. 6 the interaction potentials for $^{16}$O + $^{154}$Sm (depending on orientation of statically deformed $^{154}$Sm nucleus) and for $^{40}$Ca + $^{90}$Zr (depending on quadrupole dynamic deformations of two nuclei) are shown. As can be seen, nuclei have to overcome multidimensional potential barrier to reach the contact configuration. The corresponding penetration probability can be calculated within the quantum or semi-empirical channel coupling models discussed below.
3.2. Quantum channel coupling approach [7,6]

Hamiltonian of two deformable nuclei rotating in reaction plane is written as

\[
H = -\frac{\hbar^2 \nabla^2}{2\mu} + V_{12}(r, \beta_1, \theta_1; \beta_2, \theta_2) + V_{12}^{\text{nc}}(r, \beta_1, \theta_1; \beta_2, \theta_2) + \sum_{i=1,2} \frac{\hbar^2 l_i^2}{2\Omega_i} + \sum_{i=1,2} \sum_{\lambda \geq 2} \left( -\frac{1}{2d_{i\lambda}} \frac{\partial^2}{\partial \beta_{i\lambda}^2} + \frac{1}{2} c_{i\lambda} \beta_{i\lambda}^2 \right).
\]

(7)

where \(\Omega_i\) are the moments of inertia, \(d_i\) are the inertia parameters of surface vibrations, and \(\lambda = 2, 3, \ldots\) is the multipolarity of these vibrations.

Decomposing the total wave function over the partial waves

\[
\Psi_k(r, \bar{\beta}; \alpha) = -\frac{1}{kr} \sum_{l=0}^{\infty} i^l e^{i\sigma_l} (2l + 1) \chi_l(r, \alpha) P_l(\cos \bar{\beta})
\]

(8)

one gets the Schrödinger equation for the partial wave function

\[
\frac{\partial^2}{\partial r^2} \chi_l(r, \alpha) + \frac{2\mu}{\hbar^2} \left[ E - V_{12}(r, \alpha) - \frac{\hbar^2 l(l+1)}{2\mu r^2} H_{\text{int}}(\alpha) \right] \chi_l(r, \alpha) = 0.
\]

(9)

Here \(\alpha = \beta_{i\lambda}, \theta_i\) are the collective degrees of freedom (surface deformations and angles of rotation of colliding nuclei), \(H_{\text{int}}(\alpha)\) is the corresponding Hamiltonian: \(H_{\text{int}} \varphi_\nu = \varepsilon_\nu \varphi_\nu\).

Decomposition of the partial waves over the complete set of the eigenfunctions of \(H_{\text{int}}\), \(\chi_l(r, \alpha) = \sum_\nu y_{l\nu}(r) \varphi_\nu(\alpha)\), gives the following set of the coupled Schrödinger equations for the channel radial wave functions \(y_{l\nu}(r)\)

\[
y''_{l\nu} + \frac{2\mu}{\hbar^2} \left[ E_\nu - V_{\nu\nu}(r) - \frac{\hbar^2 l(l+1)}{2\mu r^2} \right] y_{l\nu} = \sum_{\mu \neq \nu} \frac{2\mu}{\hbar^2} V_{\nu\mu}(r) y_{l\mu},
\]

(10)

which can be solved numerically. Here \(E_\nu = E - \varepsilon_\nu\), \(\varepsilon_\nu\) is the nucleus excitation energy in the channel \(\nu\), and \(V_{\nu\mu}(r) = \langle \varphi_\nu | V_{12}(r, \alpha) | \varphi_\mu \rangle\) are the coupling matrix.

To solve Eqs. (10) the boundary conditions should be formulated reflected the physics process under study. We are interested here in the probability for colliding nuclei to overcome the Coulomb barrier and reach the contact configuration, that is we need to calculate incoming flux at \(r = R_{\text{cont}} < R_B\) in all the channels \(\nu\) excited on the way to this configuration. Thus the partial wave function \(y_{l\nu}(r)\) has only incoming component at \(r < R_{\text{cont}}\) and ordinary behavior of scattering wave at \(r \to \infty\): incoming and outgoing waves in the elastic channel \(\nu = 0\) and outgoing waves in all other channels

\[
y_{l\nu}(r \to \infty) \approx i \frac{1}{2} \left[ h_l^{-}\left( \eta_\nu, k_\nu r \right) \cdot \delta_{l0} - \left( \frac{k_0}{k_\nu} \right)^{1/2} S_{l0\nu} \cdot h_l^{+}(\eta_\nu, k_\nu r) \right].
\]

(11)

Here \(k_\nu^2 = \frac{2\mu E_\nu}{\hbar^2}, \eta_\nu = \frac{k_\nu Z_1 Z_2 e^2}{2E_\nu}\) is the Sommerfeld parameter, \(h_l^{(\pm)}\) are the partial Coulomb wave functions with the asymptotic behavior \(\exp(\pm i x_{l\nu})\), \(x_{l\nu} = k_\nu r - \eta_\nu \ln 2k_\nu r + \sigma_{l\nu} - l\pi/2, \sigma_{l\nu} = \arg \Gamma(l + 1 + i\eta_\nu)\) is the Coulomb partial phase shift, and \(S_{l0\nu}\) are the partial scattering matrix elements. Similar expression is written also for the closed channels \((E_\nu < 0 )\) with imaginary argument of the function \(h_l^{(+)}\).
The sought-for partial transmission probability is defined by the ratio of the passed and incoming fluxes

$$P_{\text{cont}}(E, l) = \sum_{\nu} \frac{j_{l,\nu}}{j_0},$$

(12)

where $j_{l,\nu} = -\frac{i\hbar}{2\mu} \left( y_{l,\nu} \frac{d y_{l,\nu}^*}{dr} - y_{l,\nu}^* \frac{d y_{l,\nu}}{dr} \right)|_{r=R_{\text{cont}}}$ is the partial flux in the channel $\nu$, and $j_0 = \hbar k_0 / \mu$.

This method of quantum channel coupling calculation of the contact cross section is realized in the CCFULL fusion code [7] as well as in the NRV CC fusion code [8] (both codes give absolutely similar results). In the last one an effective algebraic method [6] is used for numerical solution of a set of coupled Schrödinger equations (10). This method has no limitation on the number of coupled channels and allows one to calculate contact cross sections of very heavy nuclei used for synthesis of super-heavy elements.

3.3. Empirical channel coupling approach

Coupling to surface vibrations or rotation of statically deformed nuclei lead to multidimensional Coulomb barriers which colliding nuclei have to overcome to reach the contact configuration (see Fig. 6). Within the empirical channel coupling model (ECC) proposed in [9] the penetration probability is calculated by averaging of the Hill–Wheeler formula (5) over the dynamic surface deformations and/or orientations of colliding nuclei taking into account dependence of the Coulomb barrier height on these parameters: $B(\beta_{1,\lambda}, \beta_{2,\lambda}; \theta_1, \theta_2)$.

Collision dynamics of spherical nuclei depends mainly on the coupling to their surface vibration degrees of freedom. Therefore, the partial penetration probability should be averaged over the deformation-dependent barrier height

$$P_{\text{cont}}(E, l) = \int f(B) P^{\text{HW}}(E, l; B) dB,$$

(13)

where dynamic deformations are assumed to take place along inter-nuclear axis, and $f(B)$ is the empirical dynamic barrier distribution function [9] normalized to unity: $\int f(B) dB = 1$. It is not the same as the conventional ("experimental") barrier distribution function $D(B) = d^2(E \cdot \sigma_{\text{fus}}) / dE^2$ [10]. For one-dimensional (no-coupling) barrier model $f(B) = \delta(B - B_0)$ whereas $D(B)$ in this case is still a smooth function with one peak at $B = B_0$ and with a width of about $0.56\hbar \omega_B$ (see, for example, [6]).

For the realistic multidimensional barrier (simulating channel coupling) one can use the Gaussian approximation for $f(B)$

$$f(B) = N_B \cdot \exp \left( -\left[ \frac{B - B_0}{\Delta_B} \right]^2 \right),$$

(14)

where $B_0 = (B_1 + B_2)/2$. Here $B_2$ is the height of the barrier at zero dynamic deformation of colliding nuclei, $B_1$ (< $B_2$) is the height of the saddle point (shown by the circle on the right panel of Fig. 6) calculated with realistic vibrational properties of nuclei, i.e., with the surface stiffness parameters obtained from the experimental values of the excited vibrational states, $\Delta_B = (B_2 - B_1)/2$, and $N_B$ is the normalization coefficient.
In the case of collisions of statically deformed nuclei ($\beta_i^{g.s.} \neq 0$) the Coulomb barrier height and its position depend on mutual orientation of nuclei, then the averaging over the orientations of both nuclei is required

$$P_{\text{cont}}^{\text{rot}}(E, l) = \frac{1}{4} \int_0^\pi \int_0^\pi P^{\text{HW}}(E, l; B(\beta_1^{g.s.}, \beta_2^{g.s.}; \theta_1, \theta_2)) \times \sin \theta_1 \sin \theta_2 \, d\theta_1 \, d\theta_2. \tag{15}$$

Eq. (15) assumes uniform distribution over the initial orientations (the corresponding dynamic barrier distribution function is unity in the region of integration).

Both quantum (QCC) and empirical (ECC) channel coupling models give rather similar results and describe quite properly near and sub-barrier contact cross sections (which are equal to fusion cross sections for light and medium mass nuclei). In Fig. 7 the results of such calculations are shown for fusion of deformed and spherical nuclei.

3.4. Role of neutron transfers

At approaching stage an exchange (rearrangement) of neutrons between nuclei is also possible. One could be thought that neutron rich nuclei can fuse much easier at near and sub barrier energies. However, experiments show that the neutron excess itself does not help nuclei to fuse. Typical example is shown in Fig. 8. These data demonstrate that at near and above barrier energies fusion cross sections are practically the same for different isotopes, whereas at sub-barrier energies fusion probability is enhanced just for the combinations with one neutron rich and one neutron deficient isotope, $^{40}\text{Ca} + ^{48}\text{Ca}$, $^{40}\text{Ca} + ^{96}\text{Zr}$, etc.

Explanation of this effect was done in [15] where coupling to neutron transfer channels was included in the empirical channel coupling scheme.

As shown above, surface vibrations and rotation of deformed nuclei at approaching stage give a gain in the barrier penetrability owing to decrease of the height of the Coulomb barrier for specific configurations. Neutron rearrangement with positive $Q$ values gives a gain in sub-barrier penetrability owing to increase of relative motion kinetic energy.
Fig. 8. Fusion cross section of \(^{40}\text{Ca}\) and \(^{48}\text{Ca}\) with \(^{48}\text{Ca}\) [12] (left panel), fusion cross section of \(^{40}\text{Ca} + ^{96}\text{Zr}\) [13] and \(^{48}\text{Ca} + ^{96}\text{Zr}\) [14] (right panel).

For example, when neutrons are transferred from \(^{96}\text{Zr}\) to calcium one has a gain in energy owing to stronger binding energies of these neutrons in acceptor nucleus \((^{40}\text{Ca} + ^{96}\text{Zr} \rightarrow ^{44}\text{Ca} + ^{92}\text{Zr} + 9.6 \text{ MeV} \rightarrow \text{CN})\). Part of this extra energy might go to the relative motion kinetic energy helping colliding nuclei to overcome the Coulomb barrier (see below). This “energy lift” mechanism will work if the neutron rearrangement takes place before nuclei reach the Coulomb barrier. Calculations performed within the time-dependent Schrödinger equation [16] fully confirm such behavior of the valence neutrons. Results of these calculations are shown in Fig. 9 for the case of \(^{40}\text{Ca} + ^{96}\text{Zr}\) near barrier collisions.

It is rather difficult to include the nucleon transfer channels to the rigorous quantum channel-coupled approach. The problem appears when, following the standard channel-coupled method, the total wave function is decomposed over collective (rotation and/or vibrational) states and simultaneously over neutron transfer states. In such a decomposition overcomplete and non-orthogonal basis functions are used, that requires special complicated technique or some simplifying assumptions. Moreover, in medium mass and heavy nuclei single particle states are spread over numerous exited states of these nuclei (with appropriate spectroscopic factors) which hardly can be included in any microscopic CC scheme.

However the neutron rearrangement channels can be easily included [15] in the ECC model of fusion reactions described above. The total penetration probability (which takes into account the rearrangement of neutrons) can be estimated again by formulas (13) or (15) in which \(P^{\text{HW}}(E, l)\) is replaced now by the following expression

\[
\tilde{P}^{\text{HW}}(E, l; B) = \frac{1}{N_{tr}} \sum_{x=0}^{4} \int_{-E}^{Q_{xn}} \alpha_k(E, l, Q) P^{\text{HW}}(E, l; B) dQ,
\]

where \(Q_{xn}\) is the \(Q\) value of the ground-to-ground transfer of \(x\) neutrons (if transferred neutrons populate excited states then \(Q < Q_{xn}\)). The probability of the transfer of \(x\) neutrons with a given \(Q\) value (less than \(Q_{xn}\)) can be estimated in semiclassical approximation

\[
\alpha_k(E, l, Q) = N_k \exp \left( -C Q^2 \right) \exp \left( -2\kappa \left[ D(E, l) - D_0 \right] \right),
\]
where $\kappa = \kappa(\epsilon_1) + \kappa(\epsilon_2) + \cdots + \kappa(\epsilon_k)$ for sequential transfer of $k$ neutrons, $\kappa(\epsilon_i) = \sqrt{2\mu_n\epsilon_i/h^2}$ and $\epsilon_i$ is the binding energy of the $i$-th transferred neutron, $D(E,l)$ is the distance of the closest approach along the Coulomb trajectory with angular momentum $l$, $D_0 = R^{(n)}_1 + R^{(n)}_2 + d_0$, $R^{(n)}_i = r^{(n)}_0 A^{1/3}$ are the orbit radii of the valence (transferred) neutrons of colliding nuclei ($r^{(n)}_0$ and $d_0$ are adjustable parameters), $N_{tr}$ is the normalization constant, $\alpha_0 = \delta(Q)$, $C = R_B\mu_{12}/4\kappa h^2 B$ and $\mu_{12}$ is the reduced mass of two nuclei in the entrance channel.

As can be seen from (16) enhancement of the fusion probability may appear at sub-barrier energies if rearrangement of neutrons leads to a gain in energy (positive $Q$ values). In the reactions with negative $Q$ values the neutron rearrangement in the entrance channel does not influence the total fusion cross section because the penetration probability $P^{\text{HW}}(E + Q; l; B)$ becomes smaller for negative $Q$. In this case $\alpha_0$ (no neutron transfer) is the only non-vanishing term in sum (16). The probability of the neutron rearrangement depends on the binding energy of the transferred neutron, see Eq. (17). The coefficients $\alpha_k$ decrease fast with increase of the binding energies in the “donor” nucleus. Because of the fast decrease of $\alpha_k$ with increasing the number of transferred neutrons $k$, only 1n and 2n transfer channels with positive $Q$ values were found to play a significant role. The experiments indicate (see, for example, [17]) that simultaneous transfer of two neutrons might be enhanced by factor 2 or 3 as compared to independent (subsequent) transfer of these neutrons.

The ECC model with incorporated coupling to neutron transfer channel (16) was found to be in excellent agreement with experimental data. Recent systematic study of the problem [18]
shows that all available experimental data on near barrier fusion reactions are described quite properly within this model. An example of such agreement is shown in Fig. 10. Coupling of relative motion to the surface vibrations of target nuclei describes quite well the fusion cross sections for the $^{40}$Ca + $^{90}$Zr and $^{16}$O + $^{60}$Ni reactions (having negative $Q$ values for neutron transfers), but it is insufficient to describe additional sub-barrier fusion enhancement for the $^{40}$Ca + $^{96}$Zr and $^{18}$O + $^{58}$Ni reactions caused by neutron rearrangements with positive $Q$ values at the intermediate reaction stage.

3.5. Concluding remarks on contact cross sections

Approaching stage of heavy ion collisions and the contact cross sections (which are equal to fusion cross sections for light and medium mass nuclei) are well studied both experimentally and theoretically and understood quite properly. Dynamic deformations and rotation of colliding nuclei significantly enhance sub-barrier fusion probability. Neutron excess itself does not increase the contact cross section. Neutron rearrangement with positive $Q$ value enhances the barrier penetrability at sub-barrier energies but not at above-barrier energies. Here only 1n and 2n transfer channels play a noticeable role. In general, one may conclude that available CC theoretical models describe quite properly experimental data and have a good predictive power for estimation of the contact cross section for any combination of nuclei. Accuracy of such predictions depends on our knowledge of the properties of colliding nuclei and does not exceed factor 2 or 3 for known nuclei at near-barrier collision energies used for synthesis of SH elements. Note, that for the practical applications (calculation of the cross sections) we use the empirical channel coupling model. At the same time, the existing approximations to the CC approach with [18] and without [20] accounting for neutron rearrangement could be also useful for data analysis and/or better qualitative understanding of the underling physics.
4. Decay of excited SH nucleus and its survival probability

Let us consider now the last stage of SH nucleus formation, the “cooling process” of excited CN by evaporation of light particles (mostly neutrons to save the SH element) and emission of gamma rays. This process competes with dominating fission decay of excited heavy CN.

The survival probability of the excited compound nucleus \( C(E^*, J) \) in the process of its cooling by means of neutron evaporation and \( \gamma \)-emission in the competition with fission and emission of light charged particles \( (C \rightarrow B + xn + N\gamma) \) is estimated usually within the statistical model of atomic nuclei [21]. The partial decay widths of the compound nucleus for the evaporation of the light particle \( a(= n, p, \alpha, \ldots) \), emission of \( \gamma \)-rays of multipolarity \( L \), and fission are given by the following simple expressions.

\[
\Gamma_{C \rightarrow B+a}(E^*, J) = g^{-1} \sum_{l,j} T_{lj}(e_a) \left[ E^* - E_a^{\text{sep}} \right] \rho_B(E^* - E_a^{\text{sep}} - e_a, I; \beta_2^{g.s.}) \cdot de_a,
\]

\[
\Gamma_\gamma(E^*, J) = g^{-1} \sum_{l=|J-L|}^{E^*} f_L(e_\gamma) \cdot e_\gamma^{2L+1} \cdot \rho_C(E^* - e_\gamma, I) \cdot de_\gamma,
\]

\[
\Gamma_f(E^*, J) = g^{-1} \cdot \frac{\hbar \omega_B}{T} \left( 1 + x^2 - x \right) \int_0^{E^*} T_{\text{fis}}(e, J) \cdot \rho_C(E^* - e, J, \beta_2^{sd}) \cdot de.
\]

Here \( g = 2\pi \rho_C(E^*, J) \), \( \rho_A(E^*, J) \) is the level density of the nucleus \( A \) with the excitation energy \( E^* \) and spin \( J \), \( T_{lj}(e_a) \) is the penetration probability of the Coulomb and centrifugal barriers by the light particle \( a \) emitted from the nucleus \( C \).

Assuming that the electric dipole radiation \((L = 1)\) dominates in high-energy \( \gamma \)-emission, one may use the following strength function \( f_{E1} = 3.31 \times 10^{-6} \) (MeV\(^{-1}\)) \( \cdot \frac{(A-Z)Z}{A} \cdot \frac{e_\gamma \Gamma_0}{(E_0^g - e_\gamma)^2 + (e_\gamma \Gamma_0)^2} \) [22], with the resonance energy \( E_0 = 167.23 / A^{1/3} \cdot \sqrt{1.959 + 14.074 \cdot A^{-1/3}} \) and \( \Gamma_0 \approx 5 \text{ MeV} \) [23] for heavy nuclei.

For the fission width one may use the Kramers correction, which takes into account the influence of nuclear viscosity \( \eta \) on the fission probability [24,25], \( x = \frac{\eta}{2\omega_0} \). Here \( \omega_0 \) and \( \omega_B \) are the characteristic frequencies of parabolic approximations of the compound nucleus potential energy depending on the deformation near the ground state and near the saddle point of the fission barrier. The use of the Kramers factor allows one to match the fission rate (width) calculated by Eq. (20) and the quasistationary fission rate obtained in dynamical calculations (see, e.g., [26–28]). The appearance of the temperature in the denominator of Eq. (20) is due to the fact that the Bohr–Wheeler formula for the fission width \( g^{-1} \int_0^{E_f^*} T_{\text{fis}}(e) \rho_C(E^* - e)de \) is proportional to \( T \cdot \exp(-B_f/T) \) at high excitation energies, whereas correct asymptotic value should be proportional to \( \omega_B \cdot \exp(-B_f/T) \) [25]. Note that this factor is not so important for the excitation energies considered below: \( E^* \sim 10–50 \text{ MeV} \) \((T \sim 0.7–1.4 \text{ MeV})\). For very low excitation energies Eq. (20) seems to be not valid and then the standard Bohr–Wheeler formula is more appropriate.
The value of nuclear viscosity is not yet well determined. Experimental and theoretical estimations yield the values of \( \eta \) in a rather wide range of \((1-30) \cdot 10^{21} \, \text{s}^{-1}\), and indicate that viscosity increases with nuclear temperature \( T = \sqrt{E^*/a} \), where \( a \) is the level density parameter (see, for example, [29]). In our calculations (shown below) we used the expression \( \eta = (1 + cT^2) \cdot 10^{21} \, \text{s}^{-1} \) for nuclear viscosity with \( c = 1 \, \text{MeV}^{-2} \).

\[
T_{\text{fis}}(E, J) = \left\{ 1 + \exp\left[ -\frac{2\pi}{\hbar\omega B} \left( E - B_{\text{fis}}^*(E^*, J) \right) \right] \right\}^{-1}
\]

is the penetrability of the fission barrier, \( B_{\text{fis}}(E^*, J) = B_0(E^*, J) - \left( \frac{\hbar^2}{2\Delta_{gs}^s} - \frac{\hbar^2}{2\Delta_{sd}^s} \right) J(J + 1) \) is the height of the fission barrier of rotating nucleus, \( \Delta_{gs}^s, \Delta_{sd}^s = k_B M R^2 \cdot \left( 1 + \beta_2^{g.s., sd} \right) \) are the moments of inertia of fissioning nucleus in its ground state and at the saddle point configuration, where \( k \approx 0.4 \), \( B_0 = B_{LD} - \delta W \cdot e^{-\gamma_D E^*} \), \( B_{LD} \) is the LDM fission barrier, \( \delta W \) is the shell correction to the nucleus ground state. The shell correction for the saddle point configuration is also not equal to zero, but it less than \( \delta W_{g.s.} \) and we ignore it here.

\( \gamma_D \) is the damping parameter describing the decrease of the influence of the shell effects on the level density with increasing the excitation energy of the nucleus. The value of this parameter is especially important in the case of super-heavy nuclei, the fission barriers of which are determined mainly just by the shell corrections to their ground states. From analysis of experimental data it was derived the value \( \gamma_D = 0.06 \, \text{MeV}^{-1} \) for this parameter [30].

It should be noted here that the calculation of the fission width and, in particular, of the fission barrier is the most uncertain part of the statistical model. The fission width calculated within the statistical model is an approximation of true dynamical calculations of this quantity. As was said above, the Kramers factor is one of the corrections aimed at the matching of the dynamical and statistical fission widths. A special attention should be also paid to the value of the fission barrier as well as the saddle point deformation, which are calculated here in a somewhat approximative way. One may alternatively consider the fission barriers obtained in more sophisticated calculations within different modern approaches (e.g., [31–33]). At the same time, since the fission barriers predicted by different approaches may substantially deviate from each other, especially for nuclei beyond the experimentally well-explored region (for example, in the region of super-heavy nuclei), one should carefully verify the model on the existing data. However even this cannot guarantee the full reliability of the model being applied to experimentally unknown domain.

For the state density, which is the main part of (18)–(20), we used the formula [34]

\[
\rho(E, J; \beta_2) = \text{const} \cdot K_{\text{coll}}(\beta_2) \cdot \frac{2J + 1}{E^2} \cdot \exp(2\sqrt{a} \cdot \sqrt{E - E_{\text{rot}}(J)}),
\]

where \( E = E^* - \delta, \delta = 0, \Delta, \) or \( 2\Delta \) for odd–odd, odd–even, and even–even nuclei, \( \Delta = 11/\sqrt{A} \, \text{MeV} \), \( K_{\text{coll}} \) is the collective enhancement factor, and the level-density parameter \( a = a_0 \left[ 1 + \frac{1 - \exp(-\gamma_D E_{\text{int}})}{E_{\text{int}}} \right] \), \( E_{\text{int}} = E - E_{\text{rot}}(J), E_{\text{rot}} = \frac{\hbar^2}{2\Delta_{gs}^s} J(J + 1) \). The asymptotic parameter \( a_0 = 0.073A + 0.095B_5(\beta_2)A^{2/3} \, \text{MeV}^{-1} \) is taken from [35] with a dimensionless surface factor \( B_5 \) from [36].

Rotational bands of deformed nuclei bring the main contribution to the collective enhancement in the level density. For spherical nuclei the collective enhancement is smaller and caused by vibrational excitations. In [37] it was proposed to use \( K_{\text{rot}} = \frac{3\sqrt{T}}{\hbar^2} \) for deformed nuclei and \( K_{\text{rot}} = 1 \) for spherical ones, where \( T \) is the nuclear temperature and \( \hbar \) is the rigid body moment of inertia perpendicular to the symmetry axis. From the analysis of experimental data on the
fission of near-spherical nuclei it was found in [38] that the “borderline” between deformed and almost spherical nuclei is somewhere at \( |\beta| \approx 0.15 \). \( K_{\text{rot}} \) changes very sharply from the value of about 150 (at \( T \sim 1 \text{ MeV} \)) to 1 when this critical deformation is passed. This sharp change may be smoothed by the function \( \phi(\beta_2) = \left[ 1 + \exp \left( \frac{\beta_2 - \beta_2^0}{\Delta \beta_2} \right) \right]^{-1} \), where \( \beta_2^0 \approx 0.15 \) and \( \Delta \beta_2 \approx 0.04 \) [39]. Following [38] we assume that for spherical nuclei the disapperearing rotational enhancement factor has to be replaced by a vibrational factor \( K_{\text{vib}} \). Its value (\( \approx 1–10 \)) is much lower than \( \frac{3\Delta T}{\hbar^2} \).

It strongly depends on proton and neutron numbers and is not as clear as the value of \( K_{\text{rot}} \). Finally one may use the following approximate formula [39] for the collective enhancement factor, which smoothly changes from the large value \( \frac{3\Delta T}{\hbar^2} \) for well deformed nuclei to the lower value \( K_{\text{vib}} \) for spherical nuclei.

\[
K_{\text{coll}}(\beta_2) = \frac{3\Delta T}{\hbar^2} \phi(\beta_2) + K_{\text{vib}}[1 - \phi(\beta_2)]
\]

(22)

In fact, the survival probability of a weakly excited compound nucleus depends only on the ratio \( \frac{\Gamma_n}{\Gamma_f} \), i.e., roughly speaking, on the ratio \( \frac{\rho_B(E - E_\text{sep}, \beta_2^\text{g.s.})}{\rho_C(E - B_{\text{g.s.}}, \beta_2^\text{g.s.})} \). It means that the collective enhancement factor does not influence at all the survival probability of deformed compound nuclei because in this case \( K_{\text{coll}}(\beta_2^\text{g.s.}) \approx K_{\text{coll}}(\beta_2^\text{g.s.}) \approx \frac{3\Delta T}{\hbar^2} \) and they practically cancel each other.

For spherical nuclei the ratio \( \frac{\Gamma_n}{\Gamma_f} \) is proportional to \( \frac{K_{\text{vib}}(\beta_2^\text{g.s.})}{K_{\text{rot}}(\beta_2^\text{g.s.})} \), i.e., the collective enhancement factor can here significantly reduce the survival probability, and the dependence of \( K_{\text{coll}} \) on the deformation plays an important role.

Estimation of the total probability for the formation of a cold residual nucleus after emission of \( x \) neutrons and \( N \) gamma rays (\( C \to B + xn + Ng \), i.e., survival probability \( P_{\text{EvR}}(C \to B) \)) is usually performed within numerical calculations based on the analysis of the multi-step evaporation cascade [40–42].

In the case of rather small number of evaporated light particles one may use an “explicit calculation” of such probability [39]. In particular, survival probability in the channel with evaporation of \( x \) neutrons is given by the following formula with \( x \) enveloped integrals over kinetic energies of these neutrons

\[
P_{\text{EvR}}(C \to B + xn)\nonumber = \int_0^{E_0^* - E_n^\text{sep}(1)} \frac{\Gamma_n}{\Gamma_{\text{tot}}} (E_0^*, J_0) P_n(E_0^*, e_1) de_1 \int_0^{E_1^* - E_n^\text{sep}(2)} \frac{\Gamma_n}{\Gamma_{\text{tot}}} (E_1^*, J_1) P_n(E_1^*, e_2) de_2 \times \cdots \times \int_0^{E_x^* - E_n^\text{sep}(x)} \frac{\Gamma_n}{\Gamma_{\text{tot}}} (E_{x-1}^*, J_{x-1}) P_n(E_{x-1}^*, e_{x-1}) \cdot G_{N\gamma}(E_x^*, J_x \to g.s.) de_x
\]

(23)

Here \( \Gamma_{\text{tot}} = \Gamma_n + \Gamma_f + \gamma_p + \cdots \), \( E_n^\text{sep}(k) \) and \( e_k \) are the binding and kinetic energies of the \( k \)-th evaporated neutron, \( E_k^* = E_0^* - \sum_{i=1}^{k} [E_n^\text{sep}(i) + e_i] \) is the excitation energy of the residual nucleus after the emission of \( k \) neutrons, \( P_n(E^*, e) = C\sqrt{e} \exp(-e/T(E^*)) \) is the probability for the evaporated neutron to have energy \( e \), and the normalizing coefficient \( C \) is found from the
condition $\int_0^{E^*} P_n(E^*, \gamma) dE = 1$. The quantity $G_{N\gamma}$ defines the probability that the remaining excitation energy and angular momentum are taken away by $\gamma$-emission after the evaporation of $x$ neutrons. It can be approximated by the expression

$$G_{N\gamma}(E^*, J \rightarrow g.s.) = \prod_{i=1}^{N} \frac{\Gamma_{\gamma}(E^*_i, J_i)}{\Gamma_{\gamma}(E^*_1, J_1)}$$

(24)

where $E_i^* = E^* - (i - 1)\langle e_\gamma \rangle$, $J_i = J - (i - 1)\langle e_\gamma \rangle$ is the average energy of a dipole $\gamma$-quantum, and the number of emitted $\gamma$-quanta $N$ is determined from the condition $E_N^* < B_{\text{fis}}$, assuming that at energies lower than the fission barrier the fission probability is very small as compared with $\gamma$-emission and $\Gamma_\gamma/\Gamma_{\text{tot}} \approx 1$ (see below Fig. 12). Numerical calculations show that a choice of the average energy of the emitted $\gamma$-quanta $\langle e_\gamma \rangle$ in the range of 0.1–1.0 MeV weakly influences the final results in all the cases except for the 0$n$ fusion channel, the cross section of which is negligibly small in the reactions considered here. Similar formulae can be easily wrote for the channels with evaporation of different light particles, for example, $xn$, $yp$ and $z\alpha$. In this case one need just to replace “neutron quantities” with the corresponding particle in appropriate integrals ($E^*_{n} \rightarrow E^*_{p}$, $\Gamma_{\gamma} \rightarrow \Gamma_{p}$, etc.). Note that evaporation of protons and/or $\alpha$ particles noticeably competes with evaporation of neutrons only for light and medium mass CN, but not for SHE (see below the corresponding decay widths).

As an illustration of Eq. (23) in Fig. 11 decay of excited $^{256}\text{No}$ nucleus (with evaporation of 2 neutrons and emission of several gamma rays) is shown schematically. This nucleus is formed, for example, in fusion reaction of $^{48}\text{Ca}$ with $^{208}\text{Pb}$. At near barrier collision energy ($V_B \approx 176$ MeV) excitation energy of CN $^{256}\text{No}$ is about 22 MeV, sufficient for evaporation of 2 neutrons.

Decay widths of excited nucleus $^{256}\text{No}$ and survival probabilities in different exit channels are shown in Fig. 12. The height of the fission barrier of this nucleus is rather low ($B_{\text{LD}} = \ldots$).
Fig. 12. Decay widths (in rel. units) and survival probabilities of excited $^{256}$No nucleus in the channels with evaporation of 1, 2, and 3 neutrons.

1.24 MeV and $\delta W = -4.5$ MeV), whereas neutron separation energy $E_{\text{sep}} = 7.1$ MeV. As a result, at excitation energies higher than 6 MeV the fission channel dominates. Probability for neutron evaporation is less by 1 or 2 orders of magnitude (depending on $E^*$). Probability for evaporation of charged particles (protons or $\alpha$ particles) is negligibly low. At excitation energy less than $E_{\text{sep}}$ evaporation of neutrons is impossible, and only gamma emission competes with fission. At $E^* < B_f$ gamma emission dominates.

Instead of rather complicated calculation of several enveloped integrals (23). An estimation of CN decay probabilities into all possible exit channels can be performed within the Monte-Carlo approach. For a given excitation energy and angular momentum of CN one of possible decay $i$ ($i = f, n, p, \alpha, \gamma$) is chosen randomly in accordance with its probability $\Gamma_i / \Gamma_{\text{tot}}$. If it is not fission ($i \neq f$) then the same procedure is performed for the next (daughter) nucleus with excitation energy reduced by the separation and kinetic energies of evaporated particle. This process of evaporation cascade is traced up to the step at which excitation energy of daughter nucleus becomes less than its fission barrier. This nucleus defines specific decay channel of CN. The process is repeated many times and the probability for formation of the “cold” residual nucleus, for example, in the channel with evaporation of $x$ neutrons, $y$ protons and $z$ $\alpha$ particles is defined as $N(xn, yp, z\alpha) / N_{\text{tot}}$ where $N(xn, yp, z\alpha)$ is the number of obtained events of such kind and $N_{\text{tot}}$ is the total number of tested events.

Thus, to get even one event with evaporation of proton or $\alpha$ particle in decay of $^{256}$No with excitation energy of 30 MeV, one need to test $N_{\text{tot}} \geq 10^7$ events (see Fig. 12).

Note, finally, that all the decay widths and corresponding survival probabilities for a given CN can be calculated online (just in a window of web browser) within the NRV statistical model code [8].
Fig. 13. Capture and EvR cross sections for the $^{12}\text{C} + ^{238}\text{U}$ (left panel) and $^{58}\text{Fe} + ^{208}\text{Pb}$ (right panel) fusion reactions. Experimental data are taken from [43] and [44] for the first reaction and from [45] and [46] for the second one. Dotted and solid curves correspond to one-dimensional and channel coupling calculations of the capture cross sections (see Section 3). EvR cross sections are calculated with $P_{\text{CN}} = 1$ in Eq. (2). In the bottom panel survival probabilities of CN are shown in neutron evaporation channels (see Section 4).

Survival probability, $P_{\text{EvR}}(E^*)$, has a simple shape (similar for all evaporation channel) as a function of excitation energy (i.e. as a function of collision energy). For a given evaporation channel it increases rather fast when excitation energy exceeds the threshold of this channel: $E_{\text{sep}}^{n\text{p}}(1)$ for $1n$ channel, $E_{\text{sep}}^{n\text{p}}(1) + E_{\text{sep}}^{n\text{p}}(2)$ for $2n$ channel, and so on (for charged evaporated particles this threshold is equal to separation energy plus the height of the Coulomb barrier which this particle has to overcome in the exit channel). Then $P_{\text{EvR}}(E^*)$ reaches its maximal value at some $E^{*\text{opt}}$ which is “optimal” for a given evaporation channel. It means that this excitation energy matches best of all the number of evaporation particles. Each evaporated particle takes away part of excitation energy equal to its separation energy plus its kinetic energy. The last one depends on type of evaporated particle: for neutrons it is about $2T$, whereas the optimal kinetic energies of evaporated protons and $\alpha$ particles are closed to the corresponding Coulomb barriers in the exit channel. Large excitation energy $E^* > E^{*\text{opt}}$ hardly can be taken away by a small number of evaporated particles having Maxwell–Boltzmann-type energy spectrum, see factor $P_{n}(E^*, e)$ in Eq. (23). As a result, $P_{\text{EvR}}(E^*)$ exponentially decreases at $E^* > E^{*\text{opt}}$ manifesting the high-energy tail of evaporated particles.

This characteristic shape of survival probability modulates by the shape of the contact cross section in the entrance channel (see typical figures in Section 3) giving finally the bell-shaped cross sections for formation of evaporation residues.
As already mentioned, in fusion reactions of light and medium mass nuclei CN is usually formed with a large probability if these nuclei overcome the Coulomb barrier and reach the contact configuration. In this case calculation of the cross section for formation of a given residual nucleus in its ground state one can be done by expression (2) in which the probability for CN formation is assumed to be 1, \( P_{CN} = 1 \). If appropriate QCC or ECC model is used for the calculation of the contact cross section and appropriate statistical model is used for the calculation of survival probabilities then quite satisfactory agreement of EvR cross sections with the corresponding experimental data are obtained for all evaporation channels (see, for example, left panel of Fig. 13).

Such kind of calculations can be easily performed at the Web site [8] and compared then with available experimental data on the production of evaporation residues in different combinations of colliding nuclei (these data can be also found at the same site).

Thus, one may conclude that a combination of CC calculation of the contact cross section with subsequent calculation of survival probability within the statistical model described above in Section 4 gives rather good description of all available experimental data on the production of evaporation residues in mass asymmetric near-barrier collisions of heavy ions. This method has a good predictive power, at least not worse than one order of magnitude.

5. Formation of SH compound nucleus in competition with quasi-fission

As can be seen in Fig. 13 for the very asymmetric reaction, \( ^{12}\text{C} + ^{238}\text{U} \), experimental data on the production of EvR are reproduced quite well if one assumes that after sticking colliding nuclei fuse (form CN) with probability \( P_{CN} = 1 \). The same assumption made for the \( ^{58}\text{Fe} + ^{208}\text{Pb} \) fusion reaction leads to overestimation of the EvR cross section by more than 2 orders of magnitude. Thus, for this reaction the quasi-fission process dominates at the second reaction stage and the probability for CN formation is very low: \( P_{CN} \sim 10^{-2} \) for this reaction (see also Fig. 2).

In contrast with the first and third reaction stages the process of the compound nucleus formation is understood much worse because of very complicated dynamics of this stage. Moreover, it is very difficult to discriminate experimentally this reaction stage. For heavy combinations of fusing nuclei (leading to formation of SH elements) the fission fragments \( f_1 \) and \( f_2 \) (formed in conventional fission of CN) hardly can be distinguished from those formed in quasi-fission process, \( f_1' \) and \( f_2' \). For such combinations the fusion (CN formation) cross section (4) cannot be measured reliably to be analyzed within one or another theoretical model.

To describe properly dynamics of CN formation one needs, first of all, to choose appropriate degrees of freedom. This set of variables must be the same for simultaneous description of strongly coupled deep inelastic, quasi-fission and fusion–fission processes. The number of the degrees of freedom should not be too large that one is able to solve numerically the corresponding set of dynamic equations. On the other hand, however, with a restricted number of collective variables it is impossible to describe simultaneously deep inelastic scattering of two separated nuclei and quasi-fission of the highly deformed mono-nucleus. Second, one has to define the unified multidimensional potential energy surface (depending on all the degrees of freedom) which regulates in general all the processes. Finally, the corresponding equations of motion should be formulated to perform numerical analysis of the studied reactions.
5.1. Main degrees of freedom and driving potential

The distance between the nuclear centers $\tilde{R}$ (corresponding to the elongation of a mononucleus), dynamic spheroidal-type surface deformations $\beta_1$ and $\beta_2$, mutual in-plane orientations of deformed nuclei $\varphi_1$ and $\varphi_2$, proton and neutron asymmetries $\eta_Z = \frac{Z_1-2}{Z_1+2}$, $\eta_N = \frac{N_1-N_2}{N_1+N_2}$ are probably the relevant degrees of freedom in fusion–fission dynamics. Taken together there are 8 degrees of freedom shown in Fig. 14.

The interaction potential of separated nuclei is calculated rather easily within the folding procedure with effective nucleon–nucleon interaction or parameterized, e.g., by the proximity potential. Of course, some uncertainty remains here, but the height of the Coulomb barrier obtained in these models coincides with the empirical Bass parametrization [47] within 1 or 2 MeV. Dynamic deformations of colliding spherical nuclei and mutual orientation of statically deformed nuclei significantly affect their interaction changing the height of the Coulomb barrier for more than 10 MeV. It is caused mainly by a strong dependence of the distance between nuclear surfaces on the deformations and orientations of nuclei (see Section 3.1). After contact the mechanism of interaction of two colliding nuclei becomes more complicated. For fast collisions ($E/A \sim \varepsilon_{\text{Fermi}}$ or higher) the nucleus–nucleus potential, $V_{\text{diab}}$, should reveal a strong repulsion at short distances protecting the “frozen” nuclei to penetrate each other and form a nuclear matter with double density (diabatic conditions). For slow collisions (near-barrier energies), when nucleons have enough time to reach equilibrium distribution (adiabatic conditions), the nucleus–nucleus potential energy, $V_{\text{adiab}}$, is quite different. Thus, for the nucleus–nucleus collisions at energies well above the Coulomb barrier one can use a time-dependent potential energy, which after contact gradually transforms from a diabatic potential energy into an adiabatic one: $V = V_{\text{adiab}}[1 - f(t)] + V_{\text{diab}} f(t)$ [48]. Here $t$ is the time of interaction and $f(t)$ is a smoothing function with parameter $\tau_{\text{relax}} \sim 10^{-21}$ s, $f(t = 0) = 0$, $f(t \gg \tau_{\text{relax}}) = 1$. The calculation of the multidimensional adiabatic potential energy surface for heavy nuclear system remains a very complicated physical problem.

All the microscopic calculations performed within the TDHF approach (see, for example, [49, 50] and Fig. 15) or by solving time dependent Schrödinger equations (see [16], and Fig. 9 above) demonstrate quite clear that the nucleons “feel” the mean fields of both nuclei even at approaching stage. At near barrier collisions, relative motion of heavy ions is very slow. In this case, much faster moving nucleons of colliding nuclei have enough time to adjust their motion over the volumes of both nuclei forming a two-center mono-nucleus, i.e., the wave functions of valence nucleons follow the two-center molecular states spreading over both nuclei. Such behavior of nucleons is confirmed by explicit solution of the time-dependent Schrödinger equation [16].
An example is shown above in Fig. 9 for the case of near barrier collision of $^{40}\text{Ca}$ with $^{96}\text{Zr}$. The same fusion dynamics with a neck formation was found also within TDHF approach (see, for example, [49,50] and Fig. 15). After contact nucleons form highly deformed *mononucleus configuration*. Its further evolution takes place in the multidimensional space of deformation parameters and mass asymmetry.

This means the following. (1) Any perturbation model based on a calculation of the sudden overlapping of single-particle wave functions of transferred nucleons (in donor and acceptor nuclei, respectively) is not applicable for description of multinucleon transfer reactions and quasi-fission in low-energy heavy-ion damped collisions. (2) The so-called DNS model [63] with two isolated mean fields is absolutely contrary to physics. In this model quite opposite scenario of the CN formation is assumed in which a diabatic (repulsive at short distances) nucleus–nucleus potential energy is used. After stopping on the bottom of potential pocket, two touching nuclei keep their relative distance and their “individuality”. The nucleons of each nucleus move independently in the non-overlapping mean fields (one-center states). The distance between two nuclei remains frozen (not included in equation of motion), and CN formation is assumed owing to nucleon transfers from lighter nucleus to heavier one. (3) One-dimensional potential energy $V(R)$ has no meaning at $R < R_{\text{contact}}$ (as well as any speculations on the depth of potential pocket of $V(R)$ [64]). In low-energy collisions of heavy ions at $R < R_{\text{contact}}$ the multidimensional potential energy (dependent on the shape parameters of the nuclear system and mass asymmetry) has to be used.

The two-center shell model [53] looks most appropriate to describe adiabatic evolution of heavy two-center nuclear system transforming from the configuration of two touching nuclei into the configuration of more or less spherical CN (fusion) or into the configuration of two deformed re-separated fragments (dominating quasi-fission process). Elongation of the system (distance between nuclear centers) is the main collective degree of freedom here, but the other variables (nucleon transfer, deformations, neck parameter) are also very important. Slightly modified version of the two-center shell model was proposed in [54] for the calculation of adiabatic multidimensional potential energy surface. An example is shown in Fig. 16 for the case of near barrier collision of $^{48}\text{Ca}$ with $^{248}\text{Cm}$. Note that the diabatic, $V_{\text{adiab}}$, and adiabatic, $V_{\text{adiab}}$, potential energies depend on the same variables and they are equal to each other for well separated nuclei. Thus, the total potential energy, $V(R, \delta_1, \delta_2, \eta_N, \eta_Z; t)$, is a quite smooth function of all the parameters providing smooth driving forces, $-\partial V/\partial q_i$, at all reaction stages. The extended version of this model [54] leads to a correct asymptotic value of the potential energy of two separated nuclei, appropriate height of the Coulomb barrier in the entrance channel (fusion), and appropriate behavior in the exit channel, giving the required mass and energy distributions of reaction products and fission fragments. This driving potential can be also calculated directly at the Web site [8].
5.2. Dynamic equations of motion

In collisions of heavy ions kinetic energy of relative motion transforms quickly into internal excitation (temperature) of nuclear system and, thus, the fluctuations play a significant role. It was shown in Ref. [48] that the multidimensional Langevin-type dynamical equations of motion can be successfully used for simultaneous description of deep inelastic scattering (multinucleon transfer), quasi-fission and fusion processes in low-energy heavy ion collisions. Finally there is a set of 14 coupled Langevin type equations (25) for 8 degrees of freedom \( \{ R, \vartheta, \beta_1, \beta_2, \varphi_1, \varphi_2, \eta_z, \eta_N \} \equiv \vec{x} \) shown in Fig. 14.

\[
\begin{align*}
\frac{dR}{dt} &= \frac{p_R}{\mu_R} \\
\frac{d\vartheta}{dt} &= \hbar\ell \frac{\mu_R R^2}{R^2} \\
\frac{d\varphi_1}{dt} &= \hbar L_1 \frac{\Im \varphi_1}{\varphi_1} \\
\frac{d\varphi_2}{dt} &= \hbar L_2 \frac{\Im \varphi_2}{\varphi_2} \\
\frac{d\beta_1}{dt} &= \frac{p_{\beta_1}}{\mu_{\beta_1}} \\
\frac{d\beta_2}{dt} &= \frac{p_{\beta_2}}{\mu_{\beta_2}}
\end{align*}
\]
\[
\frac{d\eta_Z}{dr} = \frac{2}{Z_{CN}} D_{Z}^{(1)}(\eta) + \frac{2}{Z_{CN}} \sqrt{D_{Z}^{(2)}(\eta) \Gamma_Z(t)}
\]
\[
\frac{d\eta_N}{dr} = \frac{2}{N_{CN}} D_{N}^{(1)}(\eta) + \frac{2}{N_{CN}} \sqrt{D_{N}^{(2)}(\eta) \Gamma_N(t)}
\]
\[
\frac{dp_R}{dr} = -\frac{\partial V}{\partial R} + \frac{\hbar^2 \ell^2}{\mu_R R^3} + \frac{p_R^2}{2 \mu_R^2 R^2} \frac{\partial \mu_R}{\partial R} + \frac{p_{\beta_1}^2}{2 \mu_{\beta_1}^2} \frac{\partial \mu_{\beta_1}}{\partial R} + \frac{p_{\beta_2}^2}{2 \mu_{\beta_2}^2} \frac{\partial \mu_{\beta_2}}{\partial R}
\]
\[
-\gamma_R \frac{p_R}{\mu_R} + \sqrt{\gamma_R T \Gamma_R(t)}
\]
\[
\frac{d\ell}{dr} = -\frac{1}{\hbar} \frac{\partial V}{\partial \ell} - \gamma_{\text{tang}} \left( \frac{\ell}{\mu_R R} - \frac{L_1}{3} a_1 - \frac{L_2}{3} a_2 \right) R + \frac{R}{\hbar} \sqrt{\gamma_{\text{tang}} T \Gamma_{\text{tang}}(t)}
\]
\[
\frac{dL_1}{dr} = -\frac{1}{\hbar} \frac{\partial V}{\partial \phi_1} + \gamma_{\text{tang}} \left( \frac{L_1}{3} a_1 - \frac{L_2}{3} a_2 \right) a_1 - \frac{a_1}{\hbar} \sqrt{\gamma_{\text{tang}} T \Gamma_{\text{tang}}(t)}
\]
\[
\frac{dL_2}{dr} = -\frac{1}{\hbar} \frac{\partial V}{\partial \phi_2} + \gamma_{\text{tang}} \left( \frac{L_1}{3} a_1 - \frac{L_2}{3} a_2 \right) a_2 - \frac{a_2}{\hbar} \sqrt{\gamma_{\text{tang}} T \Gamma_{\text{tang}}(t)}
\]
\[
\frac{dp_{\beta_1}}{dr} = -\frac{\partial V}{\partial \beta_1} + \frac{p_{\beta_1}^2}{2 \mu_{\beta_1}^2} \frac{\partial \mu_{\beta_1}}{\partial \beta_1} + \frac{p_{\beta_2}^2}{2 \mu_{\beta_2}^2} \frac{\partial \mu_{\beta_2}}{\partial \beta_1} + \frac{\hbar^2 L_1^2}{2 \beta_1^2} \frac{\partial \beta_1}{\partial \beta_1} + \left( \frac{\hbar^2 \ell^2}{2 \mu_R^2 R^2} + \frac{p_R^2}{2 \mu_R^2} \right) \frac{\partial \mu_R}{\partial \beta_1}
\]
\[
-\gamma_{\beta_1} \frac{p_{\beta_1}}{\mu_R} + \sqrt{\gamma_{\beta_1} T \Gamma_{\beta_1}(t)}
\]
\[
\frac{dp_{\beta_2}}{dr} = -\frac{\partial V}{\partial \beta_2} + \frac{p_{\beta_1}^2}{2 \mu_{\beta_1}^2} \frac{\partial \mu_{\beta_1}}{\partial \beta_2} + \frac{p_{\beta_2}^2}{2 \mu_{\beta_2}^2} \frac{\partial \mu_{\beta_2}}{\partial \beta_2} + \frac{\hbar^2 L_1^2}{2 \beta_2^2} \frac{\partial \beta_2}{\partial \beta_2} + \left( \frac{\hbar^2 \ell^2}{2 \mu_R^2 R^2} + \frac{p_R^2}{2 \mu_R^2} \right) \frac{\partial \mu_R}{\partial \beta_2}
\]
\[
-\gamma_{\beta_2} \frac{p_{\beta_2}}{\mu_R} + \sqrt{\gamma_{\beta_2} T \Gamma_{\beta_2}(t)}
\]  

Here $\Gamma_q(t)$ are the normalized random variables with Gaussian distribution, zero average value and dispersion equal to 2: $\langle \Gamma(t) \rangle = 0$, $\langle \Gamma(t) \Gamma(t') \rangle = 2 \delta(t - t')$. The standard Box–Muller method [52] can be used to generate the normally distributed random numbers at each time step of numerical solution of this set of equations.

$D_{Z}^{(1)}$, $D_{Z}^{(2)}$ in Eqs. (25) are the transport coefficients. We assume that sequential nucleon transfers play a main role in mass rearrangement. In this case

$$D_{N,Z}^{(1)} = \lambda_{N,Z}^{(+)}(A \rightarrow A + 1) - \lambda_{N,Z}^{(-)}(A \rightarrow A - 1),$$

$$D_{N,Z}^{(2)} = \frac{1}{2} \left[ \lambda_{N,Z}^{(+)}(A \rightarrow A + 1) + \lambda_{N,Z}^{(-)}(A \rightarrow A - 1) \right],$$

where the macroscopic transition probabilities $\lambda_{N,Z}^{(+)}(A \rightarrow A' = A \pm 1)$ depend on the nuclear level density [55,56], $\lambda_{N,Z}^{(\pm)} = \lambda_{N,Z}^{(0)} \sqrt{\rho(A \pm 1) / \rho(A)}$ and $\lambda_{N,Z}^{(0)}$ are the neutron and proton transfer rates. The nuclear level density $\rho \sim \exp(2\sqrt{aE^*})$ depends on the excitation energy $E^*$ and, thus, the transition probabilities, $\lambda_{N,Z}^{(\pm)}$, are also coordinate and time dependent functions. The first terms in equations for the proton and neutron asymmetries, $D_{N}^{(1)} \sim \partial V / \partial N$ and $D_{Z}^{(1)} \sim \partial V / \partial Z$, drive the system to the configuration with minimal potential energy in the $(Z,N)$ space, i.e., to the configurations with optimal Q value (see deep valleys in Fig. 16). The second terms in these equations, $\sim D_{N,Z}^{(2)}$, describe a diffusion of neutrons and protons in the system of overlapped nuclei.
For separated nuclei the nucleon exchange is still possible (though it is less probable) and has to be taken into account (see Section 3.4). We use the following final formula for the transition probabilities

\[ \lambda_{N,Z}^{(\pm)} = \lambda_{N,Z}^{0} \sqrt{\frac{\rho(A \pm 1)}{\rho(A)}} P_{N,Z}^{tr}(R, A \rightarrow A \pm 1). \] (27)

Here \( P_{N,Z}^{tr}(R, A \rightarrow A \pm 1) \) is the probability of one nucleon transfer (neutron or proton), which depends on the distance between the nuclear surfaces and on the nucleon separation energy. This probability goes exponentially to zero at \( R \rightarrow \infty \) and it is equal to unity for overlapping nuclei. The simple semiclassical formula [15] can be used for the calculation of \( P_{N,Z}^{tr} \). Thus, Eqs. (25), beside other variables, define a continuous change of charge and mass asymmetries during the whole process (obviously, \( d\eta_{N,Z}/dt \to 0 \) for far separated nuclei).

The neutron and proton transfer rates, \( \lambda_{N,Z}^{0} \), are the fundamental quantities of low-energy nuclear dynamics. However, their values are not well determined. For the first time the nucleon transfer rate, \( \lambda_{0} \), was estimated in Refs. [55,56] to be about \( 10^{22} \) s\(^{-1} \). In our previous study we found that the value of \( 0.1 \cdot 10^{22} \) s\(^{-1} \) of the nucleon transfer rate is quite appropriate to reproduce experimental data on the mass distributions of reaction products for different combinations of heavy-ion damped collisions [48]. However this quantity is still rather uncertain. Its energy (and temperature) dependence was not studied yet. A systematic analysis of experimental data on multinucleon transfer reactions at different collision energies is needed to determine the nucleon transfer rate more carefully. In our approach we distinguish the neutron and proton transfers (it is important for prediction of the yields of different isotopes of a given element in multinucleon transfer reactions).

At the approaching stage (for separated nuclei) the probabilities for neutron and proton transfers are different. The Coulomb barrier for protons leads to faster decrease of their bound state wave functions outside the nuclei, and, in general, \( P_{Z}^{tr}(R > R_{1} + R_{2}, A \rightarrow A \pm 1) < P_{N}^{tr}(R > R_{1} + R_{2}, A \rightarrow A \pm 1) \). However, for well overlapped nuclei single particle motions of protons and neutrons are rather similar, and we assume that the neutron and proton transfer rates are equal to each other, i.e., \( \lambda_{N}^{0} = \lambda_{Z}^{0} = \lambda^{0}/2 \), and both are the parameters of the model (i.e., they are not derived from some microscopic calculations). The model describes quite properly [57] experimental difference of the cross sections for pure neutron and proton transfers [17].

Another rather uncertain quantity of low-energy nuclear dynamics is the nuclear friction (nuclear viscosity) responsible for the kinetic energy loss in heavy ion damped collisions. A great interest to these processes was shown 30 years ago. Those time, however, there was not appropriate theoretical model for overall quantitative description of available experimental data on the mass, charge, energy and angular distributions of reactions products. A number of different mechanisms have been suggested in the literature to be responsible for the energy loss in heavy ion collisions. A discussion of the subject can be found, e.g., in [47,58,26,27]. The uncertainty in the strength of nuclear viscosity (as well as its form-factor) is still large. Moreover, microscopic analysis shows that nuclear viscosity may also depend strongly on nuclear temperature [59].

For overlapping nuclei (mono-nucleus configuration) the two-body nuclear friction can be calculated within the Werner–Wheeler approach [60]. The corresponding viscosity coefficient \( \mu_{0} \) is estimated to be of the order \( 10^{-23} \) MeV s fm\(^{-3} \) [60]. The one-body dissipation mechanism [61, 62] leads in general to even stronger nuclear friction. Note, that in all the approaches the nuclear viscosity was found to be rather large, leading to the so-called overdamped collision dynamics. For well overlapped nuclei kinetic energy in all degrees of freedom is rather low and excited
nuclear system “creeps” along the potential energy surface in the multidimensional configuration space. For such kind of overdamped dynamics the Langevin equations can be reduced to the Smoluchowski equations, which might be solved more easily (in our calculations we do not use such reduction).

Similar in physics (but more simplified) approach is used also in the “fusion-by-diffusion model” [65] basing on the approximative (analytical) solution of the Smoluchowski equation. The probability for CN formation, $P_{CN}(E^*, I)$, is found in this model as a simple diffusion process over the (parameterized) one-dimensional intrinsic barrier (with advantage of analytical solution). The cross sections calculated by this model often substantially deviate from those obtained within the present approach (see below), but the model can be used for faster estimations of the fusion probability.

The mass, energy and angular distributions of binary reaction products depends stronger on the form-factor (e.g., on the radius) of friction forces, and not so much on its strength. The strength parameter of nuclear friction (for separated nuclei) as well as its form-factor are discussed in [48].

The double differential cross-sections of all the binary reaction channels are calculated as follows

$$\frac{d^2\sigma_{N,Z}(E, \theta)}{d\Omega dE} = \int_0^\infty b db \frac{\Delta N_{N,Z}(b, E, \theta)}{N_{tot}(b)} \frac{1}{\sin(\theta)\Delta\theta \Delta E}. \tag{28}$$

Here $\Delta N_{N,Z}(b, E, \theta)$ is the number of events at a given impact parameter $b$ in which a nucleus $(N, Z)$ is formed in the exit channel with kinetic energy in the region $(E, E + \Delta E)$ and with center-of-mass outgoing angle in the interval $(\theta, \theta + \Delta\theta)$, $N_{tot}(b)$ is the total number of simulated events for a given value of the impact parameter. This number depends strongly on low level of the cross section which one needs to be reached in calculation. For predictions of rare events with the cross sections of 1 µb (primary fragments) one needs to test not less than $10^7$ collisions (as many as in real experiment). In the case of near-barrier collision of heavy nuclei only a few trajectories (of many thousands tested) reach the CN configuration (small values of elongation and deformation parameters). All others go out to the dominating deep inelastic and/or quasi-fission exit channels. One of such trajectories (demonstrating appearance of the fluctuations of all degrees of freedom when nuclei come in contact) is shown in Fig. 17 in the three-dimensional space of “elongation–deformation–mass-asymmetry” used in the calculations.

Expression (28) describes the mass, charge, energy and angular distributions of the primary fragments formed in the binary reaction. Subsequent de-excitation cascades of these fragments via emission of light particles and gamma-rays in competition with fission are taken into account explicitly for each event within the statistical model (see above) leading to the final distributions of the reaction products. Simultaneous description of all the strongly coupled processes of deep inelastic scattering, quasi-fission and fusion allows one to avoid big errors in absolute normalization of the corresponding cross sections owing to conservation of the total flux (no one collision event is lost). All the cross sections are calculated in a quite natural way, just by counting the events coming into a given reaction channel.

All the features of HI damped collisions (mass, energy and angular distributions) are described quite properly within the model formulated above. As an example, in Fig. 18 the calculated and measured distributions of reaction fragments formed in collisions of $^{86}$Kr with $^{166}$Er are shown.

Quasi-fission channels are included in the model in a natural way as all the other binary exit channels. A typical trajectory of the nuclear system in collision of $^{48}$Ca + $^{248}$Cm at $E_{c.m.} =
Collision of $^{48}$Ca + $^{248}$Cm at $E_{\text{c.m.}} = 210$ MeV. One of typical trajectories calculated within the Langevin equations and going to the quasi-fission exit channel (lead valley) is shown in the three-dimensional space (a), and projected onto the “deformation–elongation” (b), and “mass-asymmetry–elongation” (c) planes. The dashed line in (b) shows the ridge of the multidimensional Coulomb barrier.

203 MeV (zero impact parameter) is shown in Figs. 19. This trajectory leads the system to the QF channel. After overcoming the Coulomb barrier the fragments become first very deformed, then the mass asymmetry gradually decreases and the system finds itself in the quasi-fission valley with one of the fragments close to the doubly magic nucleus $^{208}$Pb (see the potential energy surface in Fig. 16). After contact the nuclear system has almost zero kinetic energy up to scission, and the regions with higher potential energy are surmounted mainly due to the fluctuations. The excitation energy of the system (temperature) gradually increases (very sharply on descent stage to the scission point).

Fig. 20 shows the calculated correlation of the total kinetic energy and mass distributions of the reaction products along with inclusive mass distribution for the $^{48}$Ca + $^{248}$Cm reaction at near-barrier energy of $E_{\text{c.m.}} = 203$ MeV. This calculations agree rather well with experimental data [67].

The probability for CN formation in this reaction was found very small and depended greatly on the incident energy. As was already mentioned, due to a strong dissipation of kinetic energy just the fluctuations (random forces) define the dynamics of the system after contact of two nuclei. At near barrier collisions the excitation energy (temperature) of the system is rather low, the fluctuations are weak and the system chooses the most probable path to the exit channel along the quasi-fission valley. However at non-zero excitation energy there is a chance for the nuclear system to overcome the multidimensional inner potential barriers and find itself in the
Fig. 18. Charge, mass and energy distributions of reaction fragments in collisions of $^{86}$Kr with $^{166}$Er at $E_{c.m.} = 464$ MeV [66].

region of CN configuration (small deformation and elongation). Within the Langevin calculations a great number of events should be tested to find this low probability. For the studied reaction, for example, only several fusion events have been found among more than $10^5$ total tested events [see Fig. 20(b)]. The cross section of CN formation at the beam energy of $E_{c.m.} = 203$ MeV was estimated to be only 0.02 mb. With accounting survival probability, $P_{EvR}$, it is in a reasonable agreement with the EvR cross section of about several picobarns for the production of element 116 in this reaction [68].

Made within the described above approach ($P_{cont} \times P_{CN} \times P_{EvR}$), the predictions for the excitation functions of SH element production with $Z = 112–118$ in $1n–5n$ evaporation channels of the $^{48}$Ca induced fusion reactions [69–71] agree well with the experimental data (see below Fig. 26). This gives us confidence in reliability of this method for estimations of other cross sections which are urgently needed for planning future experiments in this field.

5.3. Cold fusion reactions

At near-barrier incident energies fusion of heavy nuclei ($^{48}$Ca, $^{50}$Ti, $^{54}$Cr and so on) with $^{208}$Pb or $^{209}$Bi targets leads to formation of low-excited superheavy CN (“cold” synthesis). In spite of this favorable fact (only one or two neutrons are to be evaporated, expected large values of $P_{EvR}$), the yield of evaporation residues sharply decreases with increasing charge of synthesized SH nucleus. There are two reasons for that. First, in these reactions neutron deficient SH nuclei are produced far from the closed shells or sub-shells. As a result, neutron separation energies of these nuclei are rather high whereas the fission barriers (macroscopic components plus shell corrections) are rather low (see Table 1). This leads to low survival probability even for 1n and 2n evaporation channels, see Fig. 21.
Fig. 19. Change of elongation, deformation, mass asymmetry, potential, kinetic and excitation energies along the trajectory shown in Fig. 17.

Table 1
Fission barriers (macroscopical part and shell correction) and neutron separation energies (MeV) of CN produced in the $^{48}$Ca + $^{208}$Pb, $^{50}$Ti + $^{208}$Pb and $^{54}$Cr + $^{208}$Pb fusion reactions [72]. The last column shows the excitations of CN at the Bass barrier [47] incident energies.

<table>
<thead>
<tr>
<th>CN</th>
<th>$B_{LD}$</th>
<th>Sh. Corr.</th>
<th>$B_{\text{fis}}$</th>
<th>$E_{\text{sep}}^{\text{CN}}$</th>
<th>$E^*$ (Bass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{256}$No</td>
<td>1.26</td>
<td>4.48</td>
<td>5.7</td>
<td>7.1</td>
<td>22</td>
</tr>
<tr>
<td>$^{258}$Rf</td>
<td>0.71</td>
<td>4.49</td>
<td>5.3</td>
<td>7.6</td>
<td>24</td>
</tr>
<tr>
<td>$^{262}$Sg</td>
<td>0.47</td>
<td>4.63</td>
<td>5.1</td>
<td>7.8</td>
<td>24</td>
</tr>
</tbody>
</table>

The second (main) reason for low yields of evaporation residues in these reactions is, however, a sharp decrease of the fusion probability with increasing charge of the projectile. In Fig. 22 the calculated contact, CN formation and EvR cross sections of the $^{208}$Pb induced fusion reactions are shown along with available experimental data on the yields of SH elements (not all experimental points are displayed to simplify the plot). The fusion probabilities $P_{\text{CN}}$, calculated for head-on collisions (which bring the main contribution to the EvR cross sections), demonstrate a sharp energy dependence (see Fig. 23), found earlier in [73]. Recently such decrease of the fusion probability at subbarrier energies was confirmed experimentally for the fusion of $^{50}$Ti with $^{208}$Pb [74].
Fig. 20. (a) Calculated TKE-mass distribution of primary reaction products in collision of $^{48}$Ca + $^{248}$Cm at $E_{c.m.} = 203$ MeV. (b) Contributions of deep inelastic scattering (DI), quasi-fission (QF) and fusion–fission processes (CN formation) to inclusive mass distribution (see the corresponding schematic trajectories in Fig. 16).

Fig. 21. Survival probability $P_{\text{CN}}(E^*, l=0)$ of $^{256}$No, $^{258}$Rf and $^{262}$Sg compound nuclei produced in the $^{48}$Ca + $^{208}$Pb, $^{50}$Ti + $^{208}$Pb and $^{54}$Cr + $^{208}$Pb fusion reactions. The arrows indicate the Bass barriers (see Table 1).
Fig. 22. Contact (upper solid curves), CN formation (short-dashed curves) and SH element production cross sections in the $^{208}$Pb induced fusion reactions. $1n$, $2n$ and $3n$ evaporation channels are shown by solid, dashed and dotted curves (theory) and by rectangles, circles and triangles (experiment), correspondingly. Experimental data are taken from [75–77].

Fig. 23. Calculated fusion probabilities, $P_{\text{CN}}(E^*, l = 0)$, for near-barrier collisions of heavy nuclei with $^{208}$Pb target. CN excitation energies at the Bass barriers are shown by the arrows. Experimental values of $P_{\text{CN}}$ obtained in [74] for the $^{50}$Ti $+$ $^{208}$Pb fusion reaction are shown by the rectangles.
Fig. 24. Above-barrier CN formation probability in the $^{208}$Pb induced fusion reactions. Results of calculation are shown by the circles, whereas the fitted curve corresponds to the expression (30).

The calculated energy dependence of the fusion probability (shown in Fig. 23) may be approximated by the simple formula

$$P_{CN}(E^*, l) = \frac{P_{CN}^0}{1 + \exp \left( \frac{E_B^* - E_{int}^*(l)}{\Delta} \right)},$$

(29)

which could be useful for a fast estimation of EvR cross sections in the “cold” fusion reactions. Here $E_B^*$ is the excitation energy of CN at the center-of-mass beam energy equal to the Bass barrier [47]. $E_B^*$ are shown in Fig. 23 by the arrows. $E_{int}^*(l) = E_{c.m.} + Q - E_{rot}(l)$ is the “internal” excitation energy which defines also the damping of the shell correction to the fission barrier of CN. $\Delta$ is the adjustable parameter of about 4 MeV, and $P_{CN}^0$ is the “asymptotic” (above-barrier) fusion probability dependent only on a combination of colliding nuclei.

The values of $P_{CN}^0$ calculated at excitation energy $E^* = 40$ MeV (well above the barriers for the “cold” fusion reactions) demonstrate rather simple behavior (almost linear in logarithmic scale), monotonically decreasing with increase of charge of CN and/or with increase of the product of $Z_1$ and $Z_2$, see Fig. 24. This behavior could be also approximated by very simple Fermi function

$$P_{CN}^0 = \frac{1}{1 + \exp \left( \frac{Z_1 Z_2 - \xi}{\tau} \right)},$$

(30)

where $\xi \approx 1760$ and $\tau \approx 45$ are just the fitted parameters. Eq. (30) is obviously valid only for the “cold” fusion reactions of heavy nuclei with the closed shell targets $^{208}$Pb and $^{209}$Bi. Unfortunately we have not enough experimental data to check this formula for other reactions (or to derive more general expression for the fusion probability).

Two important remarks can be done after analysis of the “cold” fusion reactions. The first is rather evident. There are no reasons (in fusion or in survival probabilities) to slow down the fast monotonic decrease of EvR cross sections with increasing charge of SH nucleus synthesized in the “cold” fusion reaction. The yield of element 114 in the 1n evaporation channel of the $^{76}$Ge + $^{208}$Pb fusion reaction is only 0.06 pb. Similar cross section (approximately twice larger) for the same reaction was obtained within the “fusion-by-diffusion” model [78]. For elements 116...
and 118, synthesized in fusion reactions of $^{82}$Se and $^{86}$Kr with lead target, we found unreachable cross sections of only 0.004 pb and 0.0005 pb, correspondingly, for 1n EvR cross sections (it is worth to note that our results sharply disagree with those obtained within the DNS model [79], which predicts the EvR cross sections at the level of 0.1 pb for all these elements including $Z = 120$ produced in “cold” fusion reactions). As already mentioned, fusion reactions with $^{208}$Pb or $^{209}$Bi targets lead to neutron deficient SH nuclei with short half-lives, that may bring an additional difficulty to their experimental detection at the available separators (not less than 1 µs is needed for CN to reach the focal plane detector).

The second conclusion is important for further experiments with actinide targets. The experimental value of EvR cross section for element 104 in the $^{50}$Ti $+$ $^{208}$Pb fusion reaction is two (!) orders of magnitude less as compared with the yield of element 102 in the $^{48}$Ca $+$ $^{208}$Pb reaction, see Fig. 22. At first sight, this fact makes the fusion reactions of titanium with actinide targets (“hot” fusion) much less encouraging as compared to $^{48}$Ca fusion reactions. However, this sharp decrease in the yield of the Rutherford isotopes is caused by the two reasons. One order of magnitude loss in the EvR cross section is due to the low survival probability of $^{258}$Rf nucleus (the fission barrier is less by 0.4 MeV and neutron separation energy is higher by 0.5 MeV as compared with $^{256}$No produced in $^{48}$Ca fusion reaction, Eq. (29)), whereas the fusion probability of $^{50}$Ti with $^{208}$Pb at energies near and above the Coulomb barrier is only one order of magnitude less than in the $^{48}$Ca $+$ $^{208}$Pb fusion reaction (see Fig. 23). This makes titanium beam still quite promising for synthesis of SH nuclei in fusion reactions with the actinide targets (see below).

### 5.4. Hot fusion reactions

Fusion reactions of $^{48}$Ca with actinide targets lead to formation of more neutron rich SH nuclei as compared to the “cold” fusion reactions. Their half-lives are several orders of magnitude longer. For example, the half-life of the SH nucleus $^{277}$112 synthesized in the “cold” fusion reaction $^{70}$Zn $+$ $^{208}$Pb [75,76] is about 1 ms, whereas $T_{1/2}(^{285}$112) $\sim 34$ s (approaching the “island of stability”). On average, these SH nuclei have higher fission barriers and lower neutron separation energies, which give them a chance to survive in the three- or four-neutron evaporation cascade.

Unfortunately, weaker binding energies of the actinide nuclei lead to rather high excitation energies of the synthesized CN (that is why these reactions are named “hot”). At beam energy close to the Bass barrier the value of $E_{CN}^* = E_{c.m.} + B(Z_{CN}, A_{CN}) - B(Z_1, A_1) - B(Z_2, A_2)$ is usually higher than 30 MeV for almost all the combinations, and at least 3 neutrons are to be evaporated to get a SH nucleus in its ground state. The total survival probability of CN formed in the “hot” fusion reaction (in the 3n and/or in the 4n channel) is much less than 1n-survival probability in the “cold” fusion reaction, $P_{3n}^{3n}(E^* \sim 35$ MeV) $\ll P_{1n}^{cold}(E^* \sim 15$ MeV).

On the other hand, for the more asymmetric “hot” combinations the fusion probability is usually much higher as compared to the more symmetric “cold” combinations leading to the same (but more neutron deficient) elements. We calculated the capture, fusion and EvR cross sections for the “cold” ($^{208}$Pb induced) and “hot” ($^{48}$Ca induced) reactions leading to SH nuclei with $Z = 102$–118 at the same excitation energies of the CN: 15 MeV for the “cold” and 35 MeV for the “hot” combinations. Of course, the beam energies, at which these CN excitations realized, are equal only approximately to the corresponding Coulomb barriers and not all them agree precisely with positions of maxima of EvR cross sections. Nevertheless, some general regularities can be found from these calculations.
The results of these calculations performed within the described above model are shown in Fig. 25. As can be seen, the contact cross sections are about one order of magnitude larger for the “hot” combinations. This is because the excitation energy $E^* = 15$ MeV corresponds to the incident energies somewhat below the Bass barriers of the “cold” combinations. Slow decrease of $\sigma_{\text{cont}}$ for the “cold” combinations at $Z_{\text{CN}} > 108$ is caused by gradual shallowing of the potential pocket (decreasing value of $l_{\text{crit}}$). Larger value of $\sigma_{\text{cont}}$ for the $^{48}\text{Ca} + ^{249}\text{Cf}$ combination is conditioned by a “colder” character of this reaction – the excitation energy of CN at the Bass barrier beam energy is only 28 MeV for this reaction (i.e., the chosen $E^* = 35$ MeV corresponds here to the above barrier initial energy).

The fusion probability for the “cold” combinations decreases much faster with increasing charge of the projectile and, in spite of evaporation of only one neutron, at $Z_{\text{CN}} \geq 112$ the “cold” $\text{EvR}$ cross sections become less than in “hot” fusion reactions.

Increasing survival probability of SH nuclei with $Z = 114, 116$ synthesized in $^{48}\text{Ca}$ induced fusion reactions as compared to $Z = 110, 112, 113$ is owing to the increase of the shell corrections to the ground state (i.e. owing to increase of the fission barriers of these nuclei) caused by approaching the closed shells predicted by the macro-microscopical model (see Table 2). Note that almost constant values of $\text{EvR}$ cross sections for the production of SH elements 112–118 was predicted first in [69]. Calculated [70,71] and experimental excitation functions for the production of SH element in the $^{48}\text{Ca}$ induced fusion reactions are shown in Fig. 26. Most of the experimental data have been obtained at the Flerov Laboratory (JINR, Dubna), appropriate
Table 2
Fission barriers (macroscopical part and shell correction) and neutron separation energies (MeV) of CN produced in the $^{48}$Ca fusion reactions with $^{232}$Th, $^{238}$U, $^{244}$Pu, $^{248}$Cm and $^{249}$Cf targets [72]. The last column shows the excitations of CN at the Bass barrier incident energies.

<table>
<thead>
<tr>
<th>CN</th>
<th>$B_{LD}$</th>
<th>Sh. Corr.</th>
<th>$B_{fis}$</th>
<th>$E_{np}^{sep}$</th>
<th>$E^*$ (Bass)</th>
</tr>
</thead>
<tbody>
<tr>
<td>280/110</td>
<td>0.21</td>
<td>4.76</td>
<td>5.0</td>
<td>7.0</td>
<td>32</td>
</tr>
<tr>
<td>286/112</td>
<td>0.10</td>
<td>6.64</td>
<td>6.7</td>
<td>7.1</td>
<td>33</td>
</tr>
<tr>
<td>285/113</td>
<td>0.02</td>
<td>7.04</td>
<td>7.2</td>
<td>7.1</td>
<td>30</td>
</tr>
<tr>
<td>292/114</td>
<td>0.04</td>
<td>8.89</td>
<td>8.9</td>
<td>7.0</td>
<td>34</td>
</tr>
<tr>
<td>296/116</td>
<td>0.01</td>
<td>8.58</td>
<td>8.6</td>
<td>6.7</td>
<td>32</td>
</tr>
<tr>
<td>297/118</td>
<td>0.00</td>
<td>8.27</td>
<td>8.3</td>
<td>6.2</td>
<td>28</td>
</tr>
</tbody>
</table>

Fig. 26. Predicted and measured excitation functions for the production of SH elements 112–118 in $^{48}$Ca induced fusion reactions with actinide targets. Most of experiments was performed in Dubna (see review paper [80]) and later at GSI and Berkeley.

references can be found in the review paper [80]. Later part of these data were confirmed in experiments performed at GSI [81] and in Berkley, where a new isotope of element 114 was also discovered [82] in 5n evaporation channel of the $^{48}$Ca + $^{242}$Pu fusion reaction (see Fig. 26).

5.5. Mass symmetric fusion reactions

The use of the accelerated neutron-rich fission fragments is one of the widely discussed speculative methods for the production of SH elements in the region of the “island of stability”. For example, in the $^{132}$Sn + $^{176}$Yb fusion reaction we may synthesize $^{308}$I20, which (after a few neutron evaporations) may reach the center of the “island of stability” in the subsequent $\alpha$-decay chain. Several projects in the world are now realizing to get the beams of neutron rich fission fragments. The question is how intensive should be such beams to produce SH nuclei. Evidently an answer depends on the values of the corresponding cross sections. Unfortunately, there are almost no experimental data on fusion reactions for mass-symmetric combinations of heavy colliding nuclei.
Experimental data on symmetric fusion reactions $^{100}\text{Mo} + ^{100}\text{Mo}$, $^{100}\text{Mo} + ^{110}\text{Pa}$ and $^{110}\text{Pa} + ^{110}\text{Pa}$ [83] show that the fusion probability sharply decreases with increasing mass and charge of colliding nuclei. However, the last studied reactions of such kind, $^{110}\text{Pa} + ^{110}\text{Pa}$, is still far from a combination leading to a SH compound nucleus. This means that further experimental study of such reactions is quite urgent.

The choice of the colliding nuclei is also important. In this connection the $^{136}\text{Xe} + ^{136}\text{Xe}$ fusion reaction looks very promising for experimental study, because the formed CN, $^{272}\text{Hs}$, should undergo just to symmetric fission. It means that two colliding $^{136}\text{Xe}$ nuclei are very close to the nascent fission fragments of $^{272}\text{Hs}$ in the region of the saddle point, and their fusion should really reflect a fusion process of two fission fragments.

The calculated within the two-center shell model adiabatic potential energy surface of the nuclear system consisting of 108 protons and 164 neutrons is shown in Fig. 27 as a function of elongation (distance between the centers) and deformation of the fragments at zero mass asymmetry, which correspond to two Xe nuclei in the entrance and exit channel. The energy scale is chosen in such a way that zero energy corresponds to the two $^{136}\text{Xe}$ nuclei in their ground states at infinite distance. The contact configuration of two spherical Xe nuclei is located very close (in energy and in configuration space) to the saddle point of CN (note that it is located behind the Coulomb barrier, though there is no pronounced potential pocket). This fusion reaction is extremely “cold”, the excitation energy of the CN (if it would be formed) at the Bass barrier beam energy is only 5 MeV. One may expect that after contact these nuclei may overcome the inner barrier due to fluctuations of collective degrees of freedom and thus reach the saddle configuration. After that they fuse (form CN) with 50% probability.

However the potential energy decreases very fast with increasing deformations of the touching nuclei and drives the nuclear system to the deep fission valley (see Fig. 27). As a result, the calculated fusion probability is very low and, in spite of rather high fission barriers of the hassium
isotopes in the region of \( A \sim 270 \) (\( \sim 6 \) MeV [72]), the EvR cross sections were found to be very low [85], see Fig. 27. They are much less than the yield of \( ^{265}\text{Hs} \) synthesized in the more asymmetric \( ^{58}\text{Fe} + ^{208}\text{Pb} \) fusion reaction (Fig. 22). It is worthy to note that the prediction of the EvR cross section for the 1n channel in the \( ^{136}\text{Xe} + ^{136}\text{Xe} \) fusion reaction, obtained within the “fusion-by-diffusion” model [84], exceeds our estimation by three orders of magnitude. This fact reflects significant difficulties appearing in the calculation of the fusion probability in such reactions. Note, that the recent calculations performed within the “fusion-by-diffusion” model for somewhat heavier symmetric systems \( ^{154}\text{Sm} + ^{150}\text{Nd} \) and \( ^{154}\text{Sm} + ^{154}\text{Sm} \) give experimentally unreachable cross sections of about \( 10^{-11} \) pb and \( 10^{-13} \) pb, respectively, for the synthesis of the element 122 and 124 [87].

Experiment on the synthesis of hassium isotopes in the \( ^{136}\text{Xe} + ^{136}\text{Xe} \) fusion reaction was performed recently in Dubna, and no one event was detected at the level of about 2 pb [86]. Thus, we may conclude that for the widely discussed future experiments on synthesis of SH nuclei in the fusion reactions with accelerated fission fragments one needs to get a beam intensity not lower than \( 10^{13} \) pps (comparable or greater than intensities of available stable beams of heavy ions). Since the experimental values of the EvR cross sections in such reactions are still unknown, attempts to synthesize a SH element in the fusion reaction of two heavy more or less equal in masses nuclei (Xe + Xe or Sn + Xe) should be continued.

5.6. Concluding remarks on predictive power of the model

Finally we ought to answer the most important question “How accurately can the cross sections for the production of SH element can be predicted today?”. Unfortunately this question has not a clear and unambiguous quantitative answer. First, one needs to distinguish real predictions (made before experiment) from a lot of “appropriate descriptions” (postdictions) appeared in literature just after an experiment on synthesis of some SHE was performed. Second, an accuracy of predictions strongly depends on the reactions chosen for synthesis of a given SHE. As shown above, for mass asymmetric fusion reactions accuracy of predictions is not worse than one order of magnitude. Main uncertainty here comes from unknown value of the fission barrier of CN: on average, 1 MeV change in the height of the fission barrier of heavy CN leads to a change of EvR cross section in 3n evaporation channel by 1 order of magnitude owing to change of \( \Gamma_n / \Gamma_f \). In this connection more accurate calculation of the fission barriers for superheavy mass region is urgently needed (see, for example, resent progress in this field in [89] and the corresponding chapter of this issue).

For \( ^{48}\text{Ca} \) induced fusion reactions the situation looks rather good (see Figs. 25 and 26). Note that most of the calculations shown in Fig. 26 [70,71] are real predictions, they were performed before the corresponding experiments were carried out. However such situation is largely due to existence of many experimental data on fusion of \( ^{48}\text{Ca} \) with heavy targets (from \( ^{172}\text{Yb} \) to \( ^{249}\text{Cf} \)), with formation of EvR with \( Z = 102–118 \). If one takes another projectile, say \( ^{50}\text{Ti} \) or \( ^{54}\text{Cr} \), uncertainty significantly increases because there is no experimental data on fusion of this nuclei with actinide targets leading to formation of CN and then to EvR: How less is \( P_{\text{CN}} \) and what is \( P_{\text{EvR}} \) for unknown CN? Much worse situation is for predictions of the cross sections for the production of SHE in mass symmetric fusion reactions (hotly discussed future experiments on fusion of accelerated neutron enriched fission fragments). Here are no experimental data at all. Even contact (sticking) cross section is not well defined in this case (both experimentally and theoretically). Thus, we may conclude that for realistic predictions of the cross sections for the
production of SH nuclei more efforts are needed not only in improvement of theoretical models but also (and first of all) in conducting of new experiments.

6. Further progress with the use of fusion reactions

One might think that the epoch of $^{48}$Ca in the production of SH nuclei was finished by the synthesis of element 118 in the $^{48}$Ca + $^{249}$Cf fusion reaction [88]. However this projectile still could be successfully used for the production of new isotopes of SH elements.

The extension of the area of known isotopes of SH elements is extremely important for better understanding of their properties and for developing the models which will be able to predict well the properties of SH nuclei located in the SH mass area (including those at the island of stability).

An important area of SH isotopes located between those produced in the “cold” and “hot” fusion reactions remains unstudied yet (see the gap in the upper part of the nuclear map in Fig. 28). Approaching the island of stability (confirmed, for example, by the fact that the half-life of the isotope $^{285}$Cn produced in the “hot” fusion reaction is longer by almost five orders of magnitude as compared to the $^{277}$Cn isotope of the same element produced in the “cold” synthesis) testifies about strong shell effects in this area of the nuclear map. Understanding these effects as well as other properties of SH nuclei is impeded significantly by the absence of experimental data on decay properties of the not-yet-synthesized isotopes of already known SH elements. Knowledge of the trends (especially along the neutron axis) of all decay properties of these nuclei (fission, $\alpha$- and $\beta$-decays) may help us to predict more accurately the properties of SH nuclei located at (and to the right of) the beta-stability line, including those which are located on the island of stability.

During recent time the synthesis of SH elements at the level of 1 pb became more or less a routine matter for several laboratories. The corresponding experiments require about two-week irradiation time to detect one or two events (decay chains) of SH element formation. This means that much more unknown isotopes of SH elements could be synthesized now, and the gap between nuclei produced in the “cold” and “hot” fusion reactions could be filled at last [90]. It can be done with the use of ordinary fusion reactions and, thus, with the use of existing recoil separators, in contrast with the mass transfer reactions (see below) for which separators of a new kind are needed. For this purpose several (rather cheap and available) isotopes of actinide elements...

![Fig. 28. Known nuclei in the upper part of the nuclear map.](image-url)
can be used as the targets (for example, $^{233,235}$U, $^{239,240}$Pu, $^{241}$Am, $^{243}$Cm and so on). Besides $^{48}$Ca, the beams of $^{36}$S, $^{44}$Ca and $^{40}$Ar are also of interest.

We found that it is more convenient (and easier) to close the gap “from above” by synthesis of new isotopes of SH elements with larger values of $Z$, their subsequent $\alpha$ decay chains just fill the gap [90]. This unexpected finding is simply explained by greater values of survival probabilities of the corresponding nuclei with $Z = 115, 116$ as compared to those with $Z = 111, 112$. In the left panel of Fig. 29 the values of $B_f - B_n$ are shown for the SH mass area, where $B_f$ is the fission barrier and $B_n$ is the neutron separation energy (an odd–even effect is smoothed here). As can be seen the values of $B_f - B_n$ are much higher, for compound nuclei with $Z \sim 116$ as compared with compound nuclei of $112$ element formed in fusion reactions of $^{48}$Ca with neutron deficient isotope of uranium. As a result, the corresponding survival probability of lighter CN is smaller by more than one order of magnitude.

As an example, the right panel of Fig. 29 shows survival probabilities of two compound nuclei, $^{283}$112 and $^{287}$114, formed in the fusion reactions $^{48}$Ca + $^{235}$U and $^{48}$Ca + $^{239}$Pu. The excitation energies of both compound nuclei (at collision energies equal to the corresponding Bass barriers, 195 and 198 MeV, correspondingly) are just the same for two reactions (they are about 30 MeV). Thus, in spite of the decrease of the fusion probability with increasing charge number of the target nucleus, one may expect that the EvR cross sections for the $^{48}$Ca + $^{239,240}$Pu, $^{48}$Ca + $^{243}$Cm reactions are higher (by about one order of magnitude for the 3n evaporation channel) due to the larger survival probability of $^{287,288}$114, $^{291}$116 compound nuclei as compared to $^{281–284}$112. This means that the new isotopes of element 112 could be easier synthesized and studied as $\alpha$ decay products of the heavier elements, 114 and/or 116. Note, however, that just for these isotopes of flerovium ($Z = 114$) the predicted shell corrections to their ground states (which define their fission barriers) change rather sharply [72] having minimal value for $^{286}$114. This facts brings additional uncertainty in the predicted cross sections.

In Fig. 30 the calculated EvR cross sections are shown for the production of new isotopes of elements 115 and 116 in the fusion reactions of $^{48}$Ca with $^{241}$Am, $^{44}$Ca with $^{243}$Am, $^{48}$Ca + $^{243}$Cm and $^{40}$Ar + $^{251}$Cf. High intensive beam of $^{40}$Ar can be obtained quite easily. This material is also much cheaper than $^{48}$Ca. However, as can be seen from Fig. 30, the use of an $^{40}$Ar
Fig. 30. Production cross sections for the new isotopes of elements 115 (a) and 116 (b) in $^{48}\text{Ca}$, $^{44}\text{Ca}$ and $^{40}\text{Ar}$ induced fusion reactions. The arrows show positions of the corresponding Bass barriers.

Fig. 31. Cross sections for the production of new elements 119 and 120 in the Ti and Cr induced fusion reactions predicted within the described above model (solid curves [71]) and by the “fusion-by-diffusion” model (dashed curves [91]). The latest calculations within the “fusion-by-diffusion” model [92] are shown by dash-dotted curves. The arrows indicate the upper limits reached in the corresponding experiments performed at GSI [93,94].

beam is less favorable as compared with $^{48}\text{Ca}$. This is owing to much “hotter” character of the $^{40}\text{Ar} + ^{251}\text{Cf}$ fusion reaction (only the cross sections for the 5n evaporation channels are comparable for both reactions). As can be seen the predicted cross sections are high enough to perform these experiments at available facilities.

6.1. Elements 119 and 120

Further progress in the synthesis of new elements with $Z > 118$ is not quite evident. Cross sections of the “cold” fusion reactions decrease very fast with increasing charge of the projectile (they become less than 1 pb already for $Z \geq 112$ [46,76]). For the more asymmetric $^{48}\text{Ca}$ induced fusion reactions rather constant values (of a few picobarns) of the cross sections for the production of SH elements up to $Z = 118$ were found [88]. However for the moment californium
(Z = 98) is the heaviest available target that can be used in experiments. The half-life of the einsteinium isotope $^{254}_{95}$Es is 276 days, sufficient to be used as target material. However, for the moment, it is impossible to accumulate the required amount of this matter (several milligrams) to prepare a target. To get SH elements with $Z > 118$ in a more realistic way one should proceed to heavier than $^{48}$Ca projectiles. $^{50}$Ti is most promising projectile for further synthesis of SH nuclei. Our calculations demonstrated that the use of the titanium beam instead of $^{48}$Ca decreases the yield of the same SH element due to a worse fusion probability by about factor 20 [71]. Nevertheless, the elements 119 and 120 can be produced in the fusion reactions of $^{50}$Ti with $^{249}$Bk and $^{249}$Cf targets (or in the $^{54}$Cr + $^{248}$Cm fusion reaction) with the cross sections of about 0.04 pb [71] which are already at the limit of the experimental possibilities. The first attempts to perform these experiments have been already made at GSI [93,94], but only the upper limits of the cross sections have been obtained. For synthesis of element 119 this limit is very close to the predicted cross sections [71] (see Fig. 31).

As already mentioned synthesis of these nuclei may encounter also another important problem. The proton rich isotopes of SH elements produced in these reactions are rather short-living due to large values of $Q_N$. Their half-lives are very close to the critical value of one microsecond needed for the CN to pass through the separator up to the focal plane detector (the distance of about 2 m). The next elements (with $Z > 120$) being synthesized in such a way might be already beyond this natural time limit for their detection [98].

6.2. Radioactive ion beams

Recently many speculations appeared also on the use of radioactive beams for synthesis and study of new elements and isotopes.

As shown above, the use of accelerated fission fragments for the production of SH nuclei in symmetric fusion reactions is less encouraging and needs beam intensities at the hardly reachable level of $10^{13}$ pps or higher.

The lighter radioactive ion beams could be quite useful to solve the two important problems. First they can be used to connect SH nuclei produced in the $^{48}$Ca induced fusion reactions with the “main continent” of already known nuclei with $Z \leq 106$. There are no combinations of stable nuclei to fill this gap in fusion reactions, while the use of radioactive projectiles may help to do this.

The second problem, which may be solved with the radioactive ion beams, is a synthesis of new more neutron rich transfermium isotopes. It is extremely important for two reasons. First, as we know from experiment, addition of only 8 neutrons to nucleus $^{277}$112 ($T_{1/2} = 0.7$ ms) increases its half-life by almost 5 orders of magnitude $– T_{1/2}(^{285}$112) = 34 s – testifying the approach of the “island of stability”. How far is it? How long could be half-lives of SH nuclei at this island? To answer these questions we need to add more and more neutrons. Second, somewhere in the region of $Z \sim 100$ and $N \sim 170$ the r-process of nucleosynthesis should be terminated by neutron-induced or $\beta$-delayed fission. This region of nuclei, however, is absolutely unknown and only theoretical estimations of nuclear properties (rather unreliable for neutron rich isotopes) are presently used in different astrophysical scenarios.

Contrary to common opinion, neutron excess of the projectile itself does not increase very much the EvR cross sections in fusion reactions of neutron rich radioactive nuclei. The neutron excess slightly decreases the height of the Coulomb barrier due to the small increase in the radius of neutron rich projectile. Neutron transfer with positive $Q$ value may significantly increase the sub-barrier fusion probability owing to “sequential fusion mechanism” (see [15,16].
Fig. 32. Excitation functions for the synthesis of the isotopes of the element 112 in 3n and 4n evaporation channels of the $^{48}\text{Ca} + ^{238}\text{U}$ ($A = 282$ and $A = 283$, dashed curves) and $^{44}\text{S} + ^{248}\text{Cm}$ ($A = 288$ and $A = 289$, solid curves) fusion reactions. Arrows indicate the corresponding Bass barriers for the two reactions.

Fig. 33. Excitation functions for synthesis of rutherfordium isotopes in the $^{18}\text{O} + ^{248}\text{Cm}$ ($A = 261$ and $A = 262$, dashed curves) and $^{22}\text{O} + ^{248}\text{Cm}$ ($A = 265$, $A = 266$ and $A = 267$, solid curves) fusion reactions. Experimental data for the $^{248}\text{Cm}(^{18}\text{O}, 5n)^{261}\text{Rf}$ reaction are from [95] (rectangles), [96] (triangles) and [97] (circles).

and Section 3.4). However, this mechanism does not increase noticeably the fusion probability at near-barrier incident energies, where the EvR cross sections are maximal (see above).

Fig. 32 shows the predicted EvR cross sections for the $^{44}\text{S} + ^{248}\text{Cm}$ reaction, in which the isotopes of the element 112 with six more neutrons (as compared with the $^{48}\text{Ca} + ^{238}\text{U}$ reaction) could be synthesized. The calculated one-picobarn cross sections mean that the beam intensity of sulfur-44 (which may be produced, for example, by 4p stripping from $^{48}\text{Ca}$) should be no less than $10^{12}$ pps to synthesize these extremely neutron rich isotopes.

In mass-asymmetric fusion reactions (with lighter than neon projectiles) there is no large suppression of CN formation: after contact colliding nuclei form CN with almost unit probability, $P_{\text{CN}} \approx 1$. This significantly increases the EvR cross sections in such reactions and, in spite of the rather difficult production of light radioactive nuclei with significant neutron excess, they could be used for the study of neutron rich transfermium nuclei.

For example, new heavy isotopes of Rutherfordium (up to $^{267}104$) might be obtained in the $^{22}\text{O} + ^{248}\text{Cm}$ fusion reaction. The EvR cross sections in this reaction (shown in Fig. 33) are rather large and the beam intensity of $^{22}\text{O}$ at the level of $10^8$ pps is sufficient to detect one decay event per week. Note that the reaction $^{22}\text{O} + ^{248}\text{Cm}$ is 3 MeV “colder” as compared to $^{18}\text{O} + ^{248}\text{Cm}$ ($E^*(\text{Bass}) = 41$ and 44 MeV, respectively) that allows one to measure even the
3n evaporation channel leading to $^{267}104$ (see Fig. 33). Half-lives of the heavy Rutherfordium isotopes ($A > 263$) should be rather long to use chemical methods for their identification.

7. Formation of superheavy nuclei in multinucleon transfer reactions

Upper part of the present-day nuclear map consists mainly of proton rich nuclei approaching the proton drip line (see Fig. 34). Very successful epoch of $^{48}$Ca induced synthesis of new superheavy (SH) elements is over. The heaviest available target material, Californium, was used to discover element 118 [88]. Due to the increasing bending of the stability line to neutron axis, in fusion reactions of stable nuclei one may produce only proton rich isotopes of heavy elements. For example, in fusion of rather neutron rich $^{18}$O and $^{186}$W isotopes one may get only the neutron deficient $^{204}$Pb excited compound nucleus, which after evaporation of several neutrons shifts even more to the proton rich side. That is the main reason for the impossibility to reach the center of the “island of stability” ($Z \sim 110–120$ and $N \sim 184$) in the superheavy mass region in fusion reactions.

At the same time neutron enriched isotopes of all heaviest elements were not synthesized and studied so far. This unexplored area of heavy neutron enriched nuclides (also those located along the neutron closed shell $N = 126$ to the right-hand side of the stability line) is extremely important for nuclear astrophysics investigations and, in particular, for the understanding of the r process of astrophysical nucleogenesis.

The multinucleon transfer processes in near barrier collisions of heavy ions, in principle, allow one to produce heavy neutron rich nuclei including those located at the island of stability. These
reactions were studied extensively about thirty years ago. Among other topics, there had been a great interest in the use of heavy-ion transfer reactions to produce new nuclear species in the transactinide region [99–105] (see also the recent review paper [106]).

The cross sections were found to decrease very rapidly with increasing atomic number of surviving heavy fragments. However, several Fm and Md isotopes have been produced at the level of 0.1 µb [103] (see Fig. 35). Renewed interest in the multinucleon transfer reactions with heavy ions is caused by the limitations of other reaction mechanisms for the production of new neutron rich heavy and SH nuclei. Multinucleon transfer processes in near barrier collisions of heavy (and very heavy, U-like) ions seem to be the only reaction mechanism (besides the multiple neutron capture process [1]) allowing one to produce and explore neutron rich heavy nuclei including those located at the SH island of stability.

Analysis of multinucleon transfer reactions in low energy collisions of heavy ions can be performed within the model described above. Energy, angular and mass distributions of primary reaction fragments are given by formula (28) and Eqs. (25). Excitation energies of these fragments are also known in all the exit channels and, thus, on the second reaction stage one need to use the same evaporation cascade (“cooling” procedure described above) to calculate the distributions of final (detected) fragments. Several examples of such calculations (rather time consuming) can be found in [48].

It was found that the shell effects may give a significant gain in the yields of heavy neutron rich nuclei formed in multinucleon transfer reactions [107,108]. Rather optimistic predictions were obtained for the production of SH nuclei at near barrier collisions of $^{238}$U with $^{248}$Cm. Cross sections higher than 1 pb have been predicted for the production of new neutron enriched isotopes of elements with $Z \leq 106$ located already at the stability line or even to the right of it.

These are the shell effects which may significantly enhance the yield of SH nuclei for appropriate projectile–target combinations. In Fig. 36 the charge and mass distributions of heavy primary reaction fragments are shown for near barrier collisions of $^{48}$Ca and $^{238}$U with curium target. The “lead peak” manifests itself in both reactions. However, for $^{48}$Ca + $^{248}$Cm collisions it corresponds to the conventional (symmetrizing) quasi-fission process in which nucleons are transferred mainly from a heavy target (here it is $^{248}$Cm) to lighter projectile. This is a well
Fig. 36. Calculated mass distributions of heavy primary reaction fragments formed in collisions of $^{48}\text{Ca}$ and $^{238}\text{U}$ with $^{248}\text{Cm}$ target at $E_{\text{c.m.}} = 220$ and 770 MeV, correspondingly. Schematic view of conventional and “inverse” quasi-fission processes are also shown.

studied both experimentally [67] and theoretically [48] quasi-fission process. It is caused just by the shell effects leading to the deep lead valley on the multidimensional potential energy surface which regulates the dynamics of the heavy nuclear system at low excitation energies (see Fig. 16).

Contrary to this conventional quasi-fission phenomenon, in low-energy collisions of $^{238}\text{U}$ with $^{248}\text{Cm}$ target nucleons may predominantly move from the lighter partner (here is uranium) to heavy one, i.e., U transforms to a Pb-like nucleus and Cm to complementary SH nucleus. In this case, appearance of the lead shoulder in the mass and charge distributions of the reaction fragments automatically leads to a pronounced shoulder in the region of SH nuclei (see Fig. 36). We named it “inverse” (anti-symmetrizing) quasi-fission process [109]. This process may really lead to enhanced yields of above-target nuclides, whereas even for rather heavy projectiles (like $^{136}\text{Xe}$) the nuclear system has a dominating symmetrizing trend of formation of reaction fragments with intermediate masses (heavier than projectile and lighter than target), see Fig. 37.

The use of actinide beams and actinide targets give us a possibility to produce new neutron enriched isotopes of transfermium elements located along the stability line and to the right of it, that is in the unexplored area of the nuclear map (see Fig. 34). Properties of neutron rich Fermium isotopes with $A > 260$ are extremely interesting by several reasons. First, as mentioned above, all known isotopes of Fermium (and of more heavy elements) are located to the left side of the beta-stability line (see Fig. 34). Second, the well known “fermium gap” ($^{258-260}\text{Fm}$ isotopes with very short half-lives for spontaneous fission) impedes formation of nuclei with $Z > 100$ by the weak neutron fluxes realized in existing nuclear reactors. It is extremely interesting to know what is the first $\beta^-$-decayed Fermium isotope and how long is its half-life. This is important not only for reactor but also for explosive nucleosynthesis in which this Fermium gap might be
bypassed [1]. As can be seen from Fig. 38 neutron rich fermium isotopes can be produced in low-energy transfer reactions with cross sections of about 0.1 µb, that is quite sufficient to be produced at available accelerators.

The yields of neutron enriched long living isotopes of superheavy elements in transfer reactions might be significantly enhanced owing to the shell effects leading to the mentioned above “inverse quasi-fission” phenomena. In Fig. 38 the results of calculations are shown for the formation of survived isotopes of some transfermium elements in reaction $^{238}$U + $^{248}$Cm at 770 MeV center-of-mass energy. The obtained results are rather optimistic. New neutron rich isotopes of transfermium elements with $Z = 100–104$ (located already at the stability line and beyond it) can be produced with the cross sections of several hundreds of picobarn. The cross sections for the production of new neutron rich isotopes of seaborgium and hassium ($Z = 106, 108$) are also higher than 1 picobarn. Predicted cross sections depend on the values of neutron and proton transfer rates which are not determined yet very accurately (see Refs. [55,56] and [48]).
If the shell effects play a noticeable role in collisions of actinide nuclei (in contrast with monotonically decreases of the cross sections with increasing number of transferred nucleons), then they really open a door to the production of neutron rich superheavy nuclei.

Unfortunately there are only scanty experimental data on the production of trans-target reaction fragments in low-energy damped collisions of heavy ions. It is very difficult to perform direct experiment on collision of $^{238}$U with $^{248}$Cm target with rather accurate measurement of the mass distribution of primary reaction fragments.

To study the shell effects leading to enhanced yield of trans-target fragments (“anti-symmetrizing” mass transfer) one needs to choose projectile–target combinations with non-magic initial nuclei (not like $^{136}$Xe + $^{208}$Pb reaction in which the nucleon diffusion dominates because initial configuration locates already at the bottom of deep potential valley due to high binding energies of these nuclei). One of such combination, $^{160}$Gd($Z = 64, \ N = 96$) + $^{186}$W($Z = 74, \ N = 112$), was proposed in Ref. [110] and studied experimentally [111]. In this system one neutron closed shell ($N = 82$) is located to the “left” of the projectile whereas the other ($N = 126$) to the “right” of the target on the mass axis. Our calculations predict quite unusual mass distribution of reaction fragments in this reaction: The shell effects lead to pronounced shoulder for Pb-like and Ba-like nuclei. The gain in the yield of trans-target nuclei with $A > 205$ (transfer more than 20 nucleons) owing to the shell effects is predicted to be more than 2 orders of magnitude as compared with normal (conventional) behavior of multinucleon transfer cross sections without any shell effects.

In the experiment it was found even more exotic mass distribution: almost constant values of the cross section for formation of nuclei with $A > A_{\text{th}}$ (see Fig. 39).

Unfortunately, the experimental technique (catcher foils + off-line radiochemistry) did not allow to measure the yields of stable (as well as short-living) isotopes and to obtain the specific (shoulder-like) shape of the mass distribution. Recently (in June of 2014) this experiment has been performed at Flerov Laboratory (JINR, Dubna) with online detection of both reaction fragments by two-arm detection system. The results of this experiment fully confirmed the predicted (inverse quasi-fission) shape of the mass distribution in this reaction. Thus, neutron enriched

![Fig. 39. Predicted [110] and observed [111] mass distributions of reaction fragments formed in collisions of $^{160}$Gd with $^{186}$W at $E_{\text{c.m.}} = 460$ MeV. Experimental data are shown only for target-like fragments.](image-url)
transfermium and superheavy nuclei can be really produced in low-energy collisions of actinide ions (such as U + U or U + Cm). Predicted and experimentally confirmed shell effects (leading to the “inverse quasi-fission” phenomena) significantly increases the yields of trans-target nuclei in these reactions.

8. Conclusion

One may conclude that our ability for the predictions of the cross sections for the production of superheavy nuclei (both in fusion and in transfer reactions) is not so good. It is different for different reactions and different combinations, but on average accuracy of such predictions is not better than one order of magnitude. To improve significantly predictive power of the models we really need more experimental data (which, of course, are difficult to be obtained).

(1) First of all this is direct experimental comparison of EvR cross sections for the $^{48}$Ca and $^{50}$Ti induced fusion reactions with actinide targets leading to the same (or very close) CN. Other combinations (leading to the same CN) are also of great interest.
(2) Any experimental information is needed for EvR formation in fusion reactions of heavy mass symmetric nuclei (Sn + Sn, Xe + Xe and so on).
(3) One need to study much better the shell effects in low energy collisions of actinide nuclei to plan the use of multinucleon transfer reactions for the production of neutron enriched SH nuclei.

References
