Semiclassical Theory of Direct
and Deep Inelastic Heavy Ion Collisions

V. I. ZAGREBAEV

Chuvash State University, Cheboksary 428000, USSR
Received May 3, 1989

A sufficiently simple approach is formulated for the quantitative description of direct and multi-step processes in heavy ion collisions with arbitrary (not only small) mass, energy, and angular momentum transfer. The simplification is achieved owing to the use of semiclassical description of the colliding particles relative motion. The semiclassical approximation is used for the three-dimensional (not partial) distorted waves with accounting of all caustic properties, and the channel coupling is simulated by the dissipative forces. The transition amplitude is reduced to the one-dimensional integral along the so-called “transition lines” where the inelastic process is localized. The comparison is done with the experimental data and with exact calculations of some direct reaction cross sections. Heavy-ion-induced multi-step (quasidirect) massive transfer processes with the fast light particle formation are analyzed.

1. INTRODUCTION

At present heavy-ion physics undoubtedly takes the leading place among all nuclear studies, inspiring us with the certain hope of not only thoroughly clarifying the dynamics of the nucleus–nucleus interaction, but also revealing the quite novel properties of nuclear matter. However, extensive experimental and theoretical studies on heavy ion collisions for more than twenty years do not fully justify these hopes. On the one hand, extremely interesting and sometimes unexpected experimental data are accumulated [1]. On the other hand, we have rather poor information on the dynamical properties and the structure of complex nuclei from the processing of these data.

Up to now, however surprising it is, we have not determined the very fundamental characteristics of the nucleus–nucleus interactions. We do not know, as before, the ion–ion potential forces near and behind the Coulomb barrier. Clearly observing a high-rate process of kinetic energy dissipation into an internal exitation of colliding nuclei we still have not determined unambiguously either the mechanism responsible for this process or the value and the character of the dissipative forces. Experimentally studying the massive transfer reactions and the heavy-ion radioactivity (the spontaneous emission from the “cold” nuclei of such ions as $^{14}$C or $^{34}$S)
we have only a very vague idea about multi-nucleon clustering, i.e., about potentials, formfactors, and spectroscopy of heavy fragments inside nuclei. It is not difficult to continue the list of our ignorance.

There are both experimental and theoretical reasons for such a state. The inclusive character of the most interesting experimental data does not allow us to "discern" and to fix unambiguously the mechanism of the process under study. The absence of adequate theoretical models capable of describing all the sets of experimental data only redoubles this uncertainty.

In "pre-heavy-ion" nuclear physics one could single out experimentally and use for the study of one or another nucleus property two limit (and, therefore, clear enough) mechanisms of nuclear reactions—the process of the compound-nucleus formation and the direct process. It was the direct reactions and elastic scattering, with the help of which such fundamental characteristics as the nucleus mean field and the single-particle states, an existence and behaviour of the few-nucleon clusters inside nuclei, etc. were determined quantitatively. However, with the increase of the colliding nuclei masses the role of direct processes and of reactions going through the formation of a compound nucleus, becomes noticeably smaller. As a result, it is not only difficult to single out the direct one-step reactions in heavy-ion collisions, but they are also "poor-informative." The latter is due to the very strong coupling of the channels that leads to the strong absorption in an elastic one. As a consequence, such processes are localized in an extremely peripheral region and do not allow us to look inside the nucleus.

Thus, the description of deep inelastic processes of collisions (DIP) became the key problem of the heavy-ion-induced nuclear reactions theory. By this term one designates in fact all varieties of reactions with mechanisms intermediate between one- or few-step quasielastic processes (with extraction of low-excited residual nuclei) and the process of complete fusion of colliding nuclei (with attainment of the full thermodynamical equilibrium and with subsequent independent decay of the compound system). In collisions of very heavy ions the total reaction cross section is almost exhausted by the deep inelastic processes. For lighter systems, DIP make the considerable contribution to the yield of most of the fragments forming in the reaction. The main feature of DIP is the kinetic energy dissipation realized in the energy spectra of the fragments spread over the wide range: from the energies corresponding to the low excitation of residual nuclei up to ones equal to the height of the Coulomb barrier in the exit channel, i.e., full relaxation of kinetic energy. Both the coherent (the self-consistent motion of nucleons in the time-dependent mean field, surface vibration, rotation of nucleus, multi-nucleon transfer) and the stochastic motion of nucleons are inherent in deep inelastic processes. Therefore, there are two approaches to describe DIP: the one from the compound nucleus side (moving from the full thermodynamical equilibration to pre-equilibrium states) and the other from the direct reaction side (moving from one-step processes to the multi-step ones). It is the first approach that is best developed at present [2]. In such an approach the main attention is concentrated on the final stage of the reaction, when the collective variables relax toward the equilibrium state and the...
internal degrees of freedom are considered on a statistical basis. The specification of transport coefficients allows us to evaluate the mean widths of mass and energy distribution of the reaction products. Here the most interesting coherent motion of nucleons and all interference phenomena are usually displaced out of consideration, since the basic equations are formulated in terms of probabilities, but not of the wave functions or transition amplitudes.

However, for the highly informative data, the detection of the particles formed just on the initial stage of reaction (the products of few-nucleon transfer, fast light particles, high-energy \( \gamma \)-rays, etc.) is of the most interest, because two colliding nuclei, relaxing to the equilibrium state and ultimately fusing, gradually forget all the past and the decay of such a compound system contains almost no information about the interaction dynamics of the initial nuclei. So, some years ago [4] it was proposed by the author to work out an alternative approach to describe DIP just from the direct reaction side, in which the multi-channel wave function would be kept and the cross section of the different processes would be defined by the corresponding transition amplitudes. Here the channel's coupling ought to be simulated with the semiclassical transport equation (for energy and, maybe, mass transfer) containing such values as friction forces, diffusion coefficients, etc.

An exact solution of multi-channel problem within quantum scattering theory meets with numerous difficulties, some of them are practically insuperable in heavy-ion collisions. The large number of channels, the complexity of interaction coupling the relative motion with the internal degrees of freedom, the unknown nature of the highly excited nuclear states, the necessity of accounting for hundreds of partial waves in the entrance and exit channels make the ordinary coupled channels approach practically inapplicable for the description of such processes. On the other hand, the small wavelength and the high level density of exciting states lead to an exhibition of the classical properties of relative motion and, therefore, validate the semiclassical approximation when evaluating wave functions and transition amplitudes.

Semiclassical approximation has been used for the description of heavy ion inelastic collisions since the study of these reactions began [5]. However, in spite of the successful application of the semiclassical approximation for description of elastic scattering (see, e.g., [6] and references therein), in the case of inelastic collisions success in achieving the essential progress only occurs for one- or few-step processes with small mass, energy, and angular momentum transfer, when the ejectile's trajectory is close enough to the elastic trajectory of an incident particle. Here the relative motion is described either quite classically [7] or the distorted waves are expanded over partial waves for which the one-dimensional WKB-approximation is used [8]. Since the semiclassical evaluation of one-dimensional partial waves is not easier than an exact evaluation of them, the noticeable simplification can be obtained here only by means of additional assumptions when

---

1 In the past few years attempts for the combined description of the nucleon coherent and stochastic motion in heavy ion collisions are have been made more and more often [3].
summing over partial waves of entrance and exit channels in transition amplitude [8].

As a result, there are many works in which the evaluation of different quasielastic direct processes cross sections is made with an exact formula (by means of the numerical solution of the Schrodinger equation for partial waves), but the qualitative analysis of the obtained results is carried out using visual semiclassical language. Here the qualitative (physical) conclusions about the reaction mechanism are not always correct. As shown below, one of the typical delusions is, for example, sometimes the unjustified use of the formula

$$\frac{d\sigma_{f}}{d\Omega} = \frac{d\sigma_{f}^{\gamma} (\Theta)}{d\Omega} \cdot P_{f}(\Theta)$$

for the cross sections of inelastic processes.

In this work the semiclassical approach for description of the direct and multi-step processes of heavy ion collisions is stated, based on the following scheme:

1. The semiclassical approximation must be constructed in the simple form just for the three-dimensional wave function of the heavy ion relative motion without resorting to its partial expansion and with accounting for all caustic singularities (Section 2).

2. Evaluating the transition amplitude of the direct process, we ought to use the multi-dimensional stationary phase method to find the spatial regions ("transition lines") that make predominant contributions to the given reaction. Transition amplitude is then written as the one-dimensional integral along these lines (Section 3).

3. Using the semiclassical approximation for three-dimensional channel wave functions and the high level density of exciting states we have to reduce, at last, a set of coupled equations to a transport equation allowing simulation of the channel coupling by the phenomenological friction forces (Section 4).²

Such an approach helps us to analyze not only the direct reactions with arbitrary mass, energy, and angular momentum transfer, but also the multi-step reactions, when the direct heavy fragment transfer, for example, is accompanied by considerable kinetic energy dissipation in the entrance and exit channels—the so-called "quasidirect" processes, that seem to help describe the sizable part of deep inelastic reactions.

2. CAUSTIC SURFACES AND DISTORTED WAVES

The de Broglie wavelength $\lambda = h/mv$ of the heavy ion relative motion is sufficiently small and, therefore, the properties of the corresponding wave function

² An attempt to prove item 2 of this scheme was made in [11], but the authors of [11] had not seen the "transition lines" in the multi-dimensional stationary phase condition that make it possible to reduce the transition amplitude to a very simple and physically obvious expression.
$\psi_{k^+}^{\lambda}(r)$ are defined to a considerable extent by a set of the colliding particle classical trajectories. In particular, the considerable increase of the wave function amplitude near the caustic surfaces is expected, where it aspires to infinity in the limit $\lambda \to 0$. If the caustic surface stretches to infinitely large distances, it manifests itself in a rainbow elastic scattering. Inelastic scattering and reactions occurring from contact of the nuclei surfaces are defined by the properties of the elastic channel wave function $\psi_{k^+}^{\lambda}$ (i.e., by a set of elastic trajectories and, in particular, by the caustics) at finite distances.

At high energies the trajectories are almost straight lines and there are not any caustic surfaces at small distances. The situation is quite different at near-barrier energies, when the two simplest caustics coalesce (forming the cusp) just near the nuclei surfaces, and a sharp enhancement of the wave function amplitude in this region is expected. The great interest in heavy ion collisions at near-barrier energies caused, in particular, by attempts to synthesize the superheavy elements forces us to clear up more carefully the properties of wave functions (including their caustic features) for such energies at small distances.

The wave functions of relative motion are used also for evaluation of the direct reaction transition amplitudes (within the framework of DWBA) and are figured as the channel wave functions in the coupled channels method as it is applied for the description of multi-step processes. In ordinary approaches (and in the different numerical codes) these functions are expanded over partial waves, so that only the one-dimensional radial wave functions remain after all. However, here the necessity to sum up the partial waves of the entrance and exit channels appears to extremely complicate evaluation of the transition amplitudes, especially in the case of large mass, energy, and angular momentum transfer. As shown below, one can significantly simplify evaluation of the inelastic process cross sections using semi-classical approximation for the three-dimensional wave function $\psi_{k^+}^{\lambda}(r)$ without its partial wave expansion. In this case greater classical clarity of the inelastic transition is also achieved.

The wave function $\psi_{k^+}^{\lambda}(r)$ describing elastic scattering of heavy ions obeys the Schrödinger equation ($E = \hbar^2 k^2/2m$)

$$\left\{ -\frac{\hbar^2}{2m} \nabla^2 + V(r) - E \right\} \psi_{k^+}^{\lambda}(r) = 0 \quad (2.1)$$

with the potential $V(r) = V_c(r) + V_N(r) - iW(r)$, where $V_c(r)$ is the Coulomb interaction of two uniformly charged spheres, $V_N(r)$ is a short-range nuclear part of the potential, and the imaginary addition $iW(r)$ describes the loss of the flux in the elastic channel due to coupling with the other reaction channels. At present we have only a qualitative idea about the values of $V_N(r)$ and $W(r)$ for relatively heavy ions. The total potential $V_c(r) + V_N(r)$ has a typical maximum in a surface region—the Coulomb barrier $V^p = V_c(R^p_c) + V_N(R^p_c)$; here all the energies (as above and below the Coulomb barrier) are of experimental interest.
In all cases we need a regular solution of Eq. (2.1) at \( r = 0 \) with the boundary condition

\[
\psi_k^+(r \to \infty, \Theta) \sim \exp\{ikZ - i\eta \ln k(r - Z)\} + \frac{F(\Theta)}{r} \exp\{ikr - i\eta \ln 2kr\}
\]  

(2.2)

corresponding to the scattering state of a particle with momentum \( \mathbf{k} \) parallel to the \( z \)-axis. Here \( F(\Theta) \) is the elastic scattering amplitude and \( \eta = k(Z_1 Z_2 e^2/2E) \) is the Sommerfeld parameter.

2.1. Distorted Waves Far from the Caustics

There are a few possibilities for constructing the three-dimensional wave function \( \psi_k^+(r) \) in the small wavelength limit. First of all is the "trajectory method" [12], in which \( \psi_k^+(r) \) is immediately presented as a sum of the contributions of different classical trajectories (including the complex ones) obeying the boundary conditions and coming to a given point \( r \). In the second method one should separate the variables of the Schrödinger equation, then write \( \psi_k^+(r) \) as an integral (over separation parameter) from the one-dimensional wave functions, and, last, evaluate this integral using the saddle point method. In the case of central potential \( \psi_k^+(r) \) is expanded on the partial waves, the sum over \( l \) is reduced to an integral over the impact parameter \( b = (1 + 1/2)/k \), which then should be treated by the saddle point method using the one-dimensional WKB-approximation for radial wave functions [6]. The final results are naturally the same in both cases. However, the second method seems preferable in concrete applications, since the amplitudes and the phases of the different items are immediately determined and the approximation of \( \psi_k^+(r) \) by the standard integrals near the caustic surfaces is easily obtained in this method.

Following this scheme, we write the wave function for heavy-ion relative motion in the form [9]

\[
\psi_k^+(r, \Theta) = \int_0^\infty \sqrt{i} g(b, r, \Theta) \{ e^{iS_L(b, r, \Theta)} - i e^{iS_R(b, r, \Theta)} \} \, db,
\]  

(2.3)

where

\[
g(b, r, \Theta) = \frac{kb}{r} \left[ 2\pi k(b, r) \cdot b \cdot \sin \Theta \right]^{-1/2}, \quad k(b, r) = k \sqrt{1 - \frac{V(r)}{E} \frac{b^2}{r^2}},
\]

\[
S_L = -C(b, r) + kb(\pi - \Theta), \quad b < b_0(r)
\]

(2.4a)

\[
S_R = -2C_0(b) + C(b, r) + kb(\pi - \Theta), \quad b < b_0(r)
\]

(2.4b)

\[
C(b, r) = kr - \eta \ln 2kr - \int_r^\infty \left\{ \frac{k(b, r) - k + \eta}{r} \right\} \, dr;
\]

(2.5)
$C_0(b) = C(b, r_0)$, $r_0(b)$ is an external turning point: $k(b, r_0) = 0$. At $b > h_0(r) = r \cdot \sqrt{1 - V(r)/E}$, the integrand in (2.3) decreases exponentially:

$$S_L = S_R = -C_0(b) + k b (\pi - \Theta) + i \int_{r}^{r_0} \frac{b^2}{r^2} \frac{V}{E} \, dr.$$  \hspace{1cm} (2.6)

$s, = s, = -C,(b) + kb(\pi - \Theta) + i (7.6)$

$hS_{L,R}(b, r, \Theta)$ are the classical action functions evaluated along the trajectory with the asymptotic momentum $k$ and impact parameter $b$. The function $S_L$ corresponds to the part of trajectory up to the turning point $r_0(b)$, where the radial velocity is negative ($\partial S_L / \partial r = -k(b, r)$), and $S_R$ is the continuation of the action function on the part of trajectory "behind" $r_0(b)$ ($\partial S_R / \partial r = k(b, r) \geq 0$).

As a rule, there are a few turning points (some of them may be complex) for the motion of a particle with angular momentum 1 in the central potential $V(r)$. All of them should be retained in order to construct the WKB-approximation for radial wave functions $\psi_1(r)$ [6]. However, in the case of the strong absorption inherent in heavy ion scattering one can neglect all the trajectories passing through the region of large $W(r)$ values, since the contribution of each trajectory like that is weakened approximately with a factor $\exp\{- (m/h^2k) \int_{r}^{r_0} W(r') \, dr'\}$. First of all it applies to the trajectories turning around the nucleus (the negative branch of deflection function in Eq. (2.7b) given below) and also to the trajectories leaking under the barrier and reflecting from the inner turning points (in this case one more damping factor arises due to the barrier penetration decreasing faster than any power of $E$). We omit in (2.3) the contribution of all such trajectories to obtain the simplest expression for $\psi_k(r)$. Note, that in collisions of relatively light ions such trajectories make a noticeable contribution to the wave function and to the cross sections of elastic scattering and reactions [6].

In the short wavelength limit $\lambda \to 0$ (or $k = 1/\lambda \to \infty$) the value of the integral (2.3) is defined by the strongly oscillating factors in the braces. The equations for the stationary points of corresponding phases ($\partial S_L / \partial b = 0$ and $\partial S_R / \partial b = 0$)

$$\varphi(b, r) = \pi - \Theta$$  \hspace{1cm} (2.7a)

$$\varphi(b, r) + \Omega(b) = \Theta$$  \hspace{1cm} (2.7b)

define apparently the impact parameters $b_i(r, \Theta)$ of the trajectories passing through the point $(r, \Theta)$. Here

$$\varphi(b, r) = \int_{r}^{r_0} \frac{k \cdot b}{r^2} \frac{dr}{k(b, r)}$$

is a turning angle along the trajectory with impact parameter $b$, and $\Omega(b) = \pi - 2\varphi(b, r_0)$ is a deflection function of a particle in the potential $V(r)$. Far from the caustics (the isolated stationary points $b_i$) the increase of the factor $g \sim \sqrt{k}$ is
exactly compensated by the oscillations of $\exp\{iS_{L,R}\}$ and the wave function
$\psi_k^{(+)}(r) \sim O(\lambda^0)$ retains its finite value

$$\psi_k^{(+)}(r, \Theta) = \sum_i A(b_i, r, \Theta) e^{iS(b_i, r, \Theta)} + O(\lambda^{1/2})$$ (2.8)

$$A(b, r, \Theta) = \frac{b}{r} \left[ \frac{k(b, r)}{k} b(r, \Theta) \cdot f(b, r) \sin \Theta \right]^{-1/2} + O(1).$$ (2.9)

Here $f(b, r) = (\partial/\partial b) \varphi(b, r)$, $S(b_i) \equiv S_L(b_i)$ if $b_i$ is the solution of (2.7a) and $f(b, r) = (\partial/\partial b)[\varphi(b, r) + \Omega(b)]$, $S(b_i) \equiv S_R(b_i)$ if $b_i$ is the solution of (2.7b). One can see that there are not any singularities in (2.8) and (2.9) nor at the $z$-axis ($\Theta = \pi, b = 0$).

In the numerical calculations we restricted ourselves to the real solutions of (2.7) obtained for the real part of the distorting potential $V(r)$. The imaginary part $iW(r)$ was taken into account only in the phase $S(b_i, r, \Theta)$, leading to the appearance of an imaginary addition to it and, accordingly, to the decrease of the contribution of all those trajectories that passed through the region of strong absorption. This leads to the shadow region inside the nucleus and behind it (see Fig. 3 below).

2.2. Distorted Waves near the Caustics

Any surface touching some set of trajectories divides the space into two parts, in which the number of trajectories passing through each point (i.e., the number of solutions of (2.7)) and, therefore, the number of terms in the sum (2.8) changes for two units. At such a surface the expansion (2.8) becomes singular and, therefore, also breaks down not far from it. This surface is called a caustic of the first kind, or the simplest caustic.

In the case of scattering by central forces all such surfaces have a cylindrical symmetry and may be denoted by $\Theta_a(r)$ or $r_a(\Theta)$. At the caustic surface two stationary points coalesce $b_1(r, \Theta_a) = b_2(r, \Theta_a) \equiv b_a(r)$, i.e., $(\partial^2 S/\partial b^2)|_{b = b_a} = 0$. Since $\varphi(b, r)$ is a monotonous function of parameter $b$, any caustic surface is formed by the sections of trajectories lying behind the turning points. Thus, the wave function is defined by the properties of $S_R(b, r, \Theta)$ which we expand in the powers of $(b - b_a)$ near such surface, keeping the lowest tree orders

$$S_R(b, r, \Theta) \approx S_R(b_a, r, \Theta) + k[\Theta_a(r) - \Theta](b - b_a) + \frac{\omega(r)}{3}(b - b_a)^3.$$ (2.10)

Here $\omega(r) = \frac{1}{2}k(\partial^2/\partial b^2)[\varphi(b, r) + \Omega(b)]|_{b = b_a}$. The cubic parabola (2.10) has two extrema (at $b_{1,2} = b_a + \sqrt{(k/\omega)(\Theta - \Theta_a)}$) on the one side from the caustic, and does not have any on the other side.

Thus, near the simplest caustic (on both sides of it) the wave function $\psi_k^{(+)}(r)$
can be approximated by the first standard integral \( \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\{i\alpha t + i(\alpha^3/3)\} dt = \text{Ai}(z) \) — the Airy function,

\[
\psi_{k+1}^{+}(r, \Theta) = g(b_{z}, r, \Theta) \omega^{-1/3} \cdot 2\pi \cdot \text{Ai}\left[ \frac{k(\Theta - \Theta_0)}{\alpha^{1/3}} \right] e^{iS_{R}(b_{z}, r, \Theta) - \pi/4}
+ \sum_{i \neq 1, 2} A(b_{i}) e^{iS_{R}(b_{i})} + O(\lambda^{1/2}).
\]

The first term in (2.11) has the order \( \lambda^{-1/6} \). It is the typical increase at \( \lambda \to 0 \) that the wave function amplitude has near the simplest caustic. The second term in (2.11) (proportional to \( \lambda^0 \)) is absent if the caustic surface \( \Theta_{z}(r) \) separates the classically forbidden region, in which Eqs. (2.7) have no real solutions. Notice, that in the concrete calculations even more accurate results can be obtained if one introduces the new curvilinear variables (instead of \( r \) and \( \Theta \)), one of which lies at the surface \( \Theta_{z}(r) \) and the other which is orthogonal to it.

When the two simplest caustic surfaces coalesce the caustic of the second type — the so-called caustic cusp — arises. The point of coalescence \( (R_{\text{cusp}}, \Theta_{\text{cusp}}) \) and the impact parameter \( b_{\text{cusp}} \) of the trajectory passing through this point are defined from three equations \( \partial S/\partial b = 0 \), \( \partial^2 S/\partial b^2 = 0 \), and \( \partial^3 S/\partial b^3 = 0 \).

Let us denote the solution of equation \( \partial^3 S/\partial b^3 = 0 \) by \( b_{m}(r) \) and also introduce the designations

\[
\Theta_{m}(r) = \varphi(b_{m}, r) + \Omega(b_{m}), \quad \chi(r) = \frac{k}{24} \left[ \frac{\partial^3}{\partial b^3} \right] \left[ \varphi(b, r) + \Omega(b) \right]_{b = b_{m}},
\]

\[
X(r) = \frac{k}{2 \sqrt{\chi}} \left[ \frac{\partial}{\partial b} \left[ \varphi(b, r) + \Omega(b) \right] \right]_{b = b_{m}}, \quad Y(r, \Theta) = \frac{k}{\chi^{1/4}} \left[ \Theta_{m}(r) - \Theta \right].
\]

Here \( b_{\text{cusp}} = b_{m}(R_{\text{cusp}}), \Theta_{\text{cusp}} = \Theta_{m}(R_{\text{cusp}}), \chi(R_{\text{cusp}}) = 0, \; Y(R_{\text{cusp}}, \Theta_{\text{cusp}}) = 0. \) Keeping the terms up to fourth order in expansion of \( S_{R}(b, r, \Theta) \) for powers of \( (b - b_{m}) \), we obtain the approximation of the wave function \( \psi_{k+1}^{+}(r, \Theta) \) near the cusp in terms of the second standard integral \( -\int_{-\infty}^{\infty} \exp\{i(yt + xt^2 + t^4)\} dt \equiv I(X, Y) \) — the Pearcey function:

\[
\psi_{k+1}^{+}(r, \Theta) \approx g(b_{m}, r, \Theta) X^{-1/4} I(X, Y) e^{iS_{R}(b_{m}, r, \Theta) - \pi/4}.
\]

Thus, the wave function amplitude increases as \( \lambda^{-1/4} \) at \( \lambda \to 0 \) near the cusp, i.e., faster than near the simplest caustic.

The accounting of trajectories turning around the nucleus (negative branch of \( \Omega(b) \)) and also the complex trajectories of the particles passing under the barrier and reflecting at the inner turning points can, in principle, lead to more complicated caustic surfaces. However, as we see below, because of the strong absorption and short wavelength such caustics do not practically manifest themselves at small distances in the elastic channel wave function of the heavy ion relative
motion. Classification of the more complicated caustics and corresponding standard integrals can be found in Ref. [14].

2.3. Subbarrier Energies and the Coulomb Wave Function

At the energies below the Coulomb barrier the real trajectories cannot penetrate inside the nucleus and are not distorted by the nuclear forces (Fig. 1a). In this case the wave function \( \psi_{k}^{(+)}(r) \) is very close to the Coulomb one,

\[
\psi_{k}^{(+)}(r) = e^{-\frac{\pi k r}{2}} (1 + i\eta) e^{ikr\cos \theta} F_{1}(-i\eta; 1; -ikr(1 - \cos \theta)), \tag{2.13}
\]

an exact evaluation of which is also rather awkward. Using the expressions (2.8)–(2.11) and an explicit form of the trajectories in the potential \( V_{C} = Z_{1}Z_{2}e^{2}/r \), we can obtain the semiclassical approximation for the three-dimensional Coulomb wave function (2.13).

![Classical trajectories and wave function amplitude](image)

**Fig. 1.** Classical trajectories (a) and the wave function amplitude for \(^{22}\text{Ne} + ^{181}\text{Ta}\) elastic scattering at \( E_{\text{cm}} = 73 \text{ MeV} \). The optical potential of the Saxon–Woods form was used with \( V_{0} = 75 \text{ MeV}, R_{V} = 10 \text{ fm}, W_{0} = 10 \text{ MeV}, R_{W} = 11 \text{ fm}, a_{V} - a_{W} = 0.5 \text{ fm} \) (\( V_{C} = 80 \text{ MeV} \)). The dashed line is the Coulomb caustic.
For the pure Coulomb interaction of the particles we have

\[ \varphi(b, r) = \arctan \frac{kb/r + \eta/b}{k(b, r)} - \arctan \frac{\eta}{kb}, \quad \Omega(b) = 2 \arctan \frac{\eta}{kb}. \]

\[ C(b, r) = \eta + k(b, r) - \eta \ln[k(b, r) \cdot r + kr - \eta] + kb \cdot \varphi(b, r). \]

The caustic surface \( \Theta_c(r) = \arccos(1 - 4\eta/kr) \) (or \( r_c(\Theta) = 4\eta/k(1 - \cos \Theta) \)) isolates the classically forbidden region from the region where two trajectories with impact parameters

\[ b_{1,2}(r, \Theta) = \frac{r \sin \Theta}{2} \left\{ 1 \pm \sqrt{1 - \frac{4\eta}{kr} \left(1 - \cos \Theta\right)} \right\} \]

pass through each point \((r, \Theta)\). In this region the surface \( \Theta_o(r) = \pi/2 + \arcsin(\eta/(kr - \eta)) \) also lies, containing all the turning points. On the left of this surface \((\Theta > \Theta_o(r))\) each of Eqs. (2.7a) and (2.7b) has one solution; on the right, only Eq. (2.7b) has two solutions. At the caustic surface \( \Theta_c(r) \) these two solutions coalesce \( b_1 = b_2 = b_c(r) = \sqrt{(2\eta/k)(r - (2\eta/k))} \), and for \( \Theta < \Theta_c(r) \), Eqs. (2.7) do not have real solutions.

Using the variables \( \sigma = r \cdot (1 - \cos \Theta) \) and \( \tau = r \cdot (1 + \cos \Theta) \) and the designations \( \alpha = 4\eta/k \sigma \) and \( \beta = 1 + \sqrt{1 - \alpha} \), we have in the region \( \Theta > \Theta_c \):

\[ \psi_k^{(1)}(r, \Theta) \approx A_1 e^{iS_1} + A_2 e^{iS_2} \]

\[ \psi_k^{(2)}(r, \Theta) \approx \frac{e^{ik/2}(\sigma + \tau)}{(1 - \alpha)^{1/4}} \left\{ \frac{\beta}{2} \exp \left[ i \left( -k \sigma + \eta \ln k \sigma + 2\eta \ln \frac{\beta}{2} + \frac{\eta \alpha}{\beta^2} \right) \right] ight\} \]

\[ + \frac{\eta}{k \sigma} \left( 1 + \frac{\alpha}{\beta^2} \right) \exp \left[ i \left( -\eta \ln k \sigma - 2\eta \left( 1 + \frac{\alpha}{2\beta^2} - \ln \left( 1 + \frac{\alpha}{\beta^2} \right) \right) - \frac{\pi}{2} \right) \right]. \]

The interference of two terms (incident and outgoing waves)

\[ A_1 \to 1, \quad A_2 \to \frac{|F_{C}(\Theta)|}{r} \to 0 \quad \text{at} \quad \sigma = r \cdot (1 - \cos \Theta) \to \infty \]

leads to damped oscillations of the Coulomb wave function amplitude in the region \( \Theta > \Theta_c(r) \) (Fig. 1b).

Near the caustic surface and in the classically forbidden region, the Coulomb wave function is defined by the expression (2.11), that can be reduced to the form

\[ \psi_k^{(1)}(r, \Theta) \approx \sqrt{\pi(2\eta)^{1/6}} \text{Ai} \left( \frac{k(\sigma_c - \sigma)}{2(2\eta)^{1/3}} \right) \exp \left\{ i \left( kr - 3\eta + \eta \ln \frac{\sigma_c - \sigma}{2} \right) \right\} \]

where \( \sigma_c = \sigma(r, \Theta_c(r)) \). At the caustic surface the wave function amplitude increases as \( \lambda^{-1/6}: |\psi_k^{(1)}(r, \Theta_c)| = \sqrt{\pi(2\eta)^{1/6}} \text{Ai}(0) \approx 0.3550 \sqrt{\pi(2\eta)^{1/6}} \). The amplitude
V. I. ZAGREBAEV

reaches its maximal value at the distance of \(2 \cdot (2\eta)^{1/3}/k\) from the caustic, and the halfwidth of the first maximum \((\approx 4((2\eta)^{1/3}/k))\) decreases as \(\lambda^{2/3}\). At \(r \to \infty\) the Coulomb caustic \(\theta_C(r) \to 0\). It leads to the increase (up to infinity) of the differential cross section at small angles.

The expressions (2.15) and (2.16) define the Coulomb wave function in the whole space in terms of elementary functions. The applicability of these approximated formulae is not seemingly limited by the usual semiclassical condition \(\eta \gg 1\). One can see that for \(\eta \to 0\) the expression (2.15) turns into the exact solution—the plane wave—corresponding to this case. Numerical comparison shows a good agreement between the exact expression (2.13) and the approximate ones (2.15), (2.16) in the wide range of \(\eta\) down to very small amplitude values [9].

2.4. Above-Barrier Energies

The field of the trajectories is significantly changed for the energies above the Coulomb barrier (Fig. 2a). There are two caustic surfaces in this case (the thick dashed lines in Fig. 2a): \(\Theta_C(r)\) coincides with the Coulomb caustic at the large distances \((\Theta_C(r) \to \Theta_C(r) \text{ at } r \to \infty)\), and \(\Theta_N(r)\) asymptotically approaches the

![Fig. 2. The same as Fig. 1, but at \(E_{\text{NC}} = 81\) MeV.](image)
rainbow angle \( \theta_N(r) \to \theta_r \equiv \Omega(b_R) \) at \( r \to \infty \). Their coalescence at the point \((R_{\text{cusp}}, \Theta_{\text{cusp}})\) forms the caustic cusp. The trajectories with impact parameters \( b_R < b < b_{\text{cusp}} \) touch the "nuclear" caustic \( \Theta_N(r) \), and the ones with \( b > b_{\text{cusp}} \) touch the "Coulomb" caustic \( \Theta_C(r) \). Three trajectories pass through each point inside the cusp, and only one passes outside it.

In the case of realistic nuclear forces one fails to obtain the simple formulae for \( \varphi(b_r, r) \) and \( \Omega(b) \), enabling analysis of the properties of \( \psi_k^{(+)}(r) \) as a function of potential parameters and energy near the point \((R_{\text{cusp}}, \Theta_{\text{cusp}})\) (numerical estimates are easily obtained here). At the near-barrier energies as a first approximation we have

\[
R_{\text{cusp}} \approx \frac{2\eta}{k} + \frac{kb_R^2}{\eta}, \quad \Theta_{\text{cusp}} \approx \Theta_C(R_{\text{cusp}})
\]

and

\[
|\psi_k^{(+)}(R_{\text{cusp}}, \Theta_{\text{cusp}})| \approx (kb_{\text{cusp}})^{1/4} \frac{\Delta^{1/4}}{2\pi \sin \Theta_{\text{cusp}}} I(0, 0),
\]

where \( \Delta = \Omega_C(b_R) - \Omega(b_R) = 2 \arctan(\eta/kb_R) - \Theta_R \), \( I(0, 0) = \frac{1}{2} I(\frac{1}{4}) \approx 1.81 \). Notice, that \( \psi_k^{(+)}(r) \) does not reach its maximal value at the very point \((R_{\text{cusp}}, \Theta_{\text{cusp}})\), but at the point \((X \approx -2, Y = 0)\), where the Pearcey function is maximal: \( I(-2, 0) \approx 2.6 \).

The amplitude of the wave function \( \psi_k^{(+)}(r) \) at the near-barrier energy is shown in Fig. 2b. The presence of several trajectories passing through each point leads to oscillations of the amplitude inside the cusp. The decrease of the amplitude for \( \Theta < \Theta_C(r) \) is caused by the sharp decrease in the density of trajectories in this region. The decrease of the amplitude at small distances and behind the nucleus is related to an absorption in the elastic channel, and because of this we do not extend the trajectories on the region \( r < R_c \) in Figs. 2a and 3a.

The realistic ion-ion potentials usually lead to a sharp skewness of the deflection function \( \Omega(b) \) near the rainbow angle: \( \Omega(b) \) decreases very fast at \( b < b_R \). As a result, the beam of the trajectories touching the nuclear caustic is rather narrow,

\[
b_{\text{cusp}} - b_R \approx \Delta \frac{\eta}{2k} \left[ \frac{\eta}{4k} \frac{(1 - \Theta_R/\pi)^2}{b_R \cdot \Delta} \right]^{1/3},
\]

and the wave function amplitude near the cusp (2.17) does not exceed the Coulomb one (2.16) at the corresponding energy.

The situation changes when the energy increases, since \( \psi_{\text{cusp}} \sim \Delta^{1/4} \) increases faster than the Coulomb wave function at the simplest caustic. However, the point \((R_{\text{cusp}}, \Theta_{\text{cusp}})\) is displaced to the larger values of \( r \) and to the smaller ones of \( \Theta \) \((R_{\text{cusp}} \approx R_c^B \cdot E/V_c^B, \quad \Theta_{\text{cusp}} \approx 2 \cdot V_c^B/E \) at \( E \gg V_c^B \)), and no singularities remain at the distances of our interest \((r \lesssim R_c^B)\) (Fig. 3b). Only one trajectory passes each
spatial point (not counting those which turn around the nucleus), and the wave function $\psi_k^{(+)}(r)$ is defined by the expression (2.8) in which only one term remains. As a result, the wave function amplitude is practically monotonous; it retains the unit value almost everywhere and sharply diminishes when the trajectories pass through the region of strong absorption, i.e., inside the nucleus and behind it (Fig. 3b).

The de Broglie wavelength corresponding to the relative motion of $^{22}\text{Ne} + ^{181}\text{Ta}$ we consider here as the illustration is small enough so that the semiclassical features of the wave function can be easily seen, but, at the same time, the exact values of $\psi_k^{(+)}(r)$ can be accurately obtained by solving the radial Schrödinger equations and by summing up the partial waves (of the order of 200 for $r \lesssim 20$ fm) for the given potential $V(r)$. Comparison of exact and semiclassical values of $\psi_k^{(+)}(r, \Theta)$ made for the fixed $r$ or $\Theta$ [9] showed their coincidence within the limits of 5%, down to the very small amplitude values $|\psi_k^{(+)}| \gtrsim 10^{-2}$ for the given energies. The functions for the same three-dimensional figure cannot be shown. Therefore, only exact values of $|\psi_k^{(+)}(r, \Theta)|$ are shown in Figs. 1–3.
One can see quite well that behaviour of $\psi_{k \pm}^{(+)}(r, \Theta)$ reflects completely the character of the real classical trajectories field for each energy value. Moreover, one can see also that the trajectories we omit (the real ones turning around the nucleus and the complex ones passing through the barrier and reflecting at the inner turning points) do not practically manifest themselves in $\psi_{k \pm}^{(+)}(r, \Theta)$ because of the strong absorption and small wavelength values ($\lambda \approx 0.1 \text{ fm}$). For the heavier ions the contribution of such trajectories is still smaller.

The irregularity of the heavy ion relative motion wave function amplitude at sub-barrier and near-barrier energies undoubtedly has to take effect in the inelastic collision processes. The reactions with relatively long-range formfactors (the Coulomb exitation and few-nucleon transfer reactions) are very sensitive to the elastic wave function value at large enough distances from the nucleus. As can be seen from Fig. 4, for the energies slightly above the Coulomb barrier such processes (and also the reactions in which these processes are the obligatory initial steps) would have a spatial localization because of the trajectory concentration in the cusp region.

Concerning atomic physics, the treatment of the nonstationary problem of wave packets scattering at the near- and sub-barrier energies is of considerable interest. The use of the semiclassical simple formulae for the stationary states $\psi_{k \pm}^{(+)}(r)$ makes this problem quite solvable. The appearance of pronounced density irregularities, that must be observed near the caustic surfaces, may lead to the new electron states and transitions.

![Fig. 4. Wave function amplitude in the peripheral region at three different energies: $E_{\text{Ne}} = 73 \text{ MeV}$ (1), 81 MeV (2), and 140 MeV (3).](image-url)
2.5. Weak Absorption

The experimental data on collisions either of not very heavy ions with each other or of the varying ions with medium nucleus testify that the trajectories turning around the nucleus and deflecting at negative angles make an appreciable contribution to the elastic scattering cross section (see, e.g., [15]). It is explicit evidence of not very strong absorption in such systems. In this case one takes into account the solutions of Eq. (2.7b) corresponding to the negative branch of $\Omega(b)$. As a result, at above-barrier energies there are two terms at least in the sum over $t$ in (2.8); one of them corresponds to the impact parameter $b_r \approx b_{\text{orb}}$. In Fig. 5, as an illustration, the wave function amplitude for the elastic scattering of deuterons by $^{68}$Zn nuclei is shown as obtained from the Schrödinger equation with the optical potential fitting the experimental angular distribution. It is obvious, that the deuterons cannot be considered as particles that are strongly absorbed by the medium nuclei. In the region behind the nucleus (small $\theta$) the interference pattern with a “half-period” $\Delta \theta \approx 17^\circ \approx \pi/k_d b_{\text{orb}}$ is observed that confirms a noticeable contribution from the trajectories turning around the nucleus.

In the same figure the wave function amplitude is shown for the relative motion of $^6$Li + $^{64}$Ni at the energy $E_{L_i}^{\text{lab}} = 28$ MeV. In this case the negative branch of the deflection function $\Omega(b)$ also exists corresponding to the orbitting of $^6$Li around the nucleus $^{64}$Ni. However, as can be seen in Fig. 5, the contribution of the trajectories turning around the nucleus for the wave function $\psi_{k+1}^{(1)}(r, \Theta)$ is very small in this case; i.e., such ions as $^6$Li are strongly absorbed from the elastic channel even by medium nuclei.

![Fig. 5. Wave function amplitude at strong and weak absorption. The corresponding optical potentials were taken from [26].](image)
3. Transition Lines and Direct Processes

The amplitude of the direct process can be written normally in the form (see, for example, Ref. [16])

$$T_{1}^{\mu}(\mathbf{k}_{\gamma}, \mathbf{k}_{i}) = \int \psi_{k_{i}}^{(-)}(x_{r}) U_{i}(r) Y_{\lambda_{\mu}}^{*}(\hat{r}) \psi_{k_{\gamma}}^{(+)}(r) d^{3}r.$$  (3.1)

Here $\psi_{k_{i}}^{(+)}$ and $\psi_{k_{r}}^{(-)}$ are distorted wave functions of the relative motion of particles in the entrance and exit channels. The factor $\alpha$ is equal to the mass ratio of the initial and residual nuclei. In what follows for clarity of exposition we omit this factor (in quantitative estimates it is, of course, included). $U_{i}(r)$ is the transition formfactor containing the entire information on the nuclear structure and on the interaction determining the transition. $\hbar l$ is the angular momentum transferred in the reaction.\(^3\)

In further calculation of the amplitude (3.1) one normally expands the wave functions $\psi_{k_{i}}^{(+)}$ and $\psi_{k_{r}}^{(-)}$ in partial waves and reduces Eq. (3.1) to a coherent sum of radial integrals. In the case of heavy ion collisions this procedure leads not only to significant numerical problems (especially for a large angular momentum transfer $l \gg 1$) but also makes it more difficult to derive simple analytic formulae for cross sections of the processes studied. The theoretical description becomes less transparent and loses its predictive power.

The direct calculation of the three-dimensional integral in the semiclassical approximation allows one to avoid this difficulty and to obtain a clear classical picture about the mechanism of the process and its localization in space without losing all interference phenomena.

In the heavy ion reactions the transferred angular momentum $L = \hbar l$ can be very large; here it is convenient to use the approximate expression for the spherical function entering (3.1). The semiclassical approximation for $Y_{\lambda_{\mu}}(\Theta, \varphi)$ at large values of $l = L/\hbar$ can be obtained from the differential equation for associated Legendre functions by the ordinary WKB-method. Here the angles $\Theta_{\mu} = \arcsin(|\mu|/\lambda)$ and $\varphi - \Theta_{\mu}(\lambda - l + \frac{1}{2})$ arc “turning points.” In consequence, we have for $Y_{\lambda_{\mu}}(\Theta, \varphi)$ the approximate expression defined for all the angles,

$$Y_{\lambda_{\mu}}(\Theta, \varphi) \approx (-1)^{\lambda} e^{i\mu\varphi} a_{\lambda_{\mu}}(\Theta) \cdot \cos y_{1_{\mu}}(\Theta)[1 + O(1/\lambda)].$$  (3.2)

with

$$a_{\lambda_{\mu}}(\Theta) = \frac{\gamma}{\pi} \left\{ \left( \sin^{2} \Theta - \frac{\mu^{2}}{\lambda^{2}} \right)^{-1/4} \sin \Theta > |\mu|/\lambda, \right.\left. \lambda - \Theta_{\mu}(\lambda - l + \frac{1}{2}) \right\}$$

$$\gamma = \lambda^{2}(\cos \Theta + \cos \Theta_{\mu}) \sin \Theta_{\mu}/\sin^{2} \Theta, \quad \xi = \gamma^{1/3}(\Theta_{\mu} - \Theta).$$  (3.3)

\(^3\) For the transfer reactions, an accounting of the recoil effect in the finite-range calculations leads to the formfactor containing the sum over $l, \mu$ [16]. In this case expression (3.1) must be considered as one term of this sum.
when $\Theta < \pi/2$, and $\zeta = \gamma^{1/3}(\Theta - \pi + \Theta)$ when $\Theta > \pi/2$,

$$y_{q}(\Theta) = \left(1 - \mu \right) \frac{\pi}{2} + \mu \cdot \arctan \frac{\mu \cdot \cos \Theta}{\sqrt{\lambda^2 \sin^2 \Theta - \mu^2}} - \lambda \cdot \arcsin \frac{\lambda \cdot \cos \Theta}{\sqrt{\lambda^2 - \mu^2}}.$$  

In the quantitative calculations one needs to connect the different solutions in (3.3) at the points $\Theta \approx \Theta + \gamma^{-1/3}$ and $\Theta \approx \pi - \Theta - \gamma^{-1/3}$, i.e., at $\zeta \approx -1$.

3.1. Conservation of Angular Momentum and Transition Lines

From the classical point of view, the direct process is the following. The projectile with asymptotic momentum $k_i$ moves along the elastic trajectory up to the point $r$ in which transference of energy $\Delta E$, mass $\Delta m$, and angular momentum $\lambda$ occurs. After that, the ejectile goes out from the collision region also along the elastic trajectory with the asymptotic momentum $k_f$. Transition probability is defined by the concrete reaction mechanism and by the structure of colliding particles and must be determined separately. However, if $k_i$, $k_f$, $\Delta E$, $\Delta m$, $\lambda$ and distorting potentials defining the elastic trajectories in the entrance and exit channels are fixed, the conservation laws lead to substantial restrictions for the spatial points in which this transition can occur.

Energy and mass conservation laws lead to fixed values of these quantities in the exit channel. The conservation law of angular momentum does not manifest itself so evidently because in real experiments we cannot fix its value for the projectile and ejectile.

The angular momentum of the particle coming to the point $r$ is $L_i(k_i, r) = [r \times \nabla S_i(k_i, r)]$, where $h S_i(k_i, r)$ is the classical action corresponding to the trajectory passing through $r$ ($\nabla S(k, r) = k(r)$ is the local momentum of the particle at point $r$). The particle going out of $r$ and reaching the asymptotic momentum $k_f$ is characterized by the action function $S_f(-k_f, r) = S_f(k_f, r)$ and by the angular momentum $L_f(k_f, r) = -[r \times \nabla S_f^{-1}(k_f, r)]$. The angular momentum conservation law

$$[r \times \nabla S_i(k_i, r)] + [r \times \nabla S_f^{-1}(k_f, r)] = \lambda$$

just defines the spatial points at which the transition may occur.

In real experiments one fixes only the absolute value of the transferred angular momentum and one of its projections $\lambda_z = \mu$. Therefore, Eq. (3.5) is more convenient in the spherical coordinates $(r, \Theta, \phi)$. Taking into account that $[r \times \nabla S]_z = \partial S/\partial \phi$, $[r \times \nabla S]_\Theta = \partial S/\partial \Theta$, $[r \times \nabla S]_\phi = -\partial S/\partial \phi/\sin \Theta$, we have

$$\frac{\partial S_i}{\partial \phi} + \frac{\partial S_i^{(-1)}}{\partial \phi} = \mu$$

$$\frac{\partial S_i}{\partial \Theta} + \frac{\partial S_i^{(-1)}}{\partial \Theta} = \pm \sqrt{\lambda^2 - \mu^2/\sin^2 \Theta},$$

where (3.5a) is the conservation law of the $z$-projection of angular momentum and (3.5b) is of its $\phi$-projection. These equations define, in general, several lines in
three-dimensional space \( \{r, \Theta_v(r), \varphi_v(r)\} \) in the vicinity of which the inelastic transition just occurs and, therefore, they can be called “transition lines.”

Substituting the semiclassical wave functions (2.8) and the spherical function (3.2) into (3.1), we write the transition amplitude at above-barrier energies in the form

\[
T^{\mu}_{ji}(k_i, k_f) = \int F(r) \left\{ e^{iZ_i(r)} + e^{iZ_f(r)} \right\} U_j(r) d^3r,
\]

where

\[
F(r) = \frac{1}{2} (-1)^\mu A_i(r) A_j^\dagger(r) a_{\lambda \mu}(\Theta)
\]

and

\[
Z_\pm(r) = S_i(k_i, r) + S_f(-k_f, r) \pm \psi_{\lambda \mu}(\Theta) - \mu \varphi.
\]

In heavy ion collisions the expression in the braces is a strongly oscillating factor and the value of integral (3.6) is mainly formed by contributions of the stationary phase points (in the case when \( F(r) \) is a sufficiently smooth function of its variables). The angular variables stationary phase condition \( \nabla_\Theta Z_\pm = 0 \) leads exactly to Eqs. (3.5a) and (3.5b); i.e., the stationary points lie on the transition lines and correspond to the angular momentum conservation law. These equations can be written in a form that is more convenient for numerical calculation (\( z \)-axis coincides with \( k_i, k_f \) is in the \( x, z \)-plane, \( \Theta_f \) is the scattering angle, \( \varphi_f = 0 \)):

\[
k_i h_i^{-1}(r, \Theta') \sin \Theta' \frac{\sin \Theta}{\sin \Theta'} \sin \varphi = \mu \quad (3.7a)
\]

\[
k_i h_i(r, \Theta) - k_f h_f^{-1}(r, \Theta') \frac{d\Theta'}{d\Theta} = \pm \sqrt{\lambda^2 - \mu^2/\sin^2 \Theta}. \quad (3.7b)
\]

Here \( \cos \Theta' = \cos \Theta \cos \Theta_f + \sin \Theta \sin \Theta_f \cos \varphi \) and \( h_i, h_f^{-1} \) are the impact parameters of the \( i \)- and \( f \)-trajectories which can be defined from Eqs. (2.7).

Fulfilling the integration on angular variables by the multi-dimensional stationary phase method \[17\] we reduce the amplitude (3.6) to the one-dimensional radial integral along the transition lines

\[
T^{\mu}_{ji} = \int_0^\infty U_j(r) I_{\lambda \mu}(\Theta_f, r) r^2 dr
\]

with

\[
I_{\lambda \mu}(\Theta_f, r) = 2\pi \sum_v F(r, \Omega_v^\circ) |\det Z_{\Omega_v^\circ}(r, \Omega_v^\circ)|^{-1/2}
\]

\[
\times \exp \left\{ i \left[ Z(r, \Omega_v^\circ) + \frac{\pi}{4} \operatorname{sgn} Z_{\Omega_v^\circ}(r, \Omega_v^\circ) \right] \right\}. \quad (3.9)
\]

Here \( \Omega_v^\circ \equiv \{\Theta_v^\circ(r), \varphi_v^\circ(r)\} \) and \( Z_\Theta \) is a two-dimensional matrix with the diagonal elements \( \partial^2 Z/\partial \Theta^2, \partial^2 Z/\partial \varphi^2 \) and with the nondiagonal elements \( \partial^2 Z/\partial \Theta \partial \varphi; \operatorname{sgn} Z_{\Theta} \)
is the number of positive minus the number of negative eigenvalues of this matrix (equal to 2, 0, or -2). In Eq. (3.9) either \( Z^+ \) or \( Z^- \) must be taken depending from which of Eqs. (3.7b) \( Q^+_n(r) \) is obtained.

The number of terms in the sum (3.9) is equal to the number of transition lines. Since the formfactor \( U_n(r) \) decreases with the increase of \( r \) and \( I_n \) falls to zero in the region of in- and out-shadow (where \( A_i \approx 0 \) or \( A_j^{(-)} \approx 0 \)), only small lengths of the transition lines bring a considerable contribution to \( T_{ji}^\mu \). Accounting for the corresponding \( \Theta^+_n(r) \) and \( \phi^+_n(r) \) values, we have the complete (not only radial, but also angular) spatial localization of the investigated process. The presence of several spatial regions (\( \nu > 1 \)) bringing considerable contributions to \( T_{ji}^\mu \) leads to the interference pattern in the angular distribution. Since with a change of \( \Theta_f \), some of these regions can go in the zone of the stronger shadow (or leave it), the interference pattern will be damped (or intensified). Thus, we obtain a clear classical picture about the reaction mechanism with preservation of all the interference phenomena.

3.2. Straight-Line Trajectory Approximation

As an obvious example illustrating the above-stated approach, we consider the case of the straight line elastic trajectories in entrance and exit channels. Notice, that the straight line trajectory approximation differs in the main from the plane wave approximation, because it allows taking into account correctly the shadow regions in the amplitudes \( A_i(k_i, r), A_j^{(-)}(k_f, r) \) and also the distorting potentials in the phases \( S_i, S_j^{(-)} \) in an eikonal form.

For the straight line trajectories \( b(r, \Theta) = r \cdot \sin \Theta \) and Eqs. (3.7) define two straight transition lines (for \( \lambda \) and \( \mu \) fixed) parallel to vector \( q \) and lying in the plane \( y = y_\mu = q \sin \Theta_q \) at a distance \( R_0 = \lambda/q \) from the origin and at a distance \( 2\sqrt{R_0^2 - y_\mu^2} \) from one another. Vector \( q \) is the transferred momentum \( q = k_i - k_f, q \sin \Theta_q = k_f \sin \Theta_f, \varphi_q = \pi \). The angular momentum conservation law has a trivial form \( [r \times q] = \lambda \) in this case.

There are two transition lines corresponding to the different values of \( \lambda_y = \pm \sqrt{\lambda^2 - \mu^2 / \sin^2 \Theta_q} \). Designating them \( \Omega^{(1)}(r) \) and \( \Omega^{(-1)}(r) \), we have at \( r > R_0 \)

\[
\begin{align*}
\Theta_{1,2}^+(r) &= \arccos(\cos \psi_{1,2}^+(r) \sqrt{1 - y_\mu^2/r^2}) \\
\phi_{1,2}^+(r) &= \arccotan(\sin \psi_{1,2}^+(r) \sqrt{r^2/y_\mu^2 - 1}) \\
\psi_{1,2}^+ &= \begin{cases} 2\pi - \Theta_q - \chi(r) \\ \pi - \Theta_q + \chi(r) \end{cases}, \quad \psi_{1,2}^- = \begin{cases} -\Theta_q + \chi(r) \\ \pi - \Theta_q - \chi(r) \end{cases} \\
\chi(r) &= \arcsin(\sqrt{(R_0^2 - y_\mu^2)/(r^2 - y_\mu^2)}).
\end{align*}
\]

The substitution (3.10) into (3.9) leads to the very simple result which can be rewritten, taking into account the obtaining expressions,

\[
\frac{1}{x} \left(1 - \frac{\lambda^2}{x^2}\right)^{-1/4} \exp \left\{ \pm i \left( x \sqrt{1 - \frac{\lambda^2}{x^2}} + \lambda \arcsin \frac{x}{\lambda} - \frac{\lambda \pi}{2} - \frac{\pi}{4} \right) \right\}
\]
HEAVY ION COLLISIONS

which are the semiclassical approximation for the Hankel functions \(h_j^{(1)}(qr)\). As a result,

\[
I_{\lambda q}(q, r) = \pi \cdot a_{\lambda q}(\Theta_q) \cdot \{e^{i\nu_{q\lambda}(\Theta_q)}[(A_i A_f^{-1})_{\Omega_1} \cdot h_j^{(1)}(qr) + (A_i A_f^{-1})_{\Omega_1} \cdot h_j^{(2)}(qr)] + e^{-i\nu_{q\lambda}(\Theta_q)}[(A_i A_f^{-1})_{\Omega_1} \cdot h_j^{(1)}(qr) + (A_i A_f^{-1})_{\Omega_1} \cdot h_j^{(2)}(qr)]\}. \quad (3.12)
\]

The first term in the braces corresponds to a transition with \(\lambda_j > 0\) and the second one to a transition with \(\lambda_j < 0\).

The expression (3.12) can be easily generalized for \(r \leq R_0 = \lambda/q\) (at \(r = R_0\) the two stationary phase points are fused together—\(\Omega_1^{(+)}\) with \(\Omega_2^{(+)}\) and \(\Omega_1^{(-)}\) with \(\Omega_2^{(-)}\)—and become complex):

\[
I_{\lambda q}(q, r) = 2\pi a_{\lambda q}(\Theta_q) \cdot \{j_{1/2}^{(1)}(qr) \cdot \{e^{i\nu_{q\lambda}(\Theta_q)}[(A_i A_f^{-1})_{\Omega_1} \cdot e^{i\nu_{q\lambda}(\Theta_q)} + (A_i A_f^{-1})_{\Omega_1} \cdot e^{-i\nu_{q\lambda}(\Theta_q)}] + \Theta_0^{(\pm)} = \arccos(\mp \sin \Theta_q \sqrt{1 - \mu^2/\lambda^2} \sin^2 \Theta_q) \}
\]

\[
\phi_0^{(\pm)} = \arccotan(\mp \cos \Theta_q \sqrt{\lambda^2 \sin^2 \Theta_q/\mu^2 - 1}) \quad (3.13)
\]

and also for \(\mu \geq \lambda \sin \Theta_q\) (when the two transition lines \(\Omega_1^{(+)}\) and \(\Omega_1^{(-)}\) are fused together):

\[
I_{\lambda q}(q, r) = 2\pi i a_{\lambda q}(\Theta_q) \cdot \{2(A_i A_f^{-1})_{\Omega_0} \cdot j_1(qr), \quad r \leq \lambda/q \}
\]

\[
\theta_{1,2} = \arccos(\pm \cos \Theta_q \sqrt{1 - \lambda^2/(qr)^2}), \quad \Theta_0 = \pi/2 \quad (3.14)
\]

\[
\varphi_{1,2} = \arccotan(\mp \sin \Theta_q \sqrt{(qr)^2/\lambda^2 - 1}), \quad \varphi_0 = \pi/2
\]

It can be easily seen, that Eqs. (3.12)-(3.14) determine the continuous function \(I_{\lambda q}(r)\). Only when absorption is absent \((A_i = 1\) and \(A_f^{-1} = 1\) for all \(r)\) are these expressions reduced to the form

\[
I_{\lambda q}(q, r) = 4\pi i Y_{1/2}(\Theta_q, \varphi_q = \pi) \cdot j_1(qr) = \int e^{iqr} Y_{1/2}^\ast(\Omega) \, d\Omega \quad (3.15)
\]

which corresponds to just the plane wave approximation.

We show now, that the expressions obtained give us a sufficiently good result even for light particle collisions (to be sure, the energies are larger than the Coulomb barrier when the trajectories are straight enough). We consider the well-known deuteron stripping reaction for the not very heavy nucleus \(^{40}\text{Ca}\) at energy \(E_d = 11\) MeV [18]. In Fig. 6 the transition lines in this reaction are shown for \(\Theta_f = 40^\circ, \, l = 3, \, \mu = 0\) in the straight line trajectory limit. In this case the shadow regions have a cylindrical form behind the nucleus (where \(A_i \approx 0\) and in front of it (where \(A_f^{-1} \approx 0\)).
For strong absorption $I_{\text{rad}}(q, r)$ is not equal to zero only for $r > R_{\text{w}}^{(\pm)}(1, 2)$ only (see Fig. 6), when the four terms in (3.12) take turns. As can be seen from Fig. 6, the cutoff radii are considerably larger than the radius of the absorptive sphere $R_w$ for small values of $\Theta_f$ and, in addition, the cutoff occurs at different distances; i.e., the contributions of each of the four terms in (3.12) are not the same. From this the straight line trajectory approximation differs from the well-known Butler's approximation in which the expression (3.15) (cut off at some radius $R_{\text{cut}}^{(1)}$) is used for $I_{\text{rad}}(q, r)$. The main shortcoming of this approximation (playing enormous role because of its very simple and clear form) is only wrong accounting of the shadow region. This leads to necessity to take $R_{\text{cut}} > R_w$ (in order to get a qualitative agreement with the experimental data) and to very strong oscillations (up to zero) in angular distributions.

Because of the exponential decrease of the transferred neutron formfactor, the regions $r \approx R_{\text{w}}^{(\pm)}(1, 2)$ bring the main contribution to the cross section. The contributions of these regions are coherent but are not equal to one another. This leads to a damped interference picture in the angular distribution. For $\lambda \neq 0$ the transferred momentum $q$ decreases with decrease of $\Theta_f$, the transition lines move aside from nucleous ($R_0 = \lambda/q$ increases), $R_{\text{w}}^{(\pm)}(1, 2)$ increase and the reaction cross section decreases. The maximum of angular distribution is reached at such $\Theta_f^{(\pm)} \neq 0^\circ$, when transition lines approach the nucleous; i.e., $R_0 = \lambda/q \approx R_w$ or $q(\Theta_f^{(\pm)}) \approx \lambda/R_w$ (Fig. 7).

Fig. 6. Transition lines for the $^{40}\text{Ca}(d, p)^{41}\text{Ca}$ reaction in the straight-line trajectory approximation (thick solid lines). $R_{\text{w}}^{(\pm)}(1, 2)$ are the intersection points of transition lines with the shadow.
Approximating the wave functions amplitudes by the simplest formula

\[ A(r, \Theta) = \exp \left\{ -\frac{k}{2E} \int \frac{r \cos \Theta}{r^2} W(\sqrt{r^2 \sin^2 \Theta + Z^2}) dZ \right\} \]

and using the expressions (3.12)-(3.14) for \( I_{\mu}(q, r) \) and (3.8) for the transition amplitude we calculated the cross section of \(^{40}\text{Ca}(d, p)^{41}\text{Ca} \) reaction for angular momentum transfer \( I = 3 \) and \( I = 1 \). The results are shown in Fig. 7 demonstrate a good enough agreement with experiment even in such a nonclassical case.

**Fig. 7.** Angular distribution of protons in the \(^{40}\text{Ca}(d, p)^{41}\text{Ca} \) reaction at \( E_d = 11 \text{ MeV} \). The solid lines show the semiclassical calculation in the straight-line trajectory approximation, and the dashed lines correspond to the Butler expression with \( R_{\text{cut}} = 5.5 \text{ fm} \). Experimental data are from [18].
In heavy ion collisions at slightly above-barrier energies the straight line trajectory approximation is too rough, of course. But in this case the condition of local linearity of the trajectories is almost always fulfilled. It means that in the π-vicinity of the stationarity point $r_*$ of the phase $Z_\pm$ (the region in which the variation of the phase $\Delta Z_\pm \leq \pi$) the expansion

$$S_i(k_i, r) + S_f(-k_f, r) \approx S_i(k_i, r_*) + S_f(-k_f, r_*) + (k_f(r_*) - k_f(r_*))(r - r_*)$$  \hspace{1cm} (3.17)

can be used, where $k_f(r_*) - k_f(r_*) = q(r_*)$ is the local momentum transferred. Thus, near the stationarity point the following integral must be evaluated

$$\int_{A\Omega} F(\Omega) e^{iq'\phi^*_\mu}(\Omega) d\Omega \equiv \mathcal{F}_{\lambda\mu}. $$

As stated above, for $r > \lambda/q$ and $\mu < \lambda \sin \Theta_q$ the phase of the integrand has four stationarity points. If none of them lies in the integration region, then $\mathcal{F}_{\lambda\mu} = O[1/(qr)^2]$ for $qr \gg 1$ and $\mathcal{F}_{\lambda\mu} = O[\sin \mu/\sin \lambda/\lambda]$ for $\lambda \gg 1$. If, however, $A\Omega_v$ contains one of the stationarity points, then

$$\mathcal{F}_{\lambda\mu}^{(s)} \approx \pi i (-1)^{1} e^{-iq\phi_q} \cdot F(\Omega_v) a_{\lambda\mu}(\Theta_q) e^{\pm i\nu_{\lambda\mu}(\Theta_q)} h_{\lambda}^{(1,2)}(qr). $$  \hspace{1cm} (3.18)

The sign of $\nu_{\lambda\mu}$ in (3.18) is opposite to the sign of $[r \times q]_v = \lambda$. The choice of the kind of Hankel function is determined by the sign of $Z_{\Theta}^{-} : Z_{\Theta}^{-} < 0$ corresponds to $h_{\lambda}^{(1)}$, and $Z_{\Theta}^{-} > 0$ to $h_{\lambda}^{(2)}$.

Thus, in the approximation of local linearity of the trajectories, the quantity $I_{\lambda\mu}(\Theta_f, r)$ also has a very simple form

$$I_{\lambda\mu}(\Theta_f, r) = \pi \sum_{v} A_i(r_v) A_f^{-1}(r_v) \exp \left\{ i \left[ S_i(k_i, r_v) + S_f(-k_f, r_v) - q(r_v) \cdot r_v - \mu(\varphi_q + \pi) + \frac{i\pi}{2} \right] \right\} a_{\lambda\mu}(\Theta_q) e^{\pm i\nu_{\lambda\mu}(\Theta_q)} h_{\lambda}^{(1,2)}(q_v \cdot r). $$  \hspace{1cm} (3.19)

The transition lines $\{r, \Omega_v(r)\}$ can be found from the relations (3.7); here it suffices to solve these equations at $\mu = 0$ (the plane $\sin \varphi = 0$) and then make use of (3.12)–(3.14) for continuation of the results to regions with $\mu \neq 0$ and $r < R_o(\lambda)$.

3.3. Quasielastic Processes

The quasielastic processes of heavy-ion-induced reactions (inelastic scattering, charge-exchange, and few-particle transfer reactions) are studied well enough both experimentally and theoretically (see, e.g., the excellent review [15, 19, 20] and numerous references therein). The most valuable results of such reactions are their one-step (or few-step) direct mechanism and the possibility of exact (not only semiclasical) evaluation of the cross sections within the framework of DWBA (or coupled channels) formalism. The only shortcoming of such reactions is their extremely peripheral localization (due to the strong absorption) which leads to
weak sensitivity for cross sections in the shape of distorting potentials and form-factors at small distances (in a region behind the Coulomb barrier) and, as a result, to the impossibility of deriving these quantities from such reactions. Notice, however, that for not very heavy ions the improvement in experimental sensitivity allows measurement of the cross sections of quasielastic reactions at high energies for large scattering angles and a view into the interior of the nucleus [21].

Nevertheless, interpretation of the main regularities of quasielastic reactions is not unambiguously and finally established. We consider, in particular, the bell-shaped angular distribution most typical for both nuclear inelastic scattering and few-particle transfer reactions (Fig. 8). The oscillations of the experimental cross

![Angular distributions in quasielastic reactions: $^{88}\text{Sr}(^{16}O, ^{15}N)^{93}Y$ [22]—(a), and $^{208}\text{Pb}(^{16}O, ^{16}O')^{208}\text{Pb}$ [23]—(b).](a)
section of inelastic scattering are due to interference of the Coulomb (a dashed line) and the nuclear (a solid line) excitations. If one neglects the Coulomb formfactor (absent in transfer reactions), the shape of the angular distribution is the same for inelastic scattering and transfer reactions: the maximum at the grazing angle (or at the rainbow angle) and the exponential fall on both sides of this angle.

There are two well-known explanations of such bell-shaped behaviour [19, 20]. In the first of them one assumes that the main contribution to the reaction comes from the partial waves lying in the narrow window $\Delta l$ close to the grazing partial wave $l_0$ corresponding to the grazing angle: $\Omega(l_0) = \Theta_{gr}$. The values of $\Delta l$ is determined by the initial energy and by the spatial localization of the process $\Delta R: \Delta l \approx k \Delta R$. As a result, the cross section reduces to the form

$$\sigma_{fi}(\Theta) \sim \exp\left[-\frac{1}{2}(\Delta l)^2 (\Theta - \Theta_{gr})^2\right]$$  \tag{3.20}

explaining, in particular, the displacement of the peak ($\Theta_{gr} \sim 1/E$) and the narrowing of it ($\Delta l \sim \sqrt{E}$). Such a treatment is equivalent to the diffraction process on the narrow $\Delta R$ (and, therefore, $\Delta l$) window with a “Gauss transparency” (i.e., $f_l \sim f_{l_0} e^{-(l-l_0)/\Delta l)^2}$) which leads to the absence of the interference pattern. The decrease of the cross section at $\Theta < \Theta_{gr}$ and at $\Theta > \Theta_{gr}$ is explained by the same reason in this case.

In the second explanation, based on the extremely classical relative motion, the reaction cross section is reduced to the form

$$\sigma_{fi}(\Theta) = \sigma_{el}^{\sigma}(\Theta) P_{fi}(\Theta)$$  \tag{3.21}

in which $\sigma_{el}^{\sigma}(\Theta)$ is the elastic scattering cross section, $P_{fi}(\Theta) \sim \int_{-\infty}^{\infty} U_{fi}(R(t)) dt$ is the probability for inelastic transition during the motion along the scattering orbit $R(t)$, and $U_{fi}$ is the transition formfactor. The absorption in the elastic channel leads to a decrease of the elastic scattering probability for the large angles: $\sigma_{el}^{\sigma}(\Theta) \approx \sigma_{el}(\Theta) \cdot [1 - P_{abs}(\Theta)]$, where $P_{abs}(\Theta) = \exp\{- (2/\hbar) \int_{-\infty}^{\infty} W(R(t)) dt\}$ is the absorption probability during the motion along the elastic trajectory. If there is the rainbow angle $\Theta_R$ then elastic cross section decreases exponentially also in a forbidden region $\Theta > \Theta_R$.

The decrease of the cross section $\sigma_{fi}(\Theta)$ for $\Theta < \Theta_{gr}$ is due to the factor $P_{fi}(\Theta) \sim e^{-\alpha \cdot r_0(\Theta)}$. Here $r_0(\Theta)$ is the closest approach distance (turning point) of the trajectory scattered in the angle $\Theta$. Parameter $\alpha$ is determined by the rate of exponential decrease of the formfactor $U_{fi}$ for large values of $r$. For grazing collisions $r_0(\Theta)$ increases with a decrease of $\Theta$ and $P_{fi}(\Theta)$ decreases for $\Theta < \Theta_{gr}$. Thus, the decrease of the reaction cross section $\sigma_{fi}(\Theta)$ for $\Theta > \Theta_{gr}$ and for $\Theta < \Theta_{gr}$ is explained by different reasons in this case.

We now consider the regularities of the quasielastic reactions within our above-stated semiclassical approach. Using the Eqs. (3.8) and (3.19) and taking into account only the nuclear part of the formfactor $U_{ja}(r)$, we evaluated the angular distribution of the inelastically scattered oxygen nuclei in $^{208}$Pb($^{16}$O, $^{16}$O') reaction.
at $E_{\text{lab}} = 104$ MeV (Fig. 9). We used the same parameters for the optical potentials and the formfactor as in [23]. First of all, we pay attention to the good agreement of semiclassical and exact calculations as in the shape of the angular distribution and at the absolute value of the cross section.

The transition lines of this reaction are shown in Fig. 10 for two angles: $\Theta_f = 50^\circ < \Theta_R^\text{in} \approx 72^\circ$ (in this case the transition lines with only $\lambda_\nu < 0$ are represented) and $\Theta_f = 90^\circ > \Theta_R^\text{in}$ (the upper line corresponds to $\lambda_\nu < 0$ and the lower one to $\lambda_\nu > 0$). In Fig. 10a three elastic trajectories are also shown scattering in angle $\Theta_f = 50^\circ$, and in Fig. 10b there are $i$- and $f$-trajectories passing through the "dominating region." The transition formfactor in the form of a derivative of the volume interaction $V(r) - iW(r)$ is localized in a surface domain between two concentric circles in Fig. 10. The strong absorption in the entrance and exit channels sharply restricts the distorted wave amplitudes in the shadow regions marked by the dashed lines in Fig. 10: $A_i < 0.2$ on the right and $A_i^{-1} < 0.2$ on the left and below the corresponding lines (the shadow region in the exit channel depends, of course, on the angle $\Theta_f$).
The main contribution to the reaction cross section comes from those regions of transition lines which lie in the region of the appreciable values of $U_\lambda(r)$, but do not lie in the zone of the strong shadow. These are hatched regions in Fig. 10. For the angles $\Theta_f < \Theta_R$ the dominant contribution comes from the small vicinity of only one transition line with $\lambda_y < 0$ (transition lines with $\lambda_y > 0$ for such angles lie either in the region of the stronger shadow or in the region of negligible values of $U_\lambda$ and are not shown in Fig. 10a). For $\Theta_f > \Theta_R$ two transition lines (with $\lambda_y > 0$ and $\lambda_y < 0$) make essentially equal contributions. In Fig. 9 we show separately the cross section of the transition with $\lambda_y > 0$, i.e., the formation of residual nuclei rotated counterclockwise in the reaction plane. We see, thus, that for $\Theta_f < \Theta_R$ the residual nuclei must be formed with $\lambda_y < 0$ (it is quite obvious from the classical point of view) and for $\Theta_f > \Theta_R$ the formation of nuclei with $\lambda_y < 0$ and $\lambda_y > 0$ must be gradually equalized. An experimental observation of this phenomenon would be highly interesting.
The simple qualitative expression for the transition amplitude can be obtained which may help to explain the observed shape of the angular distribution. For the angles $\Theta_f < \Theta_R$ only one term in (3.19) can be retained (corresponding to the fourth term in the braces in (3.12)), and for $\Theta_f > \Theta_R$ two terms in (3.19) bring approximately equal contributions to $I_{\mu\nu}(\Theta_f, r)$ (corresponding to the second and fourth terms in (3.12)). In first case the factor $e^{-i\mu(\Theta_f)}$ appears, which can be omitted, because we need only the transition amplitude modules. In the second one the factor $\cos y_{\lambda\mu}(\Theta_q)$ appears, which also can be omitted because of the summing up on $\mu$ when the cross section is being evaluated:

$$
\sum_{\mu = -\lambda}^{\lambda} a_{\lambda\mu}(\Theta_q) \cos^2 y_{\lambda\mu}(\Theta_q) = \lambda/2\pi.
$$

As a result, the transition amplitude may be given as the qualitative expression

$$
T_{\mu\nu}^{\mu}(\Theta_f) \sim \int_{0}^{\infty} a_{\lambda\mu}(\Theta_q) A_i(r^*_f) A_j^{(-)}(r^*_i) U_j(r) e^{-i\mu(\Theta_f)} e^{-i\nu(\Theta_q)} \frac{r^* d r^*}{q(r^*_f)},
$$

where $r^*_i = \{r, \Theta^*_i(r), \varphi^*_i(r)\}$ is the point of the transition line. The function $A_i A_j^{(-)} U_j$ in the integrand is localized on the surface, because of the rapid decrease of $A_i(r) A_j^{(-)}(r)$ at small distances and of $U_j(r)$ at large distances. Assuming this function reaches the maximum at the same point $R_m$ (lying somewhere in the hatched area in Fig. 10) and approximating it by a Gaussian with the width $\Delta R$, we can easily estimate the value of the entire integral

$$
T_{\mu\nu}^{\mu}(\Theta_f) \sim a_{\lambda\mu}(\Theta_q) A_i(R^*_m) A_j^{(-)}(R^*_m) U_j(R_m) e^{-[q(R^*_m + AR^*_2)]^2}.
$$

The meaning of the individual terms in (3.23) is absolutely clear. $A_i^*(R)$ determines the probability for the incident particle to reach the point $R$ staying in the elastic channel, and $A_j^{(-)}(R)$ is the probability for the scattered particle to leave the collision region without changing the state of the target formed by the inelastic transition at the point $R$. The probability of this transition is proportional to $U_j(R)$. The last factor in (3.23) characterizes the ability of the system to receive the transferred momentum $q$. Conservation of angular momentum manifests itself as the fact that the point $R^*_m$ lies on the corresponding transition line $\Omega^*_m(r)$.

In Fig. 9 we show the change of $A_i A_j^{(-)}$ and $q$ with the increase of $\Theta_f$. For small angles the transition lines lie in the region of the strong shadow and the cross section is small. With the increase of $\Theta_f$ the transition lines come out of the shadow and the quantity $A_i A_j^{(-)}$ rapidly increases (see insert in Fig. 9) and the cross section increases. As long as $\Theta_f < \Theta_R$, the local momenta $k_i(r^*_i)$ and $k_f(r^*_f)$ are almost parallel, and the momentum transfer $q(r^*_f)$ remains small. For $\Theta_f > \Theta_R$ the transition lines do not lie in the shadow and $A_i A_j^{(-)} \approx 1$. Though there are no classical elastic trajectories which lead to such angles (except the strong absorbed trajectories around the nucleus and leading to the negative angles $-\Theta_f$), no classical limitation whatever exist for the inelastic scattering in angles greater than the...
rainbow angle $\Theta_R$. In this case, however, the trajectories of incident particle and ejectile are not tangential now, the momentum transfer increases $q(r^*, \Theta_f) \approx 2k_s(r) \sin((\Theta_f - \Theta_R)/2)$ and the cross section sharply decreases (Fig. 9). The rate of the cross section decrease for $\Theta_f > \Theta_R$ depends on the spatial localization of the process $dR$. This, in particular, also explains the faster decrease of the contribution to the considered reaction cross section of the Coulomb exitation with a long-range formfactor (Fig. 8).

Thus, the decrease of the quasielastic reaction cross section for $\Theta_f < \Theta_R$ is the result of the transition lines either getting in the strong absorption region (for the trajectories with smaller values of $b_s(\Theta_f)$) or in the region of negligible values of formfactor $U_s(r)$ (for the trajectories with larger $b_s(\Theta_f)$). Just the latter case reflects the expression (3.21). The cross section decrease for $\Theta_f > \Theta_R$ is caused by the inability of the residual nucleus with the sufficiently “soft” formfactor $U_s(r)$ to take the large transferred momentum. Just this (though not obviously) is reflected by the expression (3.20).

Notice, at last, that in case when several transition lines (with the same $\lambda_s$) make commensurable contributions, the sum on $\nu$ (with accounting of phases) must be kept in (3.19) and in (3.23), and the interference pattern can be observed in the angular distribution. This is possible, in particular, in the reactions with more extensive formfactor and with a smaller rainbow angle, when the contribution of the transition line “under nucleus” (Fig. 10a) becomes noticeable. Observation of the interference for $\Theta_f > \Theta_R$ should prove the existence of weak absorption in the system and the possibility of “elastic” orbiting (characteristic, apparently, only for relatively light ions collisions).

3.4. Massive Transfer Processes

The reactions of heavy cluster stripping from the incident ion (also named the “massive transfer reactions” or “incomplete fusion”) are very important for a complete understanding of the deep inelastic reactions mechanisms. On the other hand, the coherent transfer of many nucleons gives the possibility to obtain and investigate the exotic nuclear states: “cold” rapidly rotating nuclei, nuclei with an anomalous $n/p$ ratio, superheavy nuclei with small excitation energy, and also to determine the spectroscopic features of heavy clusters in nuclei. The significant contribution from direct mechanisms to these reactions has been repeatedly emphasized. As shown below (Section 5), in most cases the massive transfer reactions have a quasidirect nature, i.e., the direct transfer of a heavy cluster is accompanied by considerable kinetic energy dissipation in the entrance and exit channels. However, in this subsection we consider the regularities only of direct one-step massive transfer process.

In contrast with the quasielastic reactions, it is very difficult to evaluate the cross section of the heavy-ion-induced massive transfer reaction not only exactly (in particular, because of the large value of the angular momentum transfer $\lambda$), but also in the frame of available semiclassical methods [7, 8], because in such a reaction there is not any closeness between the incoming and outgoing trajectories. In the
above-formulated approach, the closeness of $i$- and $f$-trajectories is not necessary; therefore, we can surely use the expressions (3.8) and (3.19) for the description of reactions with arbitrary values of the transferred $\Delta m$, $\Delta E$, and $\lambda$.

In order to compare the results not only with experimental data, but also with the “exact” calculations (by means of the code DWUCK), we chose as an illustration the reaction induced by the not very heavy ion $^{64}$Ni($^6$Li, $d$) $^{68}$Zn at the energy $E_{li}^{th} = 28$ MeV with angular momentum transfer $\lambda = 0$ and $\lambda = 3$. The parameters of the distorting potentials and formfactors are taken from [26] in which this reaction had been studied. The transition lines of this reaction are shown in Fig. 11. Their
location hardly depends on the emission angle $\Theta_f$; thick solid lines are the transition lines for $\Theta_f = 30^\circ$, and thick dashed ones for $\Theta_f = 5^\circ$. This is due to the fact the solutions of Eq. (3.7b) are in the region of the orbiting impact parameters in exit channel $b_f^{-1} \approx b_f^{\text{orb}}$ for $\Theta_f, \lambda$ and $r$ values we are interested in.

In the case $\lambda = 0$ there are two almost symmetric transition lines $\Theta_{v=1}(r) \approx \Theta_{v=-1}(r)$ in the $x, z$-plane, and the expression (3.19) has the form

$$I_{00}(\Theta_f, r) = \sqrt{\pi} A_f(r, \Theta_f) e^{iS(r, \Theta_f)} \frac{1}{qr} \times \{ A_f^{(-1)}(r, \Theta_f - \Theta_f) e^{i\delta f^{(-1)}(r, \Theta_f - \Theta_f)} + A_f^{(-1)}(r, \Theta_f + \Theta_f) e^{i\delta f^{(-1)}(r, \Theta_f + \Theta_f)} \}.$$  

(3.24)

Since, as mentioned above, $b_f^{(-1)}(r, \Theta_f) = b_f(r, \pi - \Theta_f - \Theta_f) \approx b_f^{\text{orb}}$, then with accounting of (2.4) we have $S_f^{(-1)}(r, \Theta_f) \approx S(r) \pm k_f b_f^{\text{orb}} \cdot \Theta_f$. Therefore, for not too large angles $\Theta_f$ the expression in the braces of (3.24) is equal to $2A_f e^{i\delta f^{(-1)}(r, \Theta_f)}$, and in the angular distribution of particles a characteristic interference pattern with half-period $\pi/k_f b_f^{\text{orb}}$ arises (Fig. 12). With increase of $\Theta_f$ one of the transition lines ($v = 2$) finds itself in the stronger shadow region (the distance of the deuteron passage along the nuclear surface from the region $v = 2$ increases, therefore, its absorption probability increase), $A_f^{(-1)}(r, \Theta_f + \Theta_f)$ decreases, and the oscillations fade. The smooth decrease of the differential cross section with the increase of $\Theta_f$ is, as earlier, due to the momentum transfer increase.

For orbital angular momentum transfer $\lambda > 0$, each of the transition lines splits into two: with $\lambda_x > 0$ ($v = 1$ and 3 in Fig. 11b) and with $\lambda_x < 0$ ($v = 2$ and 4). The lines $v = 1$ and $v = 4$ lie in the region of outgoing deuteron strong absorption and make a small contribution to the cross section, which is mainly determined by the contributions of the hatched areas on the transition lines $v = 2$ and $v = 3$. With increase of $\Theta_f$ the relative contribution from $v = 3$ decreases and from $v = 2$ (and subsequently from $v = 1$) increases.

The stripping cross section for $\lambda \neq 0$ is determined by the incoherent sum

$$\Sigma_{\lambda \neq 0} \left| T_{\lambda}^{(\ell)} \right|^2,$$

which significantly smoothes out the oscillations in the angular distribution with increase of $\lambda$, although each term in this sum oscillates with the change of $\Theta_f$. The cross section decrease at $\Theta_f \to 0$ for $\lambda = 3$ is due to the entering of the transition lines ($v = 2$ and $v = 3$) into the weak shadow of the forward-emitted deuteron. With the angular momentum transfer increase the transition lines move "upward" (to the smaller angles $\Theta_f$) and can find themselves outside the $f$-shadow at $\Theta_f = 0^\circ$. In this case the angular distribution maximum is again at $\Theta_f = 0^\circ$ (see below Fig. 13 and Fig. 15).

We now consider the direct stripping of a heavy fragment from the incident particle in reactions induced by heavier ions. As an example, we use the reaction $^{181}\text{Ta}(^{22}\text{Ne}, \alpha)$, for which the extensive experimental data has been accumulated \cite{27, 28}.

We will see below (Section 5) that the multi-step processes accompanied by the
Fig. 12. Angular distribution of deuteron in $^{64}$Ni($^6$Li, $d$)$^{68}$Zn reaction. The solid lines show the semiclassical calculations, the dashed lines are the exact DWBA-calculations, and the dotted line is the contribution of the transition line $\nu = 2$ for $\lambda = 3$.

Considerable kinetic energy dissipation in the entrance and exit channels make the dominating contribution to the cross section of $\alpha$-particle formation (and of other light particles) almost over the whole range of their energies and angles. However, even in the multi-step process the direct (coherent) transfer of several nucleons just determines the main regularities of the light particles angular and energy distributions. Therefore, in this subsection we consider these regularities in the frame of the direct one-step mechanism, meaning also an applicability of obtaining results to the extreme high-energy tail of the spectrum, where the residual nucleous energy excitation is small and the role of multi-step processes must be weakened.

The transition amplitude of the direct transfer of heavy cluster 2 from the incident ion $I$.

$$I(1 + 2) + A \rightarrow 1 + (2, A)^{\mu}$$

(3.25)
can be reduced to the form [24]

\[ T_{\mu}^{I}(k_1, k_i) \approx \mathcal{D}_I^2(Q) \, t_{\mu I}^{I}(k_1, k_i). \]  

(3.26)

Here \( \mathcal{D}(Q) = (\varepsilon_{12} - \hbar^2 Q^2/2\mu_{12}) \, \varphi_{12}^I(Q) \) defines the momentum distribution of particles 1 in the ground state of the incident ion \( I \), \( Q(\Theta_f) = k_1(R_m) - (m_1/m_i) \, k_i(R_m) \) is the momentum of particle 1 in the system connected with ion \( I \), \( t_{\mu I}^{I} \) have the form of (3.1), in which \( U_2(e^2_{2A}, r) \) is the relative motion formfactor of particles 2 and \( A \) in the residual nucleus \( B(=2+A) \) with the excitation energy \( \varepsilon^f_B \), \( \varepsilon^{rel}_{2A} = \varepsilon_{2A}^f + \varepsilon^f_B \). The factor \( \mathcal{D}_I^2(Q) \sim \exp(-Q^2/\sigma^2) \) (where \( \sigma^2 = \sigma_0^2 (A_1 \cdot A_2/(A_x - 1)) \) and \( \sigma_0 = P_F/\sqrt{5} \approx 0.51 \text{ fm}^{-1} \) [29]) is of great importance only for high incident energy, and for energies \( E_i \lesssim 5 \text{ MeV/nucleon} \) the angular and energy distributions of light particles are defined basically by the amplitude \( t_{\mu I}^{I} \) [24].

---

**Fig. 13.** Contributions of the different transition lines to the cross section of \( ^{181}\text{Ta}(^{22}\text{Ne}, \alpha)^{199}\text{Tl} \) reaction at \( E^\text{lab}_\text{Ne} = 155 \text{ MeV} \).
The uncertainty of the spectroscopic factors of initial \((1, 2)\) and final \((2, \Lambda)\) states for heavy fragments and the non-accounting of multi-step processes do not allow us to claim on obtaining the absolute values of cross sections, but only on the regularities of the angular, energy, and transferred angular momentum distributions of light particles.

In Fig. 13 the transition lines and their relative contributions to the cross section of \(^{181}\text{Ta}(^{22}\text{Ne}, \alpha)\) reaction at \(E_{\text{lab}} = 155\) MeV are shown. Only for very forward angles are the contributions of the regions \(v = 2\) and \(v = 3\) approximately equal. Just here (and also for angles \(\Theta_f \approx 80^\circ\), where the contribution from the region \(v = 1\) becomes noticeable) it is possible to observe an interference pattern in the direct massive transfer process. For its observation the channel with the fixed value of \(\mu = \lambda\) must be singled out; but here it is not obligatory to fix the values of \(\lambda\) and \(E_1\) (isolation of comparatively narrow intervals \(\Delta \lambda < \lambda\) and \(\Delta E_1 < E_1\) is enough), since the period of oscillations will be approximately \(\Delta \Theta_f \approx \pi/k_i h^{\text{rb}}\).

The main contribution to the reaction over the wide range of angles comes from the transition line \(v = 2\) and the amplitude \(t_{1,2}^{\mu}\) has the form (3.23). Thus, three main factors determine the regularities of direct massive transfer process: (1) The shadow region of the outgoing particle, which depends strongly on \(\Theta_f\) and weakly enough on \(E_1\). (2) The formfactor \(U_{\lambda}\) value, which depends strongly on \(e_2, \lambda,\) and \(R_m\) in subbarrier region. (3) The dynamic factor \(\exp\{-\frac{q}{d \mathcal{R}/2\lambda}\}\) defining the ability of the residual nucleus to take the transferred momentum \(q = k_i(R_m) - k_i(R_m)\). This factor depends strongly on \(\Theta_f\) and not very strongly on \(E_1\). For the reaction with a large angular momentum transfer it is more correct to use the quantity \(q_{\lambda} = q \sqrt{1 - (\lambda/qr)^2}\) instead of \(q\) in this factor.

Transferred angular momentum distribution. For small \(\lambda\) values the transition lines dispose nearer to \(z\)-axis in the shadow region of outgoing in the forward angles light particles, and \(d\sigma/\lambda\) is small. For large \(\lambda\) values the transition lines move up. \(R_m\) increases, \(U_{\lambda}(R_m)\) sharply decreases in the subbarrier region, and \(d\sigma/\lambda\) decreases again. In this way some peak-shaped angular momentum transfer distribution is formed with the maximum at \(\lambda = \lambda_m(E_1, \Theta_f)\) (Fig. 14). With the decrease of \(E_1\), the transition lines move down (nearer to the \(z\)-axis) and \(\lambda_m\) increases. With the increase of \(\Theta_f\), the transition lines for small \(\lambda\) go out from the shadow. \(\lambda_m\) decreases, and the \(\lambda\)-distribution is not peak-shaped any more.

Angular distribution. For small \(\lambda\) with increase of \(\Theta_f\) the transition lines go out from the shadow, \(A_{\lambda}(R_m, \Theta_f)\) increases and the cross section increases. However, with the subsequent increase of \(\Theta_f\) the cross section begins to fall sharply because of the factor \(\exp\{-\frac{q}{d \mathcal{R}/2\lambda}\}\). As a result, for small \(\lambda\) values the maximum of angular distribution will be observed at \(\Theta_f \neq 0^\circ\) (Fig. 15). For large \(\lambda\) values the angular distribution is strongly peaked in the forward direction due to the fast increase with \(\Theta_f\) of momentum transfer \(q\).

Thus, the angular distribution in the direct massive transfer reactions must essentially differ from the analogous ones in light-ion-induced reactions. Unfortunately, the channels with the fixed value of \(\lambda\) (or in a sufficiently narrow interval \(\Delta \lambda\) at
FIG. 14. Transferred angular momentum distribution in $^{181}\text{Ta}(^{22}\text{Ne}, \alpha)^{199}\text{Tl}$ reaction at $E_{\text{lab}} = 178$ MeV. (1) $E_\alpha = 90$ MeV, $\theta_\alpha = 10^\circ$; (2) $E_\alpha = 120$ MeV, $\theta_\alpha = 10^\circ$; (3) $E_\alpha = 120$ MeV, $\theta_\alpha = 50^\circ$; the dashed line is for $E_{\text{Ne}} = 141$ MeV, $E_\alpha = 90$ MeV, $\theta_\alpha = 10^\circ$.

FIG. 15. Partial angular distributions of $\alpha$-particles with $E_\alpha = 120$ MeV in $^{181}\text{Ta}(^{22}\text{Ne}, \alpha)^{199}\text{Tl}$ reaction at $E_{\text{lab}} = 178$ MeV.
least) have not been singled out experimentally in such reactions till now. Note, that the cross section falling by several orders of magnitude (from $\Theta_f = 0^\circ$ to $\Theta_f = 60^\circ$) is caused just by the direct transfer of a heavy fragment, because only in this case does such strong dependence on momentum transfer appear. The experimental data [27] show exactly such sharp forward direction of light particles, which hardly may be explained in any pre-equilibrium evaporative models. The experimental distributions of $\alpha$-particles formed in the reaction investigated had been obtained only for $E_\gamma$ not exceeding 80 MeV ($e^*_\gamma \approx 60$ MeV) [27], i.e., in that region where the multi-step processes prevail. Therefore, we will accomplish the comparison with an experimental angular distribution below (Section 5) when we take such processes into account.

Energy distribution. Practically all the factors stated above (and also $Z_2^2(Q)$) influence the dependence on excitation energy (i.e., on $E_1$) of the cross section of the direct heavy fragment stripping process. Let us introduce the designation $\gamma_\alpha(r) = [(2\mu_2A/h^2)(V_{2A}^0(r) - e^\text{rel}_{2A})]^{1/2}$ for a quantity defining the rate of the formfactor $U_\alpha(r)$ exponential change in the subbarrier region; $V_{2A}^0(r)$ is the effective interaction of particle 2 with the nucleus A, $r_0(e^\text{rel}_{2A}, \lambda)$ is the inner turning point (the second from the side of large $r$) in this potential, $V_{2A}^0(r) = (d/dr) V_{2A}^0(r)$.

The energy spectrum slope of light particles 1 forming in the direct stripping process depends on the ratio between the rate of absorption in the entrance channel $((d/dr) \ln A_\gamma \equiv \omega_\gamma(r))$ and the rate of the formfactor decrease $\gamma_\alpha(r)$. For $\omega_\gamma(R_w) > \gamma_\alpha(R_w)$ (the strong absorption or not very heavy fragment stripping) the point $R_m$ lies in the subbarrier region and

$$\frac{d}{dE_1} \ln[A_{1}(U_\gamma)] \approx -2 \frac{\gamma_\alpha(R_m)}{F_{2A}(r_0)}.$$

In the other case, the process is localized near the turning point $r_0$ and

$$\frac{d}{dE_1} \ln[A_{1}(U_\gamma)] \approx -2 \frac{\omega_\gamma(r_0)}{F_{2A}(r_0)}.$$

The terms in the r.h.s. of Eqs. (2.27) change weakly enough with $E_1$, ensuring approximately constant slope of the energy distribution. This slope (as well as the cross section absolute value) depends strongly on the absorption in the entrance channel, the value of which are determined very badly at present for such heavy ions as $^{22}$Ne.

With a change of $E_1$, the transferred momentum changes, and the factor $\exp[-(q \Delta R/2)^2]$ also has an influence on the energy distribution

$$\frac{d}{dE_1} \ln \left[ \exp \left\{ -2 \left( \frac{q \Delta R}{2} \right)^2 \right\} \right] \approx +4 \frac{m_1}{\hbar^2} \frac{\Delta R}{2} \left[ k_{1}(R_m) \cos \Theta_{1}^{\text{loc}} \frac{k_{1}(R_m)}{k_{1}(R_m)} - 1 \right].$$

Here $\Theta_{1}^{\text{loc}}$ is the angle between vectors $k_{1}(R_{1m}, \Theta_f)$ and $k_{1}(R_{1m})$. As is obvious
from (3.28), the factor \( \exp\left\{ -(q A R/2)^2 \right\} \) decreases the slope of the energy distribution for small angles \( \Theta_f \), and for large \( \Theta_f \), it makes them steeper.

In Fig. 16 the \( \alpha \)-particle energy distribution obtained with the help of (3.26), (3.8), and (3.19) is shown in comparison with the experimental data for the high-energy tail of the spectrum. The hatched area corresponds to the different choice of the absorption potential's parameters: \( \omega_f(R_w) = 3.4 \text{ fm}^{-1} \) for the upper border of this area and \( \omega_f(R_w) = 1.7 \text{ fm}^{-1} \) for the lower one (the different curves are matched at the energy \( E_\alpha = 120 \text{ MeV} \)). The breakdown in the slope of the high-energy part of the spectrum (\( E_\alpha \gtrsim 120 \text{ MeV} \)) can be explained by the going out on the yrast-line, i.e., by the inability of the almost unexcited residual nucleus to "absorb" the large angular momentum [24].

In spite of the fact that the direct stripping mechanism can qualitatively explain the regularities of the given reaction, the discrepancy of the theoretical results with experimental ones when the excitation energy increases (observing for all reasonable \( \omega_f(r) \) and \( F_{2A}^s(r) \) values) indicates the significant contribution of multi-step processes to the light particle formation (see Section 5).
4. MULTI-STEP PROCESSES AND DISSIPATIVE FORCES

The consistent enough description of multi-step processes could be made within the coupled channels approach (CC). However, for heavy ion collisions one succeeds in realizing CC only in the case of the excitation of a relatively small number of the simple structure nuclear states, for example, rotation ones [30]. The difficulties arising here are well known: the rapidly oscillating behaviour of the channel wave functions, the long range of integration, the large number of coupled equations, and the partial waves that should be considered. To these computational difficulties one needs to add the essential physical problems arising when DIP are described: we do not know the nature of highly excited nuclear states, and the coupling interaction being immensely complicated is known to us only in the common form as a sum of two-particle forces.

Noticeable progress within the semiclassical approach for the multi-step processes description was achieved only in the case of quasielastic reactions, where the relative motion is treated as extremely classical (R(t) is a C-number) [7] or it is reduced to the same in the final expressions for cross sections [31], or when the coupling of only a few channels is considered within the eikonal approximation [32].

The authors of Ref. [33] develop a more general approach, in which the multi-channel wave function $\Psi_k^{l+1}(r, \xi)$ (\(\xi\) are the internal degrees of freedom) is constructed directly in the form $A(r, \xi) e^{i\Phi(r, \xi)}$ without expanding over the target states. Here $h\Phi(r, \xi)$ is the full action function of the corresponding classical problem. A certain amount of progress had been attained in solving some atomic physics problems in this way. However, in our case, besides the numerous technical difficulties the physical problems stated above appear to have in such approach. If one is limited by the simple degrees of freedom only (for example, by rotational or vibrational states), the number of problems under consideration will be sharply reduced. Furthermore, in heavy ion collisions the eikonality condition allowing the use of straight line trajectories when evaluating $A(r, \xi)$ and $S(r, \xi)$ is not usually valid. Therefore, an accurate solution of the multi-particle classical problem becomes unattainable.

4.1. Semiclassical Limit of the Coupled Method

Let us consider the collision of ion I having the momentum $k_0$ and the energy $E_0 = h^2 k_0^2 / 2m$, with the target A, the intrinsic states of which with the energies $\varepsilon_\nu$ are described by the wave functions $\phi_\nu(\xi)$: $H_A \phi_\nu = \varepsilon_\nu \phi_\nu$. For the sake of clarity, we neglect, for the time being, the projectile internal structure and we write the interaction of it with the target in form $U(r) + V(r, \xi)$, where $U(r)$ distorts only the relative motion of the nuclei and $V(r, \xi)$ couples their relative and intrinsic motions. Expand the total wave function over the target states

$$\Psi_{k_0}^{l+1}(r, \xi) = \sum \psi_\nu(r) \phi_\nu(\xi). \quad (4.1)$$
The channel wave functions satisfy the set of coupled equations

\[
\left[ -\frac{\hbar^2}{2m_i} \nabla^2 + U(r) + V_{\nu}(r) - E_\nu \right] \psi_\nu(r) = - \sum_{\mu \neq \nu} V_{\nu\mu}(r) \psi_\mu(r),
\]

where \( E_\nu = E_0 - \varepsilon_\nu \) and \( V_{\nu\mu} = \langle \phi_\nu(\xi) | V(r, \xi) | \phi_\mu(\xi) \rangle \). We will look for these functions in the form

\[
\psi_\nu(r) = \sum_t C_t(b, r) A_t(b, r) e^{iS_t(b, r)},
\]

where \( \sum_t A(t) e^{iS(t)} \) solves the semiclassical problem of projectile motion in the channel \( \nu \) without coupling with other channels, i.e.,

\[
(\nabla S_\nu)^2 = \frac{2m_i}{\hbar^2} \left[ E_\nu - U(r) - V_{\nu\nu}(r) \right]
\]

\[
\nabla S_\nu \cdot \nabla A_\nu + \frac{1}{2} A_\nu \nabla^2 S_\nu = 0.
\]

At the above-barrier energies the caustic surfaces are far enough from the target nucleus and the trajectories with the different impact parameters do not intersect with each other in the region where we need the wave functions \( \psi_\nu(r) \). In this case only one term remains in (4.3) and the subscript \( t \) is omitted in the following for simplicity. An accounting of several trajectories in the case of a weak absorption or at the near-barrier energies can be made in the same way as above (Section 2).

Inserting the expansion (4.3) into (4.2) and using the smallness of the relative motion wavelength we get the set of coupled equations for the quantities \( C_\nu(r) \) [4]

\[
k_\nu(r) \nabla C_\nu(r) = -i \sum_{\mu \neq \nu} \frac{m_i}{\hbar^2} V_{\nu\mu}(r) e^{i\Delta S_{\nu\mu}(r)} \cdot C_\mu(r),
\]

where \( \Delta S_{\nu\mu} = S_\mu(k_\mu, r) - S_\nu(k_\nu, r) \), \( k_\nu(r) = \nabla S_\nu \). These equations are valid for relatively small energy transfer \( (\varepsilon_\nu < E_0) \), when we can neglect the contribution of the closed channels in (4.1). The similar equations for the quantities like \( C_\nu(r) \) was obtained in [7, 31, 32].

The quantities \( C_\nu(r) \) contain the information about channels coupling and fulfill the condition

\[
\sum_{\nu} \frac{k_\nu}{k_0} |C_\nu(r)|^2 = 1,
\]

reflecting the conservation of flux. They also obey the boundary conditions ensuring the incoming wave in the channel \( \nu = 0 \) only. \( |C_\nu(r)|^2 \) is the probability to excite the target into the states \( \phi_\nu(\xi) \) while the incident ion reaches the point \( r \).
The next step in simplifying the coupled equations (4.5) can be based on the usage of the fact that the large number of intrinsic states are excited in heavy ion collisions, and the density of these states is extremely high (i.e., $\epsilon_{v+1} - \epsilon_v = \rho^{-1}(\epsilon_v) \ll E_0$). Before doing this step, we consider, in general, the outline behaviour of the quantities $C_v(r)$ at large $v$ values. For this purpose we had solved exactly the model one-dimensional multi-channel problem, approximating $V_{\nu\mu}(x)$ by the simple functions $V_{\nu\mu}(x) e^{-(x-x_0)^2} e^{-(y/\lambda)^2}$ at $x > 0$ ($V_{\nu\mu}(x) = 0$ at $x < 0$) and taking into account 200 complex equations in (4.5). Distribution of the system over the channels as the projectile moves along the $x$-axis ($x = 0$ corresponds to the nuclear surface, $C_v(x < 0) = \delta_{\nu 0}$), is shown in Fig. 17.

The model calculations and the direct analysis of Eqs. (4.5) lead to the following conclusions:

1. Two phases of reaction are distinguished: the "initial" one when the occupation of excited states occurs mainly due to their coupling with the entrance channel $v = 0$ (here $|C_{v0}| \leq |C_{00}|$), and the subsequent "friction phase," in which energy transfer over the channels is observed ($d\nu(x)/dx \sim F_{\nu\mu}$, where $\nu(x)$ is the $\nu$ value at which $C_{\nu\mu}(x)$ is maximal—see Fig. 17), the energy loss is considerable and, in particular, $C_{0\mu}(x)$ is small; i.e., the system is mainly in the states $\nu \gg 1$.

---

**Fig. 17.** Evolution of $C_{\nu\mu}(x)$ obtained from Eqs. (4.5). Solid lines correspond to the channels coupling parameter $\lambda_1 = 1$ ($V_0 = 4$), and the dashed lines are for $\lambda_1 = 2$ ($V_0 = 2$).
2. The character of the solution is defined basically by the channel coupling parameter \( \Delta \nu \). At large values of \( \Delta \nu \) there is even distribution over the channels within \( 0 \leq \nu \leq \nu(x) \). At small \( \Delta \nu \) the irregular distribution occurs with the maximum at some \( \nu(x) \).

3. Representing the quantities \( C_\nu(x) \) in the form \( |C_\nu(x)| e^{i\varphi(x)} \) we see that their phases are correlated (Fig. 17b). This means that one ought to retain the coherent summation over the intermediate states in transition amplitudes when evaluating the cross sections of multi-step processes. However, there are definite reasons, in fact, for allowing completion of the incoherent summation when DIP are considered (see below). At the “friction phase,” when the distribution over \( \nu \) becomes wider than \( \Delta \nu \), the set of equations for phases \( \varphi_\nu(r) \) follows from Eqs. (4.5):

\[
\mathbf{k}_\nu(r) \mathbf{\nabla}_\nu(r) = - \sum_{\mu \neq \nu} \frac{m}{\hbar^2} \text{Re}\{ V_{\nu\mu}(r) e^{i(\Delta S_{\nu\mu} + \gamma_\nu - \gamma_\mu)} \}.
\]

(4.7)

Independence of these equations on \( |C_\nu(r)| \) means that some fixed distribution of the phases over \( \nu \) is settled; here \( \mathbf{\nabla}_\nu \ll \mathbf{\nabla}_\nu ; \) i.e., the heavy projectile’s momentum in channel \( \nu \) is not changed considerably by coupling with the other channels.

4. It is convenient to convert from the quantities \( |C_\nu(r)| \) to \( C(\epsilon_\nu, r) = \sqrt{\rho(\epsilon_\nu) |C_\nu(r)|} \) at the “friction phase.” Then in the first approximation over a small quantity \( 1/\rho(\epsilon_\nu) E_\nu \), we obtain for \( C(\epsilon_\nu, r) \) the simple transport equation [4]

\[
\frac{\partial C(\epsilon_\nu, r)}{\partial t} + F(\epsilon_\nu, r) \frac{\partial C(\epsilon_\nu, r)}{\partial \epsilon_\nu} = 0
\]

(4.8)
in which \( \partial/\partial t \) is a curvilinear derivative along the trajectory passing through the point \( r \), and the quantity

\[
F = -\mathbf{k}_\nu(r) \sum_\mu \frac{\epsilon_\mu - \epsilon_\nu}{2E_\nu(r)} \text{Im}\{ V_{\nu\mu}(r) e^{i(\Delta S_{\nu\mu} + \gamma_\nu - \gamma_\mu)} \}
\]

(4.9)
determines the rate of energy transfer over the channels while the projectile moves along the trajectory: \( \epsilon_{\nu(r)} \equiv \epsilon(r) = \int_{r'}^{r} F(r') \, dl' \). We do not call for the time being, the quantity \( F \) the “friction force” (although this is what it is) because the initial equation (4.5) describes, in principle, the time-reversible process.

5. The rate of excitation transfer \( F \) depends strongly on the channel coupling parameter \( \Delta \nu \). For the straight line trajectories,

\[
\Delta S_{\nu\mu} \approx \frac{\epsilon_\nu - \epsilon_\mu}{2E_\nu(r)} k_\nu(r) \cdot x = \frac{\sqrt{m_i/2\hbar^2} \cdot x \cdot (\nu - \mu)}{\rho(\epsilon_\nu) \sqrt{E_\nu - U(r)}} \equiv \alpha(\nu - \mu).
\]

At the large relative velocities and large level density the quantity \( x \) is small, the oscillations of \( \exp(i \Delta S_{\nu\mu}) \) within the interval \( \Delta \nu \) of the noticeable values of \( V_{\nu\mu} \) are
negligible, and \( F \sim V_0 \cdot A_{IV}^3 \). This result means, for example, that the particle-hole excitations (for which \( A_{IV} \) is seemingly larger than unity) shall pass ahead of the excitation of collective states (for which the nearest levels \( \nu \) and \( \nu \pm 1 \) are coupled with the largest intensity). In slow collisions the excitation of the states with a low level density ("hard" degrees of freedom) becomes hardly probable because of the strong oscillations of \( \exp(i \Delta S_{\nu\rho}) \) within the interval \( \Delta_{IV} \), and \( F \sim V_0 \cdot A_{IV}^3 e^{-(1/2)\Delta_{IV}^2} \).

This well-known (both in classical and quantum mechanics) theoretical result about small energy losses in adiabatic collisions of the simple systems is, however, inconsistent with the experimental data on slow atomic and nuclear collisions.

4.2. Destruction of Coherence

There is one well-known reason for allowing summation over the intermediate states to go from coherent to incoherent when evaluating the transition amplitude (5.3) or the cross section (5.9) of the multi-step process. This reason is that in the heavy-ion experiments one measures the cross sections summed over the final states \( f \) within some interval \( \Delta f \) containing the large enough number of residual nuclei's states: \( d\sigma \sim \sum_{f \in \Delta f} |T_f|^2 \). Designate by \( \{\nu\} \) the set of quantum numbers \( \nu, \nu, \ldots \); then \( T_f = \sum_{\{\nu\}} t_{ij}^{[x]} \) and \( \sum'_{\{\nu\}} = |t_{ij}^{[x]}| \rho^{\nu_i\nu_j(f)} \). If the phases of the channel amplitudes are uncorrelated over \( f \) (they may be correlated over \( \{\nu\} \)) and the interval \( \Delta f \) is narrow enough (i.e., \( |t_{ij}^{[x]}| \) is almost unchanged within it), then

\[
\frac{1}{N_f} \sum_{f \in \Delta f} \exp\{i\tau_{\nu_i}(f) - i\tau_{\nu_i}(f)\} \approx \delta_{\nu_i\nu_j}
\]

and

\[
\frac{1}{N_f} \sum_{f \in \Delta f} \left| \sum_{\nu} t_{ij}^{[x]} \right|^2 \approx \sum_{\nu} |t_{ij}^{[x]}|^2.
\]

Here \( N_f \) is the number of states within the interval \( \Delta f \).

Now we consider one more reason that destroys the coherence primarily when evaluating the multi-step process transition amplitude (5.3). In realistic cases the relative motion of nuclei is not coupled equally with the different internal degrees of freedom. Let the relative motion be strongly coupled with some intrinsic degree of freedom \( \xi_1 \) that, in turn, is coupled with other degrees of freedom \( \xi_2 \); i.e., the total Hamiltonian may be written as

\[
H(\mathbf{r}, \xi_1, \xi_2) = -\frac{\hbar^2}{2m_r} \nabla^2 + U(\mathbf{r}) + H_1(\xi_1) + H_2(\xi_2) + V(\mathbf{r}, \xi_1) + V(\mathbf{r}, \xi_2).
\]

We expand the total wave function over eigenstates of \( H_1 \) and \( H_2 \) \( (H_1 \varphi_v = \varepsilon_v \varphi_v, H_2 \phi_i = \varepsilon_i \phi_i) \),

\[
\Psi_{k_0}^{(-)}(\mathbf{r}, \xi_1, \xi_2) = \sum_{\nu} \sum_i \psi^{(i)}(\mathbf{r}) \varphi_v(\xi_1) \phi_i(\xi_2)
\]

and will look for the channel wave functions \( \psi^{(i)}(\mathbf{r}) \) in the form (4.3).
Two natural enough assumptions can be made: (1) The level density of exciting states is much smaller than the total level density. It means that $\varepsilon_{i+1} - \varepsilon_i \gg \varepsilon_i$, i.e., the incident ion strongly interacts with the "hard" degree of freedom $\xi_1$ which is coupled with the other more "soft" internal degrees. (2) Matrix elements $v_{\mu k}^{ik} = \langle \phi_i | \nu | \phi_{\mu} \phi_k \rangle$ decrease more rapidly with the increase of $(\nu - \mu)$ than of $(i - k)$ (because the states $\phi_i$ and $\phi_{\nu \pm 1}$ are farther from each other in energy than $\phi_i$ and $\phi_{i \pm 1}$ are) and they can be approximated by the expression $v_{\nu k}^{ik} \approx \delta_{\nu k} v_{\nu k}^{ik}$. Notice, that the assumption (2) is not of principle character and is made in order that a set of equations for $C_{\nu}^{(i)}$ will have a simple enough form,

$$k_{\nu}(r) V C_{\nu}^{(i)} = -i \sum_{\mu \neq \nu} \frac{m_i}{\hbar^2} V_{\nu \mu}(r) e^{i \Delta S_{\nu \mu}(r)} C_{\mu}^{(i)}(r)$$

$$- i \sum_{k \neq i} \frac{m_i}{\hbar^2} v_{\nu k}^{ik} C_{\nu}^{(k)}(r). \tag{4.5b}$$

The accounting of the coupling of the exciting degree of freedom $\xi_1$ with the other "soft" degrees leads to two interesting results. Since $\xi_1$ and $\xi_2$ are different degrees of freedom, the number $v_{\nu}^{ik}$ ought to depend on $\nu$ weakly enough and sufficiently random (their dependence on $(i - k)$ is the dependence of $V_{\nu \mu}$ on $(\nu - \mu)$ may be quite regular). The second sum in the r.h.s. of Eq. (4.5b) corresponds to a "noise" that destructs the coherence of $\gamma_{\nu}(r)$ over $\nu$ (but not over $r$!). It means that when evaluating the quantities $C_{\nu}(r)$ we may not care about their phases $\gamma_{\nu}$, because they drop out of the final expressions (due to incoherent summation) and because in the radial integration the phases $S_{\nu}(r)$ play the main role (due to $\nabla S_{\nu} \ll \nabla S_{\nu}$).

On the other hand, the chaotic variation of the phases $\gamma_{\nu}(r)$ has to weaken the role of the $\exp(i \Delta S_{\nu \mu})$ oscillations in (4.9) for slow collisions. Therefore, the probability of excitation of the "hard" degrees of freedom shall increase and the probability of their de-excitation in the respiration stage decrease. Thus, Eqs. (4.5) and (4.8) (with regard to (4.5b)) describe, in fact, the time irreversible processes of heavy ion collisions and the quantity $F$ defined by expression (4.9) acquires the meaning of the friction force.

4.3. Mass Transfer

One would like to generalize the approach stated above for the nucleon transfer channels that are spread over the continuous spectrum of $H_A$ in expansion (4.1). Let $\phi_{\nu}^{(N)}(\xi^{(N)}) - \phi(I - N) \phi(A + N)$ are the intrinsic states, $N = 0, \pm 1, \pm 2, \ldots$ is the number of nucleons transferred from ion $I$ to target $A$. The total wave function can be expanded over $\phi_{\nu}^{(N)}$,

$$\Psi_{k_0}^{(+)}(r, \xi) = \sum_N \sum_\nu C(N, \nu, r^{(N)}) A(N, \nu, r^{(N)}) e^{i S(N, \nu, r^{(N)})} \phi_{\nu}^{(N)}(\xi^{(N)}), \tag{4.10}$$

where $A(N, \nu) e^{i S(N, \nu)}$ solves, as before, the semiclassical problem of relative motion of heavy fragments $I_N = I - N$ and $A_N = A + N$ without channel coupling.
When the relative motion is not very fast \( (v_{rel} \ll v_F) \) and the number of transferred nucleons is small \( (N \ll I, A) \), one can neglect the difference of relative motion coordinates: \( r^{(N)} \approx r^{(N=0)} \equiv r = r_I - r_A \). Inserting the expansion (4.10) in the Schrödinger equation and neglecting the non-orthogonality of the channels (the detailed consideration of this problem can be find in [7]), we obtain the equations similar to (4.5) in the semiclassical limit

\[
\mathbf{k}(N, v, r) \nabla C(N, v, r) = -i \sum_{N', \nu} \frac{m(N)}{\hbar^2} V_{Nv, N'v'}(r) \times \exp[i \Delta S_{Nv, N'v'}(r)] \cdot C(N', v', r). \tag{4.11}
\]

These equations also retain the total flux \( \sum_{N, \nu} (k(N, v)/k_0) |C(N, v, r)|^2 = 1 \) along the trajectory and it ensures the avoidance of some gross mistakes due to neglecting channel non-orthogonality.

Now, as before, at the "friction phase" we can expand the quantities \( C(N', v', r) \) into a series over \( (N - N') \) and \( (v - v') \) powers, assuming that \( V_{Nv, N'v'} \) decays quickly enough with the increase of \( (N - N') \) and \( (v - v') \). As a result, a more general equation than (4.8) arises for the quantity \( C(N, \epsilon, r) = \sqrt{\rho(N, \epsilon)} |C(N, v, r)| \) simulating the channel coupling,

\[
\frac{\partial C(N, \epsilon, r)}{\partial t} + F_\epsilon \frac{\partial C}{\partial \epsilon} + F_N \frac{\partial C}{\partial N} + \sum_{n, m \geq 2} \mathcal{F}_{(n, m)} \frac{\partial^{n+m} C}{\partial \epsilon^n \partial N^m} = 0. \tag{4.12}
\]

The drift and diffusion coefficients of this equation may be evaluated microscopically or derived from the experimental distributions of DIP products. The arguments stated above in Subsection 4.2 are valid also for Eqs. (4.11) and (4.12). Therefore, these equations describe the uncorrelated nucleon exchange between nuclei. The heavy fragment transfer (i.e., essentially coherent transfer of several nucleons) must be described in quite a different way (see Section 5).

4.4. Quanta1 Friction

Here we briefly touch upon the problem of the quantum mechanical treatment of particle motion in a dissipative medium, i.e., upon the construction of the Schrödinger equation that takes into account potential (conservative) and dissipative forces. The question of a principle for the possibility of a quantum description for dissipative motion with pure states (i.e., the question about existence of corresponding wave function and, therefore, the Schrödinger equation for them) has been discussed, for example, in [34] and we do not touch upon it here.

Several possible variants of such an equation were proposed [35]. They use either the explicitly time-dependent Hamiltonian or the non-local and non-linear one of the form

\[
H = \frac{\hat{p}^2}{2m} + U(x) + \gamma(x - \langle x \rangle)[\epsilon \hat{p} + (1 - \epsilon)|\hat{p}|] - \frac{i \hbar c}{2},
\]
where $c$ is a free real parameter, $\gamma$ is a friction coefficient, and $\langle p \rangle$, $\langle x \rangle$ are expectation values of the corresponding quantum operators. All the proposed variants are not without definite shortcomings. For instance, at $c = 0$ this Hamiltonian leads to the appearance of undamping stationary states, and at $c > 0$ there is no localizable ground state. A more detailed consideration of the problems in this way can be found in [34, 35].

The approach to the multi-channel problem considered above, in which the friction forces defining the evolution of the multi-component wave function $\{\psi(r)\}$ arise in a natural way, allows one to assume that the one-body wave function describing the motion in a dissipative medium can also be seen in the form $\psi(\varepsilon, r, t)$. The additional variable $\varepsilon$ stands for the lost (dissipated) energy. The desired Schrödinger equation will describe the evolution of the wave function in the $(\varepsilon, r)$-space; i.e., it ought to contain the additional operator acting on the variable $\varepsilon$, $\varepsilon$, and $E$

$$i\hbar \frac{\partial \psi}{\partial t} = \left( -\frac{\hbar^2}{2m} + \hat{U} + \varepsilon \right) \psi + \int_{-\infty}^{\infty} \beta(\varepsilon', r) \psi(\varepsilon', r, t) d\varepsilon'. \quad (4.13)$$

In order to make this equation non-contradictory, the following basic conditions must be fulfilled:

1. Conservation of normalization, i.e., $\frac{d}{dt} \int |\psi(\varepsilon, r, t)|^2 d\varepsilon d^3r = 0$.
2. $\frac{d}{dt} \langle \hat{\varepsilon} \rangle = 0$; i.e., $\langle \hat{\varepsilon} \rangle = 0$.
3. $\frac{d}{dt} \langle \varepsilon \rangle = 2\mathcal{D}$, where $\mathcal{D}$ is the Rayleigh dissipative function.
4. $\frac{d}{dt} \langle \hat{\varepsilon} \rangle = -\langle \nabla U \rangle + \langle F_d \rangle$. In a simplest case $F_d = -\gamma p$, $\mathcal{D} = (\gamma/2m)p^2$.
5. Time-irreversibility that follows, in fact, from 4.

These conditions being satisfied, the wave packet moves in the $(\varepsilon, r)$-space along the classical trajectory $\varepsilon(t)$, $r(t)$ and $\varepsilon(r) = \int_{r'}^{r} F_d(r') dl'$. Here the quantum description of the process is feasible up to the particle’s standing. This equation also describes the stationary scattering states, i.e., the plane wave incoming on the refractive and dissipative target. In this case, as can be seen, the amplitude of the wave $\psi(\varepsilon = 0, r)$ will be decreasing due to appearance of the waves $\psi(\varepsilon \neq 0, r)$, while it enters the medium. Here all the refractive and reflective phenomena will be valid.

For a weak non-locality of the $\hat{\varepsilon}$-operator that can be expected in the classical limit it is reasonable to make the desired equation in the differential form:

$$i\hbar \frac{\partial \psi}{\partial t} = \left[ -\frac{\hbar^2}{2m} \nabla^2 + U(\varepsilon, r) + \varepsilon \right] \psi - i\hbar \hat{\alpha} \frac{1}{|\psi|^2} \frac{\partial |\psi|^2}{\partial \varepsilon} \cdot \psi \quad (4.14)$$

could be one of such non-contradictory equations. Here the operator $\hat{\alpha}$ must not depend on $\varepsilon$ explicitly and $\langle \alpha \rangle = \langle \mathcal{D} \rangle$; for instance, $\alpha = -F_d(p/2m)$. The non-linear equation (4.14) undoubtedly needs additional analysis. It is possible that there are other similar equations, based on the idea stated above. Note, that it
would be very simple and visual to analyse the fusion–fission processes by such an equation.

An interesting elaboration of this idea for the motion of the system along the variable \( v \) for numbering the channels has been made in [43].

5. **Quasidirect Transfer Processes**

As it was emphasized above, the detection of particles formed on the initial stage of the reaction, before thermodynamical equilibration of the system is reached, is of special interest, because such particles are more informative than evaporated ones. However, in order to extract the reliable and unambiguous information about the dynamics of nucleus–nucleus collisions, it is necessary, on the one hand, among a great number of processes to single out (experimentally!) the simplest ones, for which the realistic theoretical models could be constructed. On the other hand, these processes must be informative enough, i.e., not have too small cross sections (to measure the energy, angular, and other distributions in wide ranges), and be sensitive to those characteristics of the nucleus–nucleus interaction we want to determine.

In our view, the so-called “quasidirect” transfer reactions

\[
I(=1+2) + A(0) \rightarrow I' + A' \xrightarrow{\text{dir}} 1' + B' (=2+A) \rightarrow 1 + B(\gamma, L, J), \quad (5.1)
\]

in which the direct transfer process of fragment 2 (heavy cluster or one nucleon) is accompanied by a considerable dissipation of kinetic energy and angular momentum of the relative motion in the entrance and exit channels, satisfy these conditions more precisely. As we will see below, for heavy fragment 2 just the stage of its direct transfer “regulates” the whole process defining the angular and energy distributions of particle 1 and the values of the preliminary dissipated energy.

An accounting of inelastic excitation channel coupling (including the excitations due to the mutual nucleon exchange) makes it possible to avoid the usage of effective absorption (that always makes the interior of the nucleus in the one-step processes “invisible”) and to keep watching the projectile to much deeper distances than it could in the elastic channel. The transfer of heavier fragments also leads to localization of the process at smaller distances due to the faster decrease of the formfactor \( U_{24}^2(r) \) with the increase of \( r \) in the subbarrier region. Thus, “quasidirect” massive transfer processes having the relatively simple mechanism to be described accurately within some theoretical model give us a chance to look into the nucleus deeply enough and to extract such characteristics of the nucleus–nucleus interaction as the potential and dissipative forces, the spectroscopic factors of multi-nucleon clusters, etc.

To be sure, in addition to the direct transfer, the uncorrelated (sequential) transfer of nucleons constituting the fragment 2 is also possible. In our approach the description of the incoherent nucleon transfer can be performed in the same
manner as the kinetic energy dissipation (see Subsection 4.3). However, the additional drift and diffusion parameters for mass transfer are required for this. Therefore, we restrict ourselves to consideration of the regularities of only the “quasidirect” transfer process (5.1).

5.1. Amplitude of Multi-step Massive Transfer Process

The exact transition amplitude of the process (5.1) is given by

$$T_{fi}(k_1, k_f) = \langle \phi_{i,k_i}^{(-)} | \mathcal{P}_f | \Psi_{i,k_i}^{(+)} \rangle,$$

(5.2)

where $\Psi_{i,k_i}^{(+)}$ is the total wave function of the entrance channel, $\phi_{i,k_i}^{(-)}$ is the asymptotic state of the exit channel ($k_i$ is the momentum of detected particle $i$, $f$ are the other quantum numbers of residual nucleous $B$ and fragment $I$), $\mathcal{P}_f$ is the interaction that was omitted when constructing $\phi_{i,k_i}^{(+)}$.

In order to simplify (5.2) we make two assumptions: (a) The transfer process of fragment 2 is direct; i.e., the coherent transfer of all the nucleons forming this fragment occurs. It means that we must take into account only the inelastic excitation (but not the rearrangement) channels in the wave functions $\Psi_{i,k_i}^{(+)}$ and $\phi_{i,k_i}^{(-)}$, and in the capacity of $\mathcal{P}_f$ we may use the interaction between fragments 1 and $2-V_{12}(r_1 - r_2)$. (b) The relative motion of $I$ and $A$ in the entrance channel and of $1$ and $B$ in the exit one is classical enough; i.e., the corresponding wavelengths are sufficiently small, the distorting potentials are smooth, and the intrinsic state level density is high.

Therefore, we can use the expansion (4.1), (4.3) for $\Psi_{i,k_i}^{(+)}$ and $\phi_{i,k_i}^{(-)}$ and Eq. (4.5) for the corresponding quantities $C_i^{(+)}$ and $C_i^{(-)}$. Inserting these expansions in (5.2) and taking into account the internal structure of all the fragments, we get

$$T_{fi}(k_1, k_f) = \sum_{v_4} \sum_{v_1} \sum_{v_8} \sum_{v_1} \sum_{v_2} \langle C_f^{(-)}(v_B, v_1, r_1) \chi_{k_i}^{(-)}(r_1) \rangle$$

$$\times \langle \phi_{v_8} \phi_{v_A} \phi_{v_2} | V_{12}(r) | \phi_{v_1} | C_i^{(+)}(v_A, v_I, R) \chi_{k_i}^{(+)}(R) \rangle.$$

(5.3)

Here $\phi_n$ are the intrinsic states of fragment $\alpha$, $\epsilon_i = \epsilon_A + \epsilon_f$ is a dissipated energy in the entrance channel, $k_f^2(\epsilon_f) = (2m_f/h^2)(E_0 - \epsilon_f)$. $k_f^2(\epsilon_f) = (2m_f/h^2) (E_1 + \epsilon_f)$. $R - (m_1 r_1 + m_2 r_2)/m_1$, $\chi^{(+)}$ are distorted waves evaluated in the one-trajectory semiclassical limit.

The six-dimensional integral in (5.3) can be reduced to the product of two three-dimensional ones by the usual local momentum approximation \cite{16,24} when $m_2 \ll m_I$ or $m_1 \ll m_f$. Here we consider just the process of the light particle formation ($m_1 \ll m_f$). As a result of such a procedure, the cross section is proportional to the quantity

$$\mathcal{D}^2(Q) = \sum_{v_2} \left( \epsilon_i - \epsilon_1 - \epsilon_2 - \frac{h^2 Q^2}{2 \mu_{12}} \right)^2 | \langle e^{iQr} \phi_{v_1} - 0 \phi_{v_2} | \phi_{v_1}(\xi_1, \xi_2, r) \rangle |^2$$

(5.4)
that defines the momentum distribution of the relative motion of particles 1 and 2 inside the ion \( I \), being in the state \( v_i \). Assuming such a distribution by the function \((\sigma / \sqrt{\pi})^3 \exp(-Q^2 / \sigma^2)\) with unit normalization, we get

\[
\mathcal{D}_I^2(Q) \approx N_I(i) \left( \frac{2 \sqrt{\pi}}{\sigma} \right)^3 \left( \frac{\hbar^2 Q^2}{2 \mu_{12}} + \tilde{\epsilon}_{12} \right)^{-\frac{3}{2}} e^{-Q^2 / \sigma^2}, \tag{5.3}
\]

where \( \sigma \) and \( Q \) are defined in Subsection 3.4, \( \tilde{\epsilon}_{12} > 0 \) is some mean value of the relative motion energy of particles 1 and 2 (of the same order of magnitude as their bound energy), and \( N_I(i) = \sum_{\nu} \int d^3r \left| \langle \phi_{v_I} \phi_{v_2} \mid \phi_{v_i}(\xi_1, \xi_2, r) \rangle \right|^2 \) is the effective number of particles 1 in the incident ion \( I \) [38].

Splitting out the integration over \( r \) from (5.3) one ought to take into account the inelasticity of the interaction of particle 1 with the heavy fragment 2 [36]. As a result, the integration on \( r \) in (5.4) must not be made over the whole space; i.e., not all the nucleons of ion \( I \) take the equal part in the formation of particle 1. Thus, the normalization factor \( N_I(i) \) can be much less than the total effective number of particles 1 in ion \( I \) and it tends to this value with an increase of the initial energy.

5.2. Parametric Representation of the Quantities \( C_{i}(r) \) Simulating the Channel Coupling

As stated above, in heavy ion collisions there is actually the incoherent summation over those quantum numbers in (5.3) that vary classically enough. In our case the relative motion of the particles in entrance and exit channels is classical and the direct transfer process of fragment 2 is not classical in the general case. In this connection it is convenient to convert from the complex quantities \( C_i^+(v_I, v_A, r) \) to the real ones \( C_i^+(\varepsilon_A, L_A, \varepsilon_I, L_I, r) = \sqrt{\rho_{L_A}(\varepsilon_A) \rho_{L_I}(\varepsilon_I)} \) \( C_i^+(v_A, v_I, r) \) with the normalization condition,

\[
\int d\varepsilon_A \int dL_A \int d\varepsilon_I \int dL_I \frac{k_i(\varepsilon_I)}{k_i} C_i^+(\varepsilon_A, L_A, \varepsilon_I, L_I, r) = 1. \tag{5.6}
\]

It is not difficult to generalize the transport equation (4.8) for the angular momentum dissipation and also to take into account the terms of the higher powers of \( 1/\rho E_0 \) that lead to the appearance of higher derivatives \( \partial^n C / \partial \varepsilon^n \) and \( \partial^n C / \partial L^n \) \((n > 1)\) defining the diffusion of \( C(\varepsilon, L, r) \) on \( \varepsilon \) and \( L \) while the projectile moves along the trajectory. However, this equation is valid only during the “friction phase” when the number of excited states is large enough and the distribution of \( C_i(r) \) on \( \varepsilon \) is much wider than the channel coupling parameter \( \Delta Y_i \). It does not hold at the “initial phase” of the reaction when such distribution is just formed and is then “transported” in accordance with (4.8). It means that the relation between the width of this distribution and the rate of its transfer may differ considerably from the ordinary thermodynamical relation between these quantities. Therefore, for practical purposes it is much simpler to represent the quantities \( C(\varepsilon, L, r) \) parametrically specifying the phenomenological friction forces and diffusion, taking
into consideration the general features of solution of Eq. (4.5) (see Fig. 17), and assuming that no anomalies occur at the "initial phase" (e.g., the excitation of high-energy states with the probability close to unity).

For the first quantitative calculations we used the dissipative forces of the simplest form $F_d(r) = -f^0_{d}\phi_1(1 + \exp[(r - R_f)/a_F])$, ignoring the difference between radial and tangential friction in order to reduce the number of parameters. Since the relative motion occurs in one plane (even when friction forces), the initial angular momentum $J_0 = k_f \beta_f(r) = L_i + \mathcal{J}(r)$, where $L_i(r) = L_A + L_f$ is the dissipated angular momentum and $\mathcal{J}(r) = [r \times k_f(\xi, r)]$ is the relative motion angular momentum at the point $r$.

The problem of energy ($\varepsilon_i = \varepsilon_A + \varepsilon_f$) and angular momentum ($L_i = L_A + L_f$) sharing between colliding nuclei is not solved at present. There are inconsistent points of view and experimental data on this question that is important for understanding of DIP [37]. Usually one shows that the projectile excitation is approximately in the interval $(m_I/m_A)\varepsilon_A \lesssim \varepsilon_i \lesssim \varepsilon_A$. It is quite obvious that the energy sharing depends on the reaction channel we single out and on the interaction time. The quantity $C^{(+)}(\varepsilon_A, L_A, \varepsilon_f, L_f, r)$ can be represented in the form $C^{(+)}(\varepsilon_i, L_i, r) f_{\varepsilon}(\varepsilon_A, \varepsilon_f) \cdot f_{L}(L_A, L_f)$, where the functions $f_{\varepsilon}$ and $f_{L}$ just define the energy and angular momentum sharing, here $\int f_{\varepsilon}^2 d\varepsilon_A = \int f_{L}^2 dL_A = 1$. In numerical evaluations we used the sharing proportional to the nuclei's masses, i.e., $f_{\varepsilon}^2 = \delta(\varepsilon_f - (m_f/m_A)\varepsilon_A)$ and $f_{L}^2 = \delta(L_f - (m_f/m_A)L_A)$. Notice, as the last experiments show [37], the equal sharing $\varepsilon_f \approx \varepsilon_A$ is seemingly more proper in the initial stage of the reaction.

Since $C^{(+)}(\varepsilon, L, r)$ is maximal at some $\varepsilon = \varepsilon(r) = \int r', F_d(r') \, dl'$, we represent it in the form

$$C^{(+)}(\varepsilon, L, r) = \frac{\sqrt{k_f}}{\sqrt{k_f(\varepsilon)}} n_{\varepsilon}(r) n_L(r) \exp\left(-\frac{1}{2}\left(\alpha - \varepsilon(r)/\mathcal{L}(r)\right)^2\right) \exp\left(-\frac{1}{2}\left(L - L(r)/\mathcal{L}(r)\right)^2\right) \quad (5.7)$$

that automatically satisfies the normalization condition (5.6) if

$$n_{\varepsilon}(r) = \left\{\frac{\pi}{2} \mathcal{D}_\varepsilon(r) \left[1 + \text{erf}\left(\frac{\varepsilon(r)}{\mathcal{D}_\varepsilon(r)}\right)\right]\right\}^{-1/2} \quad (\alpha = \varepsilon \quad \text{or} \quad L).$$

The quantities $\varepsilon(r)$ and $L(r)$ are defined unambiguously by the friction forces, and the coefficients $\mathcal{D}_{\varepsilon, L}(r)$ we specified with accounting of the quantity $C(\varepsilon = 0, L = 0, r)$ determine the wave function damping in the elastic channel. Within the optical model such a damping is defined by the imaginary part of the potential and is proportional to

$$\exp\left[-\int_{r'}^{r} \frac{m_f}{\hbar^2 k_f(r')} W(r') \, dl'\right] \sim \exp\left[-\frac{k_f}{2E_f} W_0 \cdot l\right],$$

where $l$ is a distance covered in the absorbing medium. Since $\varepsilon(r) \sim F_d \cdot l$, we used the dependence of the form $\mathcal{D}_{\varepsilon, L} \sim \mathcal{D}_{\varepsilon, L}^0 \cdot \sqrt{l}$ for diffusion coefficients to retain an accordance with the optical model. Representing the quantity $C^{(-)}(\varepsilon, L, r)$, we sup-
posed that the de-excitation process in the exit channel is hardly probable, i.e.,
\[ C_{f}^{(-)}(e, L, r) = 0 \text{ at } e > e_f \text{ and at } L > L_f. \]
Therefore, the outgoing particle 1 can only excite still more of the residual nucleus \( B \) from the energy \( e_B = e_A + e_{2A}^{\text{rel}} \) that it has, just after the stripping of fragment 2 onto the excited nucleus \( A \), up to the final excitation energy \( e_B^{\text{f}} \geq e_B \).

5.3. Fast Light Particles Formation in Deep Inelastic Processes

Using the designation

\[
\langle \varphi_{v_B} | \varphi_{v_4} \varphi_{v_2} \rangle = \sum_{\lambda} \left( L_A M_A \lambda_{\mu}/L_B M_B \right) S_{B}^{1/2}(v_{B}^{T}, v_{4}, v_{2}, \lambda) \left( \sum_{\mu} U_{\mu}^{\text{rel}}(e_{2A}^{\text{rel}}, r_{2}) \right). \tag{5.8}
\]

where \( U_{\mu} \) is a formfactor of the relative motion of fragments 2 and \( A \) with angular momentum \( \lambda \) and with energy \( e_{2A}^{\text{rel}} \), \( S_{B} \) is the spectroscopic factor of this motion in the state \( v_{B}^{T} \), and neglecting the spins and the excitation energies of fragments 1 and 2 in comparison with \( L_A, L_B \gg 1 \) and \( e_A, e_B \), we get the final expression for the cross section of the light particles formation in the “quasidirect” massive transfer process (5.1)

\[
\frac{d^{2}\sigma(e_{B}^{*}, L_{B}^{*}, \Theta_{f})}{d\Omega_{f} dE_{f}} = (2\pi)^{-2} \frac{m_{1} m_{2} k_{1}}{h^{4}} \int d\epsilon_{f} \int dL_{f} \int d\epsilon_{i} \int dL_{i} \int d\lambda \int d\mu \times \rho_{L_{f}}(\epsilon_{B}^{*}) S_{B}(\epsilon_{B}, L_{B}; L_{A}, \epsilon_{A}, \lambda) \mathcal{D}_{f}^{2}(Q) \times \left( L_{A} \lambda_{\mu}/L_{B} \mu \right)^{2} \left( r_{\mu}^{B}(k_{1}(\epsilon_{f}), k_{f}(\epsilon_{f})) \right)^{2} \tag{5.9}
\]

\[
r_{\mu}^{B} = \left( C_{f}^{(-)}(e_{f}, L_{f}, r) \chi_{k_{f}(\epsilon_{f})}^{-1}(r) U_{\mu}^{\text{rel}}(e_{2A}^{\text{rel}}, r) C_{i}^{(+)}(e_{i}, L_{i}, r) \chi_{k_{i}(\epsilon_{i})}^{(+)}(r) \right). \tag{5.10}
\]

The experimental and theoretical investigation of the fast light particle formation in heavy ion collisions has been progressing for many years. However, there is not yet a complete understanding of the mechanism of their formation. Undoubtedly, there are several mechanisms leading to light particle emission, the contributions of which depend on masses and energies of colliding nuclei and also on the energy and emission angle of the light particle. The definite causes force us to think that the massive transfer process gives a dominant contribution to the high-energy tail of the inclusive light particle spectrum (at small energies \( E_f \leq 5 \text{ MeV}/n \)—almost to the total energy spectrum). With the increase of initial energy the processes of inelastic breakup must play an ever-growing role [24, 25, 39]. However, the detailed calculations of such processes within an adequate theoretical model, accounting for multi-step mechanisms and reproducing experimental energy and angular distributions have not been carried out till now. The approach formulated above makes it possible.

It must be admitted, that we have insufficient information at present about some quantities defining the cross section of the quasidirect transfer process, i.e., about heavy fragments spectroscopy, formfactors of their relative motion, potential and
dissipative forces. It complicates considerably the choice of the corresponding parameters when the numerical evaluations are made. However, it is such processes that may be the instrument which allows us to determine (or to refine) these fundamental quantities for nuclear physics. It is important to note, that accounting for the inelastic channels and the large mass of the transferred fragment it is possible to look over, not only the peripheral region, but also over much interesting and practically unexplored range behind the Coulomb barrier.

Regarding the parameters of the used quantities, we did not aspire at this stage to determine immediately their exact values from the fitting of experimental data. We were interested first of all in a sensitivity of the cross section to the different quantities and also in the mechanism of the massive transfer reaction at the different energies and emission angles of the light particles.

We computed the energy and angular distributions of the α-particles formed in the reaction $^{22}$Ne + $^{181}$Ta [27] (for which there are the most complete experimental data obtained in Dubna) and in the reaction $^{14}$N + $^{93}$Nb [40]. Since in these reactions one detects the light particles corresponding to the excitation energy of the residual nucleus about tens of millions of electron Volts, it is quite reasonable to use the thermodynamical equiprobable distribution of relative “one-body” motion of fragments 2 and Α over the states of nucleus B. In this case the spectroscopic factor $S_B = 1/d_0 R_{LB}(\varepsilon_B)$, where $d_0$ is the mean distance between “one-body” states. For the level density $\rho_L(\varepsilon)$ we used the routine expression [41] with the parameter $a = A/10$ MeV$^{-1}$. The normalization factors $d_0$ and $N_I(I)$ in (5.5) have an influence only upon the absolute value of the cross section which we cannot assure with total responsibility because of the uncertainty of the other parameters. Nevertheless, in order to see how much the cross section (5.9) corresponds to the experimental data we chose realistic enough values for these quantities: $d_0 = 10$ MeV and $N_I(I) = 0.1$.

Since we did not have experimental data on the elastic scattering of $^{22}$Ne + $^{181}$Ta at the required energies, we fitted the available data on the scattering of neon nuclei from $^{189}$Tb and $^{209}$Bi at $E_{lab} = 126, 164, and 198$ MeV in order to determine the averaged parameters of the real part of the optical potential. Just as expected, the satisfactory agreement with experiment was achieved, as with the deep and with the shallow potentials, giving approximately the same height of the Coulomb barrier (at $R_C^B \approx R_I + R_A + 2$ fm), but having the very different gradient $dV_{IA}/dr$ at the point $r = R_I + R_A$ (from zero up to several tens of MeV/fm). It reflects the well-known insensitivity of the heavy ion elastic scattering to the form of the potential behind the Coulomb barrier ($r < R_C^B$). Therefore, we calculated (with obtained parameters) and fixed the value of the Coulomb barrier ($V_C^B \approx 80$ MeV for $^{22}$Ne + $^{181}$Ta) and subsequently varied the value of the potential force $F_{pot} = -dV_{IA}/dr$ at the point $r = R_A + R_I$ to discover the sensitivity of the process under consideration to this quantity.

The most uncertain characteristic of the nucleus–nucleus interaction is the nuclear friction responsible for the dissipation of the kinetic energy. The experimental data on the inelastic scattering do not allow us to determine this quantity unambiguously. The values of the coefficient $f_d^0$ obtained by the authors in [42]
vary over a wide range: from \(10^{22}\) up to \(50 \times 10^{22}\) MeV·S·fm\(^{-2}\). In our case \((^{22}\text{Ne} + ^{181}\text{Ta}, E_{\text{kin}}^{\text{in}} = 178\text{ MeV})\) it corresponds to the friction force \(F_d = 2 - 100\text{ MeV/fm}\) at the barrier region. In the exit channel we put \(f_d^{\text{out}}(\text{out}) = (m_l/m_t) f_d^{\text{in}}(\text{in}).\)

Taking into account the inelastic channels in the wave function (4.1) and in the cross section (5.9) we need not the absorptive potential (more exactly, much fraction of it). The main channel leaved out of account in (4.1) is the complete fusion one, breaking the condition (4.6) at small distances. In order to simulate somehow the decrease of the particles flux due to their complete fusion we inserted into the distorted waves \(\chi_{k_l}^{i} \) and \(\chi_{k_l}^{e} \) (evaluated with the real distorting potentials) the damping factor simulating the absorptive sphere of the radius \(R_{\text{fusion}} = r_{\text{fusion}}^{\text{in}} (A_{\text{n}}^{1/3} + A_{\text{p}}^{1/3}), r_{\text{fusion}}^{\text{in}} = 1.0\text{ fm}.\) However, at those potential and dissipative forces that we used the quasidirect massive transfer process (5.1) is still localized at the distances larger than \(R_{\text{fusion}}(\text{in}, \text{out})\). Therefore, the inserting of the additional damping factor exerts a weak influence on obtaining results.

The amplitude (5.10) of the fragment 2 transfer onto the excited nucleus \(A\) has an ordinary form of the direct process amplitude and can be evaluated in a semiclassical limit that accounts for the process localization at the "transition lines" (Section 3). Therefore, those main factors, as before, define the regularities of the given reaction: the momentum distribution of the particles 1 inside the projectile \(-\mathcal{D}_{1}^{\text{in}}(Q = k_1(\Theta_1, e_1, R_{v,m})) - (m_l/m_t) k_1(e_1, R_{v,m}),\) the formfactor value at the transition point \(-U_1(e_{2A}^{\text{rel}}, R_m),\) and the dynamical factor \(\exp[-(\frac{1}{2} qA)R^2]\) defining the target's ability to receive the transferred momentum \(q = k_1(e_1, R_{v,m}) - k_1(\Theta_1, e_1, R_{v,m}).\) The distinction of the direct process is that the transferred momentum \(q,\) the point of localization \(R_m,\) and the relative motion energy \(e_{2A}^{\text{rel}} = e_{2A}^{\text{rel}} + e_{1} - e_{1}\) now depend on the value of the dissipated energy.

The light particle angular distribution is defined mainly by two factors: \(\exp[-(\frac{1}{2} qA)R^2]\) and \(\mathcal{D}_{1}^{\text{in}}(Q)\) (the momenta \(q \) and \(Q \) increase with \(\Theta_1)),\) and the energy distribution—by all three factors. The increase of the dissipated energy means that colliding nuclei penetrate deeper into each other and the transfer process occurs at smaller distances \(R_m\) (i.e., at the larger values of \(U_1(e_{2A}^{\text{rel}}, R_m)\) for subbarrier energies \(e_{2A}^{\text{rel}}\)) where the relative motion velocity decreases and so does the transferred momentum. It means that the preliminary dissipation of the kinetic energy must be especially favourable in the formation of high-energy particles (the small values of \(e_{2A}^{\text{rel}},\) i.e., deeply subbarrier stripping of the fragment 2) and also of the particles ejected in large angles \(\Theta_1\) (the large values of \(q\)).

Figure 18 shows the energy distribution of \(\alpha\)-particles formed in the reaction \(^{181}\text{Ta}(^{22}\text{Ne}, \alpha)\) and also the sensitivity of this distribution to the value of the dissipative force. One can see that the value of friction in the entrance channel has a strong influence on the absolute value of the cross section, and that too small friction and the small radius of the friction forces obviously do not agree with experiment.

Having fixed the parameters on the friction forces in accordance with the curve 2 in Fig. 18, we calculate the angular distribution of the \(\alpha\)-particles (Fig. 19). The falling
FIG. 18. Inclusive $\alpha$-particles spectrum from $^{181}\text{Ta}(^{22}\text{Ne}, \alpha)$ reaction at $E_{\text{lab}}^{\text{Ne}} = 178$ MeV [27]. The theoretical curves correspond to the different values of the friction force in entrance channel: $f_0 = 10 \times 10^{-22}$ MeV·s·fm$^{-2}$ (1), $20 \times 10^{-22}$ (2), $40 \times 10^{-22}$ (3) at $R_P = R_I + R_A + 1.5$ fm, $a_F = 0.5$ fm, $\mathcal{Q}_0 = 5$ MeV·fm$^{-1/2}$. The dashed line is for the smaller friction radius $R_P = R_I + R_A$ and $f_0 = 20 \times 10^{-22}$ MeV·s·fm$^{-2}$. The arrow indicates the maximal $\alpha$-particle energy in the case of two-body exit channel.

of the experimental cross section by a factor of $10^6$ from $\Theta_f = 10^0$ to $\Theta_f = 90^0$ obviously points to the direct character of the transfer process as just in this case there is a strong dependence of the cross section on the transferred momentum $q$. The angular distribution of the light particles was found sensitive to the value of the potential forces at the range behind the Coulomb barrier, i.e., at $r < R_C^B$. Here the deep potentials with $F_{\text{pot}}(R_I + R_A) > F_d(R_I + R_A)$ agree poorly with the experimental data.

Having also fixed the parameters of the potential forces (in accordance with curve 2 of Fig. 19) we made the calculation of the cross sections for smaller incident ion energies (Fig. 20). The satisfactory agreement obtained with the same set of parameters convinces us that the multi-step massive transfer process seems to predominate in the formation of light particles almost over the whole range of their energies for $E_{\text{lab}}^{\text{Ne}} \lesssim 5$ MeV/nucleon. In Fig. 21 the average angular momenta $\langle L_B' \rangle$ transferred in such a reaction are shown in comparison with experimental data obtained by the measurement of $\gamma$-ray multiplicity in coincidence with $\alpha$-particles at $E_{\text{lab}}^{\text{Ne}} = 155$ MeV [28]. The slight excess of the theoretical $\langle L_B' \rangle$ values over the experimental ones can be explained by the fact that highly excited states of the residual nucleus $B$ with large $L_B'$ values decay predominantly into the channel of fast fission and, thus, are lost when only the $\gamma$-rays are detected.
Fig. 19. Angular distribution of α-particles in $^{181}$Ta($^{22}$Ne, α) reaction at $E_{\text{lab}}^{\text{Ne}} = 178$ MeV and $E_\alpha = 60$ MeV [27]. $F_{\text{pol}}(R_1 + R_2) = -20$ MeV/fm (1), $-5$ MeV/fm (2), $+4$ MeV/fm (3). In the inset the corresponding potentials are shown.

Fig. 20. The same as Fig. 18, but at the different incident energies. The calculations were made with parameters corresponded to the curves (2) in Fig. 18 and in Fig. 19.
Figure 21. Average values of the angular momentum transfer in $^{181}$Ta($^{22}$Ne, $\alpha$) reaction at $\Theta_a = 10^\circ$. The experimental data at $E_{Ne}^{lab} = 155$ MeV are from [28].

Figure 22 gives us an insight on the reaction mechanism itself and into the role of the multi-step processes. The quantity obtained after the evaluation of all the integrals in (5.9) except for the integral over the dissipated energy $\varepsilon_i$ is shown in this figure. The dissipated energy varies from zero (one-step process, $\varepsilon_{2A}^{rel} = \varepsilon_{2A}^{s} + \varepsilon_{2A}^s$) up to $\varepsilon_{2A}^f$ (stripping of fragment 2 onto the excited nucleus $A$ with $\varepsilon_{2A}^{rel} = \varepsilon_{2A}^{s} + \varepsilon_{2A}^s$). As can be

Figure 22. Contribution of the processes with the different kinetic energy dissipation in entrance channel to the $\alpha$-particle formation at $E_{Ne}^{lab} = 178$ MeV, $L_B^f = 30$. (1) $\Theta_a = 20^\circ$, $E_a = 60$ MeV; (2) $\Theta_a = 20^\circ$, $E_a = 100$ MeV; (3) $\Theta_a = 90^\circ$, $E_a = 60$ MeV. The arrows indicate the maximal allowable energies $\varepsilon_i(E_a, \Theta_a)$. 

\[ \langle L_B^f \rangle \]
seen, at relatively low energies $E_\alpha$ (the large values of $\varepsilon_{2A}^f$ and $\varepsilon_{2A}^{rel}$) the $\alpha$-particles are ejected predominantly at the stage when not all the kinetic energy available for the given $E_\alpha$ has been dissipated. "Disadvantageousness" of further dissipation is due to the fact that the relative motion energy $\varepsilon_{2A}^{rel} = \varepsilon_{2A}^f + \varepsilon_{B}^i - \varepsilon_i - \varepsilon_f$ falls into the deep subbarrier range and the formfactor $U_A(\varepsilon_{2A}^{rel}, R_m)$ begins to decrease. With the increase of $E_\alpha$ and for the large outgoing angles $\Theta_f$ the mechanism dominates, in which the maximal (available for given $E_\alpha$) preliminary dissipation of kinetic energy occurs; here the local transferred momentum decreases and the formfactor $U_A(R_m)$ increases due to the deeper penetration.

Figure 23 demonstrates the role of the energy dissipation in the exit channel. One can see, that energy losses in the exit channel are much smaller than in the entrance one. The outgoing into the forward angle $\alpha$-particle excites the residual nucleus $B$ several million electron volts more; here for the smaller angular momentum transfer such additional excitation is more considerable. With the increase of the outgoing angle the role of the $\alpha$-particle inelastic interaction with the residual nucleus sharply decreases and it is quite clear. Notice, that qualitative behaviour of the curves shown in Figs. 22 and 23 is not changed by the variation of the potential and the dissipative forces over the range that is in Figs. 18 and 19. This means that the conclusions made above about the reaction mechanism are reliable enough.

We also calculated the cross section of $\alpha$-particle formation for higher energy

\[ \frac{d^3\sigma}{d\Omega dE_\alpha} \]

\[ \frac{d^3\sigma}{d\Omega dE_\alpha}(18^1\text{Ta}^{(22Ne, \alpha)}_{\text{lab}}, E_{\text{Ne}} = 178 \text{ MeV}, E_{\alpha}^{\text{lab}} = 60 \text{ MeV}) \]

\[ L_B^f = 50, \Theta_\alpha = 20^\circ \]

\[ L_B^f = 30, \Theta_\alpha = 20^\circ \]

\[ L_B^f = 30, \Theta_\alpha = 60^\circ \]

\[ \varepsilon_f (\text{MeV}) \]

FIG. 23. Role of the energy dissipation in exit channel. The cross section $d^3\sigma/d\Omega dE_\alpha$ is obtained as the area under the corresponding curve.
\[ E_{\text{lab}}^{\text{lab}} = 360 \text{ MeV} \ (E_{\text{cm}}^{\text{cm}} - V_C^{B} \approx 12 \text{ MeV/n}) \] (Fig. 24a), and for the reaction \(^{93}\text{Nb}^{(^{14}\text{N}, \alpha)}\) at \( E_N^{\text{lab}} = 208 \text{ MeV} \ (E_{\text{cm}}^{\text{cm}} - V_C^{B} \approx 10 \text{ MeV/n}) \) (Fig. 24b). Unfortunately, there are not experimental data for the extreme high-energy tail of the \(\alpha\)-spectra in both reactions (in the first case \(E_x^{\text{max}} \approx 300 \text{ MeV}\); in the second one \(E_x^{\text{max}} \approx 200 \text{ MeV}\) for \(\theta_f = 0^0\)). It limits to some extent the possibility of the theoretical study of these data.

As can be seen from the comparison of the theoretical results with the experimental ones, the quasidirect massive transfer process makes, as before, the dominant contribution to light particle formation at the high-energy tail of their spectrum and especially for \(\theta_f > \theta_{gr}\). However, at the forward angles \((\theta_f < \theta_{gr})\) the mechanism of massive transfer is obviously insufficient to explain the yield of light particles.

![Diagram](image-url)

Fig. 24. The \(\alpha\)-particle energy spectra in \(^{181}\text{Ta}^{(^{20}\text{Ne}, \alpha)}\) reaction at \(E_{\text{Ne}}^{\text{lab}} = 360 \text{ MeV}\) [44]—(a), and in \(^{93}\text{Nb}^{(^{14}\text{N}, \alpha)}\) reaction at \(E_N^{\text{lab}} = 208 \text{ MeV}\) [40]—(b). The calculations were made with the same parameters as in Fig. 20, but with the normalization factor \(N_\psi(I) = 0.8\).
HEAVY ION COLLISIONS

with velocities \( v_x \leq v_{\text{beam}} \). It means that the break-up process contributes significantly in this region. The absence of the pronounced bump at the beam velocity indicates that this break-up process is also a multi-step one with the preliminary dissipation of kinetic energy into the internal excitation of both the target and the projectile.

For these higher incident energies we were able to reproduce the absolute values of the cross sections only by increasing the normalization factor \( N_1(I) \) (with the fixed other parameters). It may be explained qualitatively by the fact that for higher excitation energy the summation is made over a larger number of the intrinsic states of fragments 1 and 2 and the \( N_1(I) \) has to increase, approaching its ultimate value.

6. SUMMARY

We propose an approach that makes it possible in a sufficiently simple manner to describe both direct and multi-step processes of heavy ion collisions including the ones with large mass, energy, and angular momentum transfer, i.e., essentially non-quasielastic reactions. This approach is based on the semiclassical conceptions of the heavy ions relative motion description justified by the smallness of the wavelength and by the high-level density of the excited states.

The semiclassical approximation is made immediately for the three-dimensional wave functions without expansion of them over partial waves. We omit the complex trajectories corresponding to the waves leaking under the barrier and reflected at the inner turning points (the contribution of such trajectories becomes less and less with the wavelength decrease and with the increase of absorption in the elastic channel, i.e., with increase of nuclei masses), but we take into account the real caustic surfaces when constructing the distorted waves. The behaviour of the wave function was studied and considerable irregularities of it were found near the ordinary caustic and in the region of the caustic cusp at the sub- and near-barrier energies. The semiclassical limit of the Coulomb wave function valid over the whole space has been obtained.

The angular momentum conservation low manifesting itself in the stationary phase condition leads to the spatial localization of the direct processes at the so-called "transition lines." The transition amplitude is reduced to the one-dimensional integral along these lines, and the whole process (including the interference phenomena) acquires the clarity that is usually concealed in multi-dimensional integration or in partial expansion.

Comparison of the evaluated cross sections of some direct processes with the experimental data and with exact DWBA-calculations has been made and it was shown that the proposed approach describes quite satisfactorily the direct collisions of even not very heavy ions. The new interpretation was made for the bell-shaped angular distributions usually observed in quasielastic reactions.

The semiclassical limit of the coupled channels method is used for description of
the multi-step process. Destruction of the channel coherence makes it possible to simulate the channel coupling by friction forces, that not only substitute for the imaginary part of the optical potential defining the exhaustion of the flux in the elastic channel, but allow us to keep watching the distribution of this flux over other channels. It substantially reduces the uncertainty in evaluating of the cross section's absolute value. One suggests looking at the problem of the quantal friction from the novel point of view.

This approach was used for the description of the multi-step (quasidirect) massive transfer processes of light particle formation in heavy ion collisions. The experimental angular and energy distributions of these particles are analyzed. The sensitivity of the differential cross sections of such processes to the values of dissipative and potential forces at the relatively small distances behind the Coulomb barrier was found. It means that quasidirect reactions are very informative for obtaining the dynamical characteristics of nucleus–nucleus interaction.

The approach we proposed can be very useful for understanding and describing the deep inelastic processes from the side of direct reactions.

ACKNOWLEDGMENTS

The author is deeply indebted to V. E. Bunakov, whose teamwork stimulated to a certain extent development of the quite simple semiclassical approach to the description of heavy ion collisions. I am grateful also to Yu. N. Demkov, G. V. Dubrovskiy, F. A. Gareev, S. G. Kadmenskiy, W. Nörenberg, and B. N. Zakhariev for valuable discussions of various aspects and for clearing up some points in the course of the working out and applying the approach stated above.

REFERENCES

5. V. M. Strutinskiy, JETP (USSR) 46 (1964), 1063.
Amsterdam, 1979.
North-Holland, Amsterdam, 1979; W. VON OERTZEN, in “Frontiers in Nuclear Dynamics” (R. A. 
20. P. D. BOND, in “Semiclassical Descriptions of Atomic and Nuclear Collisions” (J. Bang and 
33. G. V. DUBROVSKII AND L. F. VUUNENKO, JETP (USSR) 80 (1981), 66; A. V. BOGDANOV AND 
38. S. G. KADMENSKII AND V. I. FURMAN, “Alpha-Decay and Related Nuclear Reactions,” 
Energoatomizdat, Moscow, 1985.
337; S. PAL AND D. M. E. GROSSE, Z. Phys. A 329 (1988), 349; see also the works of W. Nörenberg 
and collaborators in Ref. [2].
press.