Near-barrier neutron transfer in reactions $^3$He+$^{197}$Au

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Experimental excitation functions for near-barrier neutron transfer in $^3$He+$^{197}$Au reactions have been measured and analyzed. Time-dependent Schrödinger equation and coupled channel equations for external neutrons of $^3$He and $^{197}$Au nuclei have been solved numerically taking into account spin-orbit interaction and Pauli exclusion principle.

Keywords: Nuclear reactions; Neutron transfer; Time-dependent Schrödinger equation; Coupled channel equations.

1. Introduction

Low-energy reactions with light He nuclides ($^3$He and halo nuclei $^6$He) attract interest due to the new possibilities of investigating the nuclear structure of light He projectiles and heavy target nuclei, e.g. $^{197}$Au [1]. We may explore target nuclei by weakly bound neutrons of $^4$He and the more strongly bound neutron of $^3$He in reactions of neutron transfer from the projectile to the target as well as study the properties of external neutrons of $^{197}$Au in the pick-up reaction $^3$He+$^{197}$Au [2, 3]. We may also test some theoretical models with both simple and complicated approximations by comparing them with experimental data.

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2. Experiments

The experiments were performed with the extracted radioactive beams of $^6$He obtained at the DRIBs accelerator complex, JINR, Dubna [2]. The beams of the $^3$He nuclei were accelerated by the U-120M cyclotron of the Nuclear Physics Institute, the Academy of Sciences of the Czech Republic [3]. The target assemblies consisting of gold foils of different thicknesses were installed in the reaction chamber and irradiated by the beam of particles. For the reaction with $^6$He the gold target assemblies were installed in the focal plane of the magnetic spectrometer MSP-144, where it was possible to obtain the lower energy beam with a good resolution. The maximum energy of the accelerated $^6$He ions was ~10 MeV/nucleon, the intensity reached $(2\text{−}5)\cdot10^7$ s$^{-1}$. The maximum energy of $^3$He was 24.5 MeV, the beam intensity varied in the range $(5\text{−}10)\cdot10^{11}$ s$^{-1}$. After each irradiation session induced $\gamma$-activity was measured in all Au targets. The measurements were carried out using HPGe-detectors with the energy resolution of ~1.8 keV for the gamma quantum energy of 1.3 MeV. The identification of the $^{196,198}$Au isotopes was performed by their gamma-radiation energies and half-lives. The measurements of the yields were performed taking into account absolute intensities of gamma-transitions and detector efficiency. The details of the experiment, data processing and results are described in [2, 3]. The experimental excitation functions for the formation of $^{196,198}$Au isotopes in the reactions of $^3$He and $^6$He with $^{197}$Au were analyzed and interpreted on the basis of the time-dependent Schrödinger equation and the coupled channel method.

3. Time-dependent model for neutron transfer

The probabilities of neutron transfer and transfer cross sections in the reactions $^3,^6$He+$^{197}$Au were calculated on the basis of the numerical solution of the time-dependent Schrödinger equation [4].

3.1. Neutron states in $^{3,6}$He

Information about the structure of loosely bound nuclei may be obtained by measuring their momentum distributions after breakup in a nuclear reaction. From the momentum distribution for the $^6$He nucleus it follows that it consists of the $^4$He core and the two-neutron cluster, see e.g. [1, 5]. There are several different approaches to the approximate analysis and solution of the three-body problem [5, 6]. Within the shell model the $^6$He nucleus may be described as the inert core plus neutrons. In Ref. [6] the three-body bound state problem for borromean nuclei was solved on the basis of the hyperspherical harmonics.
expansion as well as the coordinate space Faddeev approach. The obtained radial density for the $^6$He valence neutron normalized to unity (see Eq. (1)) is shown in Fig. 1.

$$\int_0^\infty \rho_1(r)r^2 dr = 1.$$ (1)

To determine the wave function of the ground state of the $^4$He three-body system the Schrödinger equation may also be solved by Feynman’s continual integrals method [7–9]. Within the simple approximation we use the shell model with the experimental value of the neutron separation energy 1.86 MeV and the root-mean-square charge radius 2.065 fm (the values were taken from [10]). The result of calculation of the neutron spatial probability distribution $|R(r)|^2$ with the radial wave function $R(r)$ is similar to the distribution $\rho_1(r)$ for the three-body bound state [6], Fig. 1a. As can be seen, all three methods provide similar results in the central part of the distribution. The neutron states in the $^3$He nucleus were calculated within the shell model taking into account the properties of the ground state of the system of three particles (p+p+n) found by Feynman’s continual integrals method, Fig. 1b. In this case the two methods also provide similar results in the central part of the distribution.

![Fig. 1. The normalized to unity radial density of the valence neutron in the $^4$He nucleus $\rho_1(r)$ for the three-body bound state [6] (dash-dotted line), within the shell model (dashed line) and Feynman’s continual integrals method (solid line) (a); $^3$He neutron radial density within Feynman’s continual integrals method (solid line) and within the shell model for $1s$ state (dashed line) (b).](image)

3.2. Time-dependent Schrödinger equation (TDSE) and neutron wave functions

For theoretical description of neutron transfer during collisions of heavy atomic nuclei several semi-classical models are used [4, 11]. They combine quantum
description of internal one-particle and collective degrees of freedom with
classical equations of motion of atomic nuclei, Eq. 2.

\[
m_{12} \ddot{\mathbf{r}}_1 = -\nabla_{\mathbf{r}_1} V_{12}(\mathbf{r}_1 - \mathbf{r}_2), \quad m_{22} \ddot{\mathbf{r}}_2 = -\nabla_{\mathbf{r}_2} V_{12}(\mathbf{r}_2 - \mathbf{r}_1). \tag{2}
\]

Here \(\mathbf{r}_1(t), \mathbf{r}_2(t)\) are the centers of nuclei with the masses \(m_1, m_2\) and \(V_{12}(r)\) is
the potential energy of nuclear interaction. We may assume that before contact
of the surfaces of spherical nuclei with the radii \(R_1, R_2\) the potential energy of a
neutron \(W(\mathbf{r}, t)\) is equal to the sum of its Woods-Saxon interaction energies
with both nuclei.

The evolution of the components \(\Psi_1, \Psi_2\) of the spinor wave function
\(\Psi(\mathbf{r}, t)\) for the neutron with the mass \(m\) during the collision of nuclei is
determined by the Eq. (3) with the operator of the spin-orbit interaction
[4, 12, 13].

\[
\frac{i\hbar}{\partial t} \Psi(\mathbf{r}, t) = \left\{ -\frac{\hbar^2}{2m} \Delta + W(\mathbf{r}, t) + \hat{V}_{12}(\mathbf{r}, t) \right\} \Psi(\mathbf{r}, t). \tag{3}
\]

In case of neutron transfer from \(^6\text{He}\) the neutron occupies free levels of the
\(^{197}\text{Au}\) nucleus. The evolution of the probability density for the external \(1\text{p}^{3/2}\)
shell neutrons of \(^6\text{He}\) was studied in Ref. [4]. In case of pick-up to \(^3\text{He}\) the
neutron occupies the partially free \(1\text{s}\) level of the \(^3\text{He}\) nucleus. The neutron
separation energies of \(^3\text{He}\) and \(^{197}\text{Au}\) are similar. In case of pick-up to \(^{197}\text{Au}\) the
neutron occupies free levels only. The Pauli exclusion principle limits transfers
to the unoccupied states. This principle was taken into account by two different
approximations providing very similar results. Within the first, simple,
approximation transfer to the occupied levels in the “frozen” shell structures of
colliding nuclei is excluded. Within the second, more complicated,
approximation the time-dependent few-body Slater determinant wave function is
used, Eq. 4.

\[
\Phi_N(\mathbf{r}_1, \ldots, \mathbf{r}_N, t) = \frac{1}{\sqrt{N!}} \det \begin{bmatrix}
\Psi_1(\mathbf{r}_1, t) & \ldots & \Psi_1(\mathbf{r}_N, t) \\
\ldots & \ldots & \ldots \\
\Psi_N(\mathbf{r}_1, t) & \ldots & \Psi_N(\mathbf{r}_N, t)
\end{bmatrix}. \tag{4}
\]

For \(N=3\) the following states were used: \(1\text{s}\) for the \(^3\text{He}\) nucleus and \(3\text{p}_{3/2}\) for
the highest occupied level of the \(^{197}\text{Au}\) nucleus with the angular momentum projections \(\Omega=1/2\) and \(\Omega=3/2\). This model is similar to the Hartree-Fock method
[14] and the time-dependent Hartree-Fock approximation (TDHF, e.g. [15]), but
it is more simple.

The evolution of the probability density during the grazing collision of
\(^3\text{He}^{+}\text{Au}\) is shown in Fig. 2.
Fig. 2. The evolution of the probability density for the external neutrons of $^3$He and $^{197}$Au nuclei during the grazing collision with the energy $E_{c.m.} = 25$ MeV. The course of time corresponds to the panel locations a, b, c, d.

The most important aspects of neutron transfer may be understood in head-on nuclear collisions. In this case the projection of the full angular momentum of the neutron on the internuclear axis is a quantum number, $\Omega = 1/2, 3/2$. The two-center neutron levels in the $^6$He+$^{197}$Au systems are shown in Fig. 3. Calculations within the two-center shell model were performed by the method based on the Bessel series [16]. Transfer occurs from the occupied neutron levels in $^3$He to the unoccupied neutron levels in $^{197}$Au. Partial populations of the neutron two-center states for the transfer from $^6$He nuclei during head-on collisions equal $|\psi_\nu|^2$ (see Fig. 4). Here $a_\nu$ are the coefficients of the series expansion of the time-dependent valence neutron wave function in Eq. (5).

$$\Psi_\Omega(r,t) = \sum_\nu a_{\nu,\Omega}(R(t)) \phi_{\nu,\Omega}(r,R(t)).$$  (5)
Two-center neutron levels in $^{6}$He+$^{197}$Au (a) and $^{3}$He+$^{197}$Au (b) systems. $R$ is the internuclear distance, $R_1$, $R_2$ are the radii of nuclei, $R_B$ is the barrier position. The projection of the full angular momentum of the neutron on the internuclear axis is $\Omega=1/2$. Dashed lines are $1p_{1/2}$ (a) and $1s$ (b) levels of $^3$He and $^6$He, respectively, at $R \to \infty$. Solid lines are unoccupied levels of the $^{197}$Au nucleus at $R \to \infty$. Dash-dotted line is the highest occupied level of the $^{197}$Au nucleus.

The partial populations of the neutron two-center states for the transfer from the $^6$He nuclei during head-on collisions $^6$He+$^{197}$Au, $E_{c.m.}=18.5$ MeV $< V_B$ (a) and $^3$He+$^{197}$Au, $E_{c.m.}=20$ MeV $< V_B$ (b). $R$ is the distance between the centers of nuclei. The projection of the full angular momentum of the neutron on the internuclear axis is $\Omega=1/2$.

After the transfer from the $^3$He nucleus (see Figs. 2, 4b) the neutron may occupy several initially vacant levels ($2f_{5/2}$, $3p_{1/2}$, $2g_{9/2}$) in the $^{197}$Au nucleus. These levels lie near the Fermi level with the energy approximately equal to the energy of the neutron in $^3$He (about $-8$ MeV, taken from [10]). After the transfer from the $^6$He nucleus neutrons may occupy several initially vacant top levels ($3d_{5/2}$, $3d_{3/2}$, $4s$) in the $^{197}$Au nucleus. The energy of these levels is approximately equal to the energy of the external neutron in $^6$He (about $-2$ MeV, taken from [10]).
3.3. Calculations of transfer cross section

The total cross section of neutron transfer in the time-dependent model is calculated by Eq. (6).

\[ \sigma = 2\pi \int_{b_0}^{\infty} w(b)bdb. \]  \hspace{1cm} (6)

Here \( b \) is the impact parameter, \( b_0 \) is the minimum collision impact parameter which is equal to zero for the energy less than the Coulomb barrier. The function \( w(b) \) is the probability of neutron transfer during the collisions of nuclei without contact between their surfaces. It was determined by integrating the probability density over the volume after the collision (Fig. 2d).

The comparison of the experimental data and theoretical calculations for the reactions \(^{197}\text{Au}(^{3}\text{He},^{4}\text{He})^{196}\text{Au}\) and \(^{197}\text{Au}(^{3}\text{He},2\text{p})^{198}\text{Au}\) is shown in Fig. 5a and Fig. 5b, respectively. Symbols are the data from Refs [3, 17]. For neutron pick-up the energy of the neutron in \(^{197}\text{Au}\) was chosen to coincide with the experimental neutron separation energy. As can be seen, in the reaction \(^{3}\text{He}+^{197}\text{Au}\) neutrons are predominantly transferred from \(^{3}\text{He}\) to \(^{197}\text{Au}\).

![Fig. 5. a) The excitation function for the reaction \(^{197}\text{Au}(^{3}\text{He},^{4}\text{He})^{196}\text{Au}\). b) The excitation function for the reaction \(^{197}\text{Au}(^{3}\text{He},2\text{p})^{198}\text{Au}\). Symbols are the experimental data from Ref. [3] (filled squares) and Ref. [17] (empty squares), dash-dotted and dashed curves are the results of the TDSE calculations with two different approximations for the probability of neutron transfer \(w(b)\). Solid lines are the results of combining CC and TDSE methods (see the next section).](image)

Dash-dotted and dashed curves in the above barrier region are the results of two different approximations for the probability of neutron transfer \(w(b)\). In the first, classical, approach the transfer probability was assumed to be zero for the minimum distances less than the barrier distance (dashed lines). In the second approach taking into account some quantum effects the transfer probability was...
assumed to be constant and equal to the transfer probability at the barrier (solid lines). The "true" values of the transfer probability and the cross section are expected to lie somewhere between the values obtained within these two approaches.

The model does not take into account the energy change associated with the \( Q \)-value of the neutron transfer. In case of \( Q > 0 \) it may lead to closer distances between nuclei and, thus, higher transfer probability resulting in higher transfer cross section. In the opposite case, \( Q < 0 \), transfer probability is lower resulting in lower transfer cross section. This also implies the change of the structure of the projectile during the neutron transfer.

The effect of the \( Q \)-value and the quantum effects related to the motion of cores, e.g. tunneling, may be taken into account in the coupled channel (CC) approach combined with the time-dependent Schrödinger equation method (TDSE). The results of combining CC and TDSE methods (solid lines, Fig. 5) demonstrate overall satisfactory agreement with the experimental data.

4. Combination of CC and TDSE methods for neutron transfer

Coupled channel equations (7) based on the perturbed stationary states method [18] were proposed for neutron transfer channels in [13, 19].

\[
y_{L,s,\Omega}^* - \frac{L(L+1)}{R^2} y_{L,s,\Omega} + \frac{2\mu}{\hbar^2} \left[ E_v(R) - V_{1s}(R) \right] y_{L,s,\Omega} = f_i \sum_{\beta} T_{\beta\nu}(R) y_{L,\beta,\Omega}. \tag{7}
\]

Here \( L \) is the orbital momentum of the partial wave, \( \mu \) is the reduced mass, \( E_v = E_{c.m.} + Q(R) \), \( E_{c.m.} = \hbar^2 k_0^2 / 2\mu \), \( Q(R) = e_0 - e_\nu(R) \) is the distance-dependent \( Q \)-value, \( e_0 = e_\nu(\infty) \) is the energy of the initial neutron state \( \alpha \) in a distant nucleus, \( e_\nu(R) \) is the two-center (molecular) energy level (see Fig. 3), \( f_i = F_i(E) \theta(R_{cut} - R) \), \( \theta(x) \) is the Heaviside step function. The reduced kinetic energy coupling matrix \( T_{\beta\nu}(R) \) is determined by Eq. (8).

\[
T_{\beta\nu}(R) = \int \frac{\partial}{\partial R} \phi_\nu^*(r, R) \frac{\partial}{\partial R} \phi_\beta(r, R) dr'. \tag{8}
\]

The coupling matrix elements \( T_{\beta\nu}(R) = T_{\beta\nu}(R) \) are determined by the two-center wave functions \( \phi_{\nu}(r, R) \) of a valence neutron. They were calculated within the two-center shell model based on the Bessel series [16]. In the simplest approximation the coupling matrix elements have nonzero values only for the states \( \nu, \beta \) corresponding to different nuclei in the limit \( R \to \infty \). The program CCFULL [20] includes pair-transfer coupling between the ground states only. It
uses the macroscopic coupling form factor given by \[21, 22\], and the value of the coupling strength \( F_c \) is adjusted manually, which makes its choice quite unobvious. In contrast, formula (8) takes into account complete shell structure of colliding nuclei. In this work to determine the value of the coupling strength \( F_c(R_{cut}) \) the solution of the time-dependent Schrödinger equation (Figs. 2, 4) was used. When nuclei move away from each other and \( |a_{c\Omega}(R)| \) stops changing \( |a_{c\Omega}(R)| \approx |a_{c\Omega}(R(\rightarrow \infty))|, \quad R > R_{cut} \) as well as for approaching nuclei when \( |a_{c\Omega}(R_{cut})| \ll |a_{c\Omega}(R(\rightarrow \infty))|, \) the factor \( f_c = 0, \quad R > R_{cut}. \) For \(^3\)He \( R_{cut} = 14 \text{ fm} \) was used. (Fig. 4). The assumptions \( F_c(E) = \max \{1; 0.43 + 0.039E_1\} \) and \( F_c(E) = \max \{0.5; -0.9 + 0.73E_1\} \) \( (E \text{ in MeV}) \) were used for \(^{197}\)Au\(^{(3}\)He,\(^{4}\)He)\(^{196}\)Au (Fig. 5a) and \(^{197}\)Au\(^{(3}\)He,\(^{2}\)p)\(^{198}\)Au (Fig. 5b) reactions, respectively. For \(^{6}\)He \( F_c \approx 10 \) was used. Near the Coulomb barrier the similarity between the values of the channel functions \( |y_{L,\Omega}^c| \) and the coefficients \( |a_{c\Omega}(R)|^2 \) (Fig. 4) of the series expansion is observed.

The transfer cross section \( \sigma_t \) and the fusion cross section \( \sigma_{ fus } \) are calculated by the Eqs. (9)- (12).

\[
\sigma_t = \frac{\pi \hbar^2}{2 \mu E_{cm} j_0} \sum_{L=0}^{\infty} (2L+1) \sum_{\Omega=\pm 1} |j_{L,\Omega}|^2.
\]

\[
j_{L,\Omega} = -\frac{i}{2 \mu} (y_{L,\Omega} \frac{dy_{L,\Omega}}{dR} - y^*_{L,\Omega} \frac{dy^*_{L,\Omega}}{dR}) |_{R \to \infty}.
\]

\[
\sigma_{ fus } = \frac{\pi \hbar^2}{2 \mu E_{cm} j_0} \sum_{L=0}^{\infty} (2L+1) \sum_{\Omega=\pm 1} |\tilde{j}_{L,\Omega}|^2
\]

\[
\tilde{j}_{L,\Omega} = -\frac{i}{2 \mu} (y_{L,\Omega} \frac{dy_{L,\Omega}}{dR} - y^*_{L,\Omega} \frac{dy^*_{L,\Omega}}{dR}) |_{R \to \infty}.
\]

Here \( j_0 = h \kappa_0 / \mu \) is the flux incoming from the infinity. \( j_{L,\Omega} \) and \( \tilde{j}_{L,\Omega} \) are the fluxes outgoing to the infinity and entering the fusion region in the channel \( v, \Omega. \)

As expected, the results of combining CC and TDSE methods (Fig. 5, solid lines) demonstrate much better agreement with the experimental data.

The results of theoretical calculations for the formation of the \(^{198}\)Au isotope shown in Fig. 6 demonstrate satisfactory agreement with the experimental data near the barrier \( V_B \approx 20 \text{ MeV}. \) Results of theoretical calculations for fusion in the reactions \(^{6}\)He+\(^{197}\)Au are shown in Fig. 7. The enhancement of the fusion
cross section for the reaction $^6$He+$^{197}$Au in comparison with the reaction $^4$He+$^{197}$Au may be explained by the decrease of the two-center energy level $1p_{3/2}$ ($^6$He) near the barrier (≈1 MeV for each neutron, which gives totally ≈2 MeV, Fig. 3a) plus the barrier difference ≈0.5 MeV.

Fig. 6. The excitation function for the formation of the $^{198}$Au isotope in the reaction $^6$He+$^{197}$Au. Experimental data (circles) is from Refs. [2, 23]. Theoretical curves were calculated within the CC approach (solid line, this work) and the TDSE method (dashed line, Ref. [4]).

Fig. 7. The fusion excitation function for the reactions $^4$He+$^{197}$Au (empty circles, solid line) and $^6$He+$^{197}$Au (filled symbols, dashed line). Experimental data for $^4$He is from Refs. [2, 23]. Experimental data for $^6$He is from Refs. [17, 24–26]. Theoretical curves were calculated within the CC method.
5. Conclusion

The results of calculation within the coupled channel approach and the time-dependent Schrödinger equation method demonstrate overall satisfactory agreement with experimental data. These methods may also be applied for calculation of reactions with cluster nuclei.

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References

10. Nuclear Reactions Video (Low-Energy Nuclear Knowledge Base),
    http://nrv.jinr.ru/nrv
26. Experimental Nuclear Reaction Data (EXFOR),