Sub-barrier fusion of weakly bound nuclei: Strong enhancement due to sequential fusion mechanism

- History
- Role of neutrons in fusion reactions
- Sequential fusion mechanism
- Huge enhancement in fusion of weakly bound nuclei
- Summary
History

Theory:

• Takigawa and Sagawa, Phys.Lett. 1991: $^{11}$Li$^{+}$208Pb, “halo, soft dipole mode” - enhancement
• Hussein et al, Phys.Rev. 1992: $^{11}$Li$^{+}$208Pb, “break-up” – suppression
• Dasso and Vitturi, Phys.Rev. 1994: $^{11}$Li$^{+}$208Pb, “both” – enhancement
• Nakatsukasa et al., Fusion-2006: fusion suppression for neutron-halo nuclei

Experiment:

• Fomichev et al., Z.Phys. 1995: $^{6}$He$^{+}$209Bi, enhancement ?
• Kolata et al., Phys.Rev. Lett. 1998: $^{6}$He$^{+}$209Bi, enhancement !
• Trotta, Sida, Alamanos et al., Phys.Rev.Lett. 2000: $^{6}$He$^{+}$238U, enhancement !
• Raabe et al., Nature 2004: $^{6}$He$^{+}$238U, no fusion enhancement, only 2n-transfer
• Di Pietro et al., Phys. Rev. 2004: $^{6}$He$^{+}$64Zn, no enhancement
Role of internal degrees of freedom

Distance between nuclear surfaces and, thus, the **Coulomb barrier** depend on vibrations and rotation: 
\[
B = B(\beta_2, \beta_3), \quad B = B(\theta, \varphi) - \text{ multidimensional barriers.}
\]

Instead of one fixed barrier \( B \) we have a "**barrier distribution function**" \( f(B) \).

Barrier penetrability is
\[
T(E, \ell) = \int f(B) P_0 (B; E, \ell) dB.
\]

\( \rightarrow \) sub-barrier fusion enhancement

**What is a role of neutrons in fusion process?**
Role of neutrons in sub-barrier fusion?

H. Stelson, 1988

no barrier for neutron transfer, neck formation

\[ V(MeV) \]
\[ R (\text{fm}) \]

\[ \sigma(E) = \pi R^2 \left( \frac{(E-T)^2}{4(B-T)E} \right) \]

N. Rowley, I.J. Thompson, and M.A. Nagarajan, PL, 1992
CC simulations:
→ \( Q_n < 0 \) - broad barrier distribution, necking
→ \( Q_n > 0 \) - “anti-necking” conditions

Ning Wang, Xizhen Wu, and Zhuxia Li, PRC, 2003
QMD calculations:
→ neutron excess plays a dominant role

More neutrons = more probable fusion

\[ \frac{\sigma_{\text{fus}}(^{48}\text{Ca} + ^{40}\text{Ca})}{\sigma_{\text{fus}}(^{40}\text{Ca} + ^{96}\text{Zr})} \mid E < B \quad \gg 1 \]

Wrong!
Neutron excess itself does not help nuclei to fuse!
Lack of theory for fusion of some nuclei

experiment: Timmers et al., NP, 1998
Wave functions of valence neutrons follow the two-center quasi-molecular states and spread over the volumes of both nuclei rather fast, before nuclei come in contact and even **before** they overcome the Coulomb barrier!
Gain due to positive $Q$-value of neutron rearrangement

In most combinations neutrons may be transferred only with negative $Q$-values.

Nevertheless, there are combinations with a chance for positive $Q$-value neutron transfer!

This gain in energy may go into the relative motion energy!

\[ E_{\text{c.m.}} + M(A) + M(B) = E'_{\text{c.m.}} + M(A-1) + M(B+1) = \text{const} \]

If \([M(A-1)+M(B+1)] < [M(A)+M(B)]\), then \(E'_{\text{c.m.}} > E_{\text{c.m.}}\).
Sequential fusion process

\[ \sigma_{\text{fusion}}(E) = \frac{\pi}{k^2} \sum (2\ell + 1) \cdot T(\ell, E) \]

\[ T(\ell, E) = \int_{0}^{\infty} f(B) \, P_{0}(B; \ell, E) \, dB \]

Subsequent neutron transfer and "sequential fusion":

- \( A + B \rightarrow (A-1) + (B+1) + Q \)
- \( E_{\text{cm}} \rightarrow E_{\text{cm}} + Q \)
- \( (A-2) + (B+2) + Q \)
- \( \ldots \)

\[ T(\ell, E) = \int_{0}^{\infty} f(B) \int_{-E}^{Q_{0}(k)} \frac{1}{N} \left[ \delta(Q) + \sum_{k \geq 1} \alpha_{k}(E; \ell, Q) \right] P_{0}(B; \ell, E+Q) \, dQ \, dB \]

- \( Q_{0}(k) \) is the Q-value of g.s. transfer
- Probability for transfer of \( k \) neutrons: \( \alpha_{k}(E; \ell, Q) = N_{k} e^{-C[Q-Q_{\text{opt}}]^{2}} e^{-2\gamma[\text{D}(E, \ell) - D_{0}]} \)
- \( D(E, \ell) \) is the distance of closest approach, and \( D_{0} = d_{0}(A^{1/3} + B^{1/3}), \quad d_{0} \approx 1.4 \text{ fm} \)
- \( \gamma = \gamma(\varepsilon_{1}) + \gamma(\varepsilon_{2}) + \ldots + \gamma(\varepsilon_{k}), \quad \gamma(\varepsilon) = \sqrt{2\mu_{\text{e}}/\hbar^{2}} \)
Fusion enhancement for stable neutron rich nuclei

Idea of “sequential fusion” mechanism with intermediate neutron rearrangement seems to work!
Test for the “neutron rearrangement” mechanism in sub-barrier fusion reactions

Stable nuclei:

\[ ^{40}\text{Ca} + ^{124}\text{Sn} \rightarrow < ^{44}\text{Ca} + ^{120}\text{Sn} + 9.5 \text{ MeV} > \rightarrow ^{164}\text{Yb} \]
\[ ^{48}\text{Ca} + ^{116}\text{Sn} \rightarrow < \text{all } Q^{\text{trans}}_n < 0 > \rightarrow ^{164}\text{Yb} \]

\[ ^{14}\text{C} + ^{40}\text{Ca} \rightarrow < ^{12}\text{C} + ^{42}\text{Ca} + 6.7 \text{ MeV} > \rightarrow ^{54}\text{Fe} \]
\[ ^{12}\text{C} + ^{42}\text{Ca} \rightarrow < \text{all } Q^{\text{trans}}_n < 0 > \rightarrow ^{54}\text{Fe} \]

Weakly bound nuclei:

\[ ^{6}\text{He} + ^{206}\text{Pb} \rightarrow < ^{4}\text{He} + ^{208}\text{Pb} + 13.1 \text{ MeV} > \rightarrow ^{212}\text{Po} \]
\[ ^{4}\text{He} + ^{208}\text{Pb} \rightarrow < \text{all } Q^{\text{trans}}_n < 0 > \rightarrow ^{212}\text{Po} \]
$^6\text{He} + ^{206}\text{Pb}$
Time-dependent analysis of $^6$He + $^{206}$Pb collision

Wave functions of valence neutrons follow the two-center molecular states and spread over both nuclei before they reach and overcome the Coulomb barrier!
Schematic picture for sequential fusion of $^6\text{He}$
Huge enhancement in deep sub-barrier fusion of weakly bound nuclei

\[
\frac{\sigma(6\text{He} + ^{206}\text{Pb} \rightarrow ^{212}\text{Po})}{\sigma(4\text{He} + ^{208}\text{Pb} \rightarrow ^{212}\text{Po})} \bigg|_{E_{c.m.} = 15 \text{ MeV}} = 10^3
\]

Raabe et al., Nature, 2004

\[ ^6\text{He} + ^{238}\text{U} : \]

No fusion enhancement!
Time-dependent Schrödinger equation

\[ i\hbar \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2m_3} \Delta_{\mathbf{r}} + V(\mathbf{r},\mathbf{R}(t)) \right] \Psi(\mathbf{r},\mathbf{R}(t)) \]

\[ i\hbar \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2m_3} \Delta_{\mathbf{r}} - \frac{\hbar^2}{2M} \frac{\partial^2}{\partial \mathbf{R}^2} + V_{12}(\mathbf{R}) + V_n(\mathbf{r},\mathbf{R}) \right] \Psi(\mathbf{r},\mathbf{R},t) \]

\[ T(t,E) = \int_{-\infty}^{R_B} d\mathbf{R} \int d^3 \mathbf{r} \ |\Psi(\mathbf{r},\mathbf{R},t)|^2 \]
Two mechanisms of fusion enhancement due to neutron exchange

single particle levels in the $^{96}\text{Zr} + ^{40}\text{Ca}$ system

$$V_{\alpha\beta}(R) = V_{12}(R) + \varepsilon_{\beta}(R) - \varepsilon_{\alpha}(\infty)$$

nucleus-nucleus potential energy of $^{40}\text{Ca} + ^{96}\text{Zr}$
Nucleosynthesis

some chains:
\[ ^1H(n,\gamma)^2H(n,\gamma)^3H(d,n)^4He(3H,\gamma)^7Li(n,\gamma)^8Li, \]
\[ ^8Li(^4He,n)^{11}B(n,g)^{12}B(\beta^-,\gamma)^{12}C(n,\gamma)^{13}C(n,\gamma)^{14}C \]

\[ ^{14}C(^4He,\gamma)^{18}O \] plays the major role in heavy element production
(Malanay and Fowler, Astroph. Journal, 1988)

\[ ^{14}C + ^4He \rightarrow < \quad all \quad Q_{\text{trans}}^{\text{trans}} < 0 \quad \rightarrow ^{18}O \]

\[ ^{12}C + ^6He \rightarrow < ^{12}C + ^4He + 12 \quad \text{MeV} \quad \rightarrow ^{18}O \]

\[ \frac{\sigma_{\text{fus}}(^{6He + ^{12}C})}{\sigma_{\text{fus}}(^{4He + ^{14}C})} \bigg|_{E < B} \ll 1 \]
Conclusion

• “Sequential fusion” mechanism plays an important role

• Rearrangement of neutrons with positive Q-value significantly increases sub-barrier fusion probability

• Huge enhancement in fusion of weakly bound nuclei is revealed

• New experiments on sub-barrier fusion of light weakly bound nuclei - like $^{12}\text{C}(^6\text{He},n\gamma)^{17}\text{O}$ - are to be performed

• Scenario of primordial nucleosynthesis may be revised