

# **Sub-barrier fusion of weakly bound nuclei: Strong enhancement due to sequential fusion mechanism**

- **History**
- **Role of neutrons in fusion reactions**
- **Sequential fusion mechanism**
- **Huge enhancement in fusion of weakly bound nuclei**
- **Summary**



# History

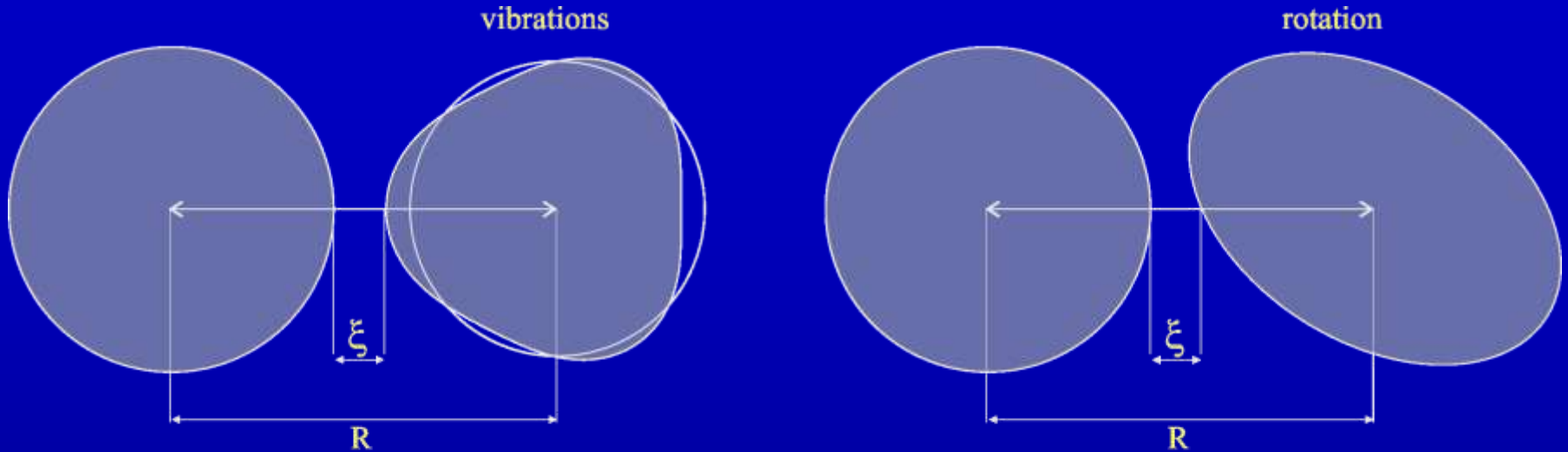
## Theory:

- Takigawa and Sagawa, Phys.Lett. 1991:  $^{11}\text{Li}+^{208}\text{Pb}$ , “halo, soft dipole mode” - enhancement
- Hussein et al, Phys.Rev. 1992:  $^{11}\text{Li}+^{208}\text{Pb}$ , “break-up” – suppression
- Dasso and Vitturi, Phys.Rev. 1994:  $^{11}\text{Li}+^{208}\text{Pb}$ , “both” – enhancement
- Hagino et al., Phys.Rev. 2000:  $^{11}\text{Be}+^{208}\text{Pb}$ , “break-up” – enhancement
- Nakatsukasa et al., Fusion-2006: fusion suppression for neutron-halo nuclei

## Experiment:

- Fomichev *et al.*, Z.Phys. 1995:  $^6\text{He}+^{209}\text{Bi}$ , enhancement ?
- Kolata *et al.*, Phys.Rev. Lett. 1998:  $^6\text{He}+^{209}\text{Bi}$ , enhancement !
- Trotta, Sida, Alamanos et al., Phys.Rev.Lett. 2000:  $^6\text{He}+^{238}\text{U}$ , enhancement !
- Raabe et al., Nature 2004:  $^6\text{He}+^{238}\text{U}$ , no fusion enhancement, only 2n-transfer
- Di Pietro et al., Phys. Rev. 2004:  $^6\text{He}+^{64}\text{Zn}$ , no enhancement

# Role of internal degrees of freedom



Distance between nuclear surfaces and, thus, the **Coulomb barrier** depend on vibrations and rotation:

$\mathbf{B} = \mathbf{B}(\beta_2, \beta_3)$ ,  $\mathbf{B} = \mathbf{B}(\theta, \varphi)$  - multidimensional barriers.

Instead of one fixed barrier  $\mathbf{B}$  we have a “**barrier distribution function**”  $f(\mathbf{B})$ .

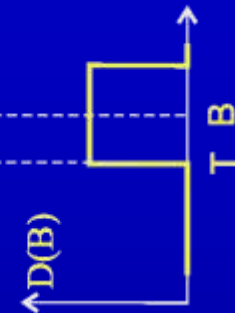
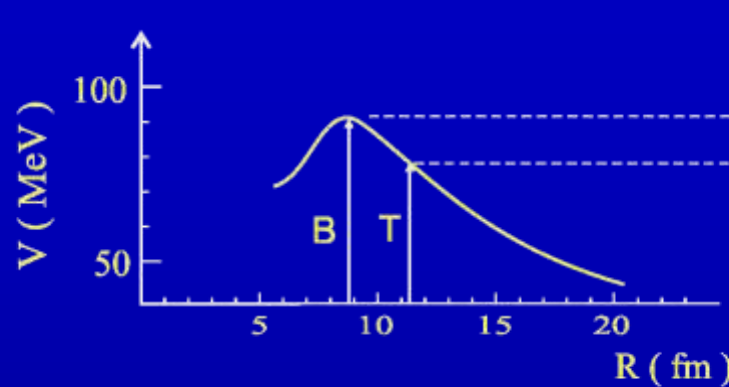
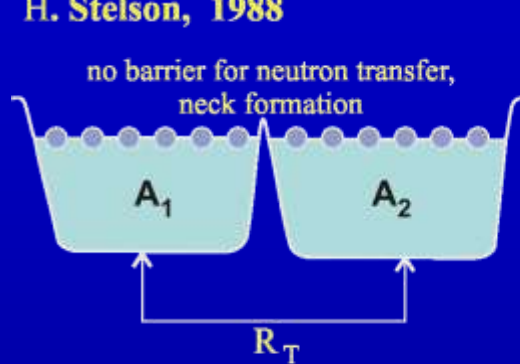
Barrier penetrability is  $T(E, \ell) = \int f(\mathbf{B}) P_0(\mathbf{B}; E, \ell) d\mathbf{B}$ .

→ sub-barrier fusion enhancement

**What is a role of neutrons in fusion process ?**

# Role of neutrons in sub-barrier fusion ?

H. Stelson, 1988



$$\sigma(E) = \pi R^2 \frac{(E-T)^2}{4(B-T)E}$$

N. Rowley, I.J. Thompson, and M.A. Nagarajan, PL, 1992

CC simulations:

- $Q_n < 0$  - broad barrier distribution, necking
- $Q_n > 0$  - "anti-necking" conditions

Ning Wang, Xizhen Wu, and Zhuxia Li, PRC, 2003

QMD calculations:

- neutron excess plays a dominant role

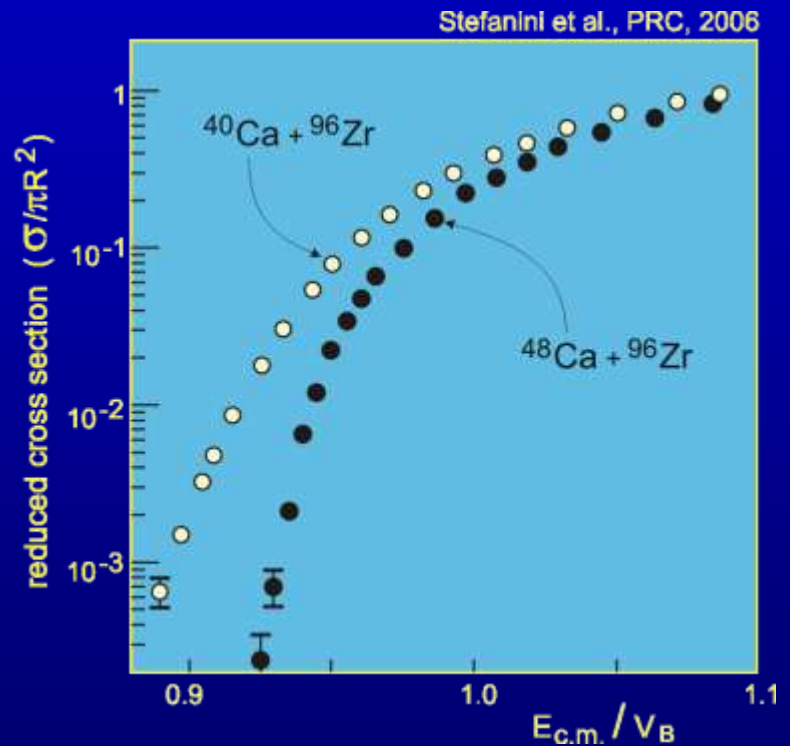
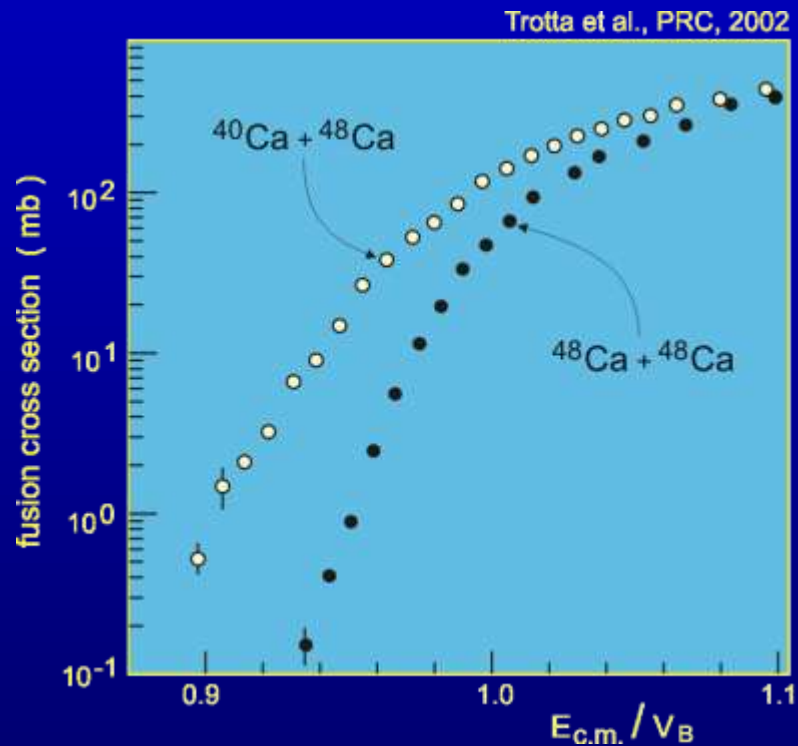
**More neutrons  
more probable fusion!**

$$\frac{\sigma_{\text{fus}}(^{48}\text{Ca} + ^{48}\text{Ca})}{\sigma_{\text{fus}}(^{40}\text{Ca} + ^{48}\text{Ca})} \Big|_{E < B} \gg 1$$

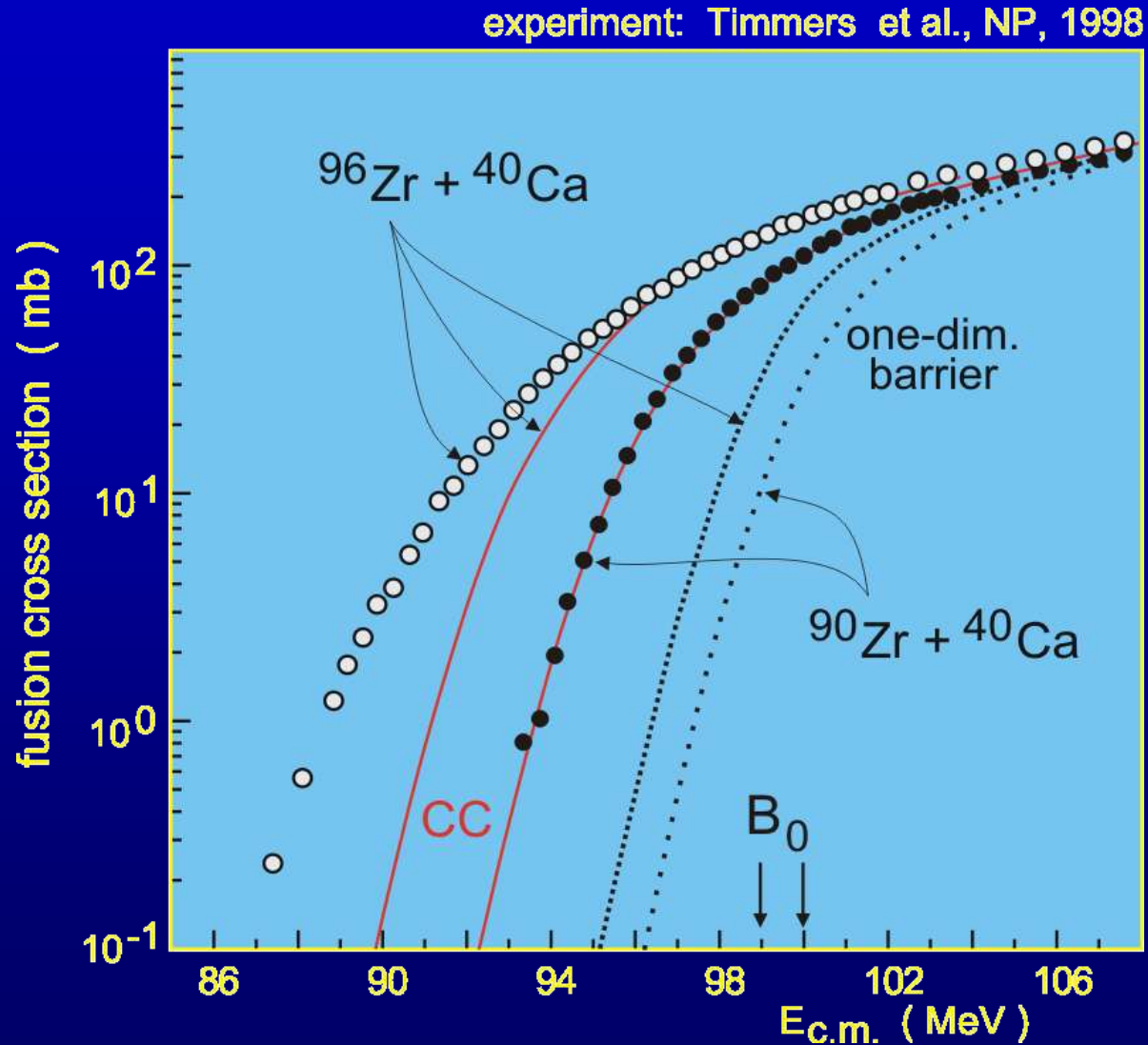
$$\frac{\sigma_{\text{fus}}(^{48}\text{Ca} + ^{96}\text{Zr})}{\sigma_{\text{fus}}(^{40}\text{Ca} + ^{96}\text{Zr})} \Big|_{E < B} \gg 1$$

**Wrong!**

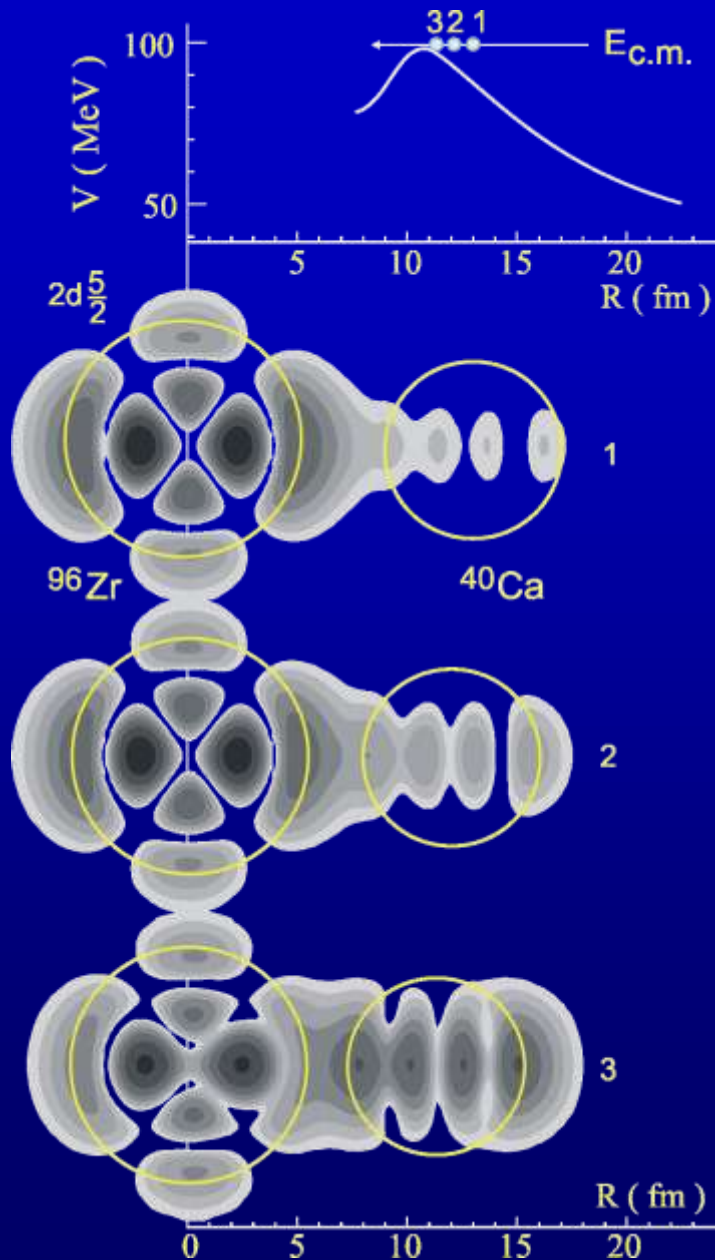
# Neutron excess itself does not help nuclei to fuse !



# Lack of theory for fusion of some nuclei



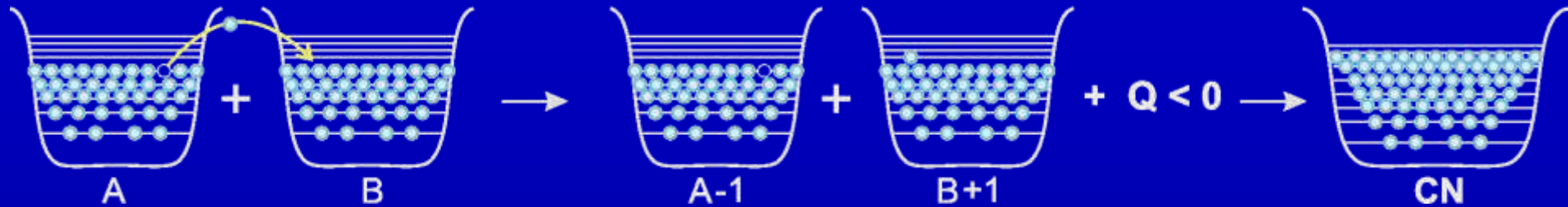
# Solution of 3-body time-dependent Schrödinger equation



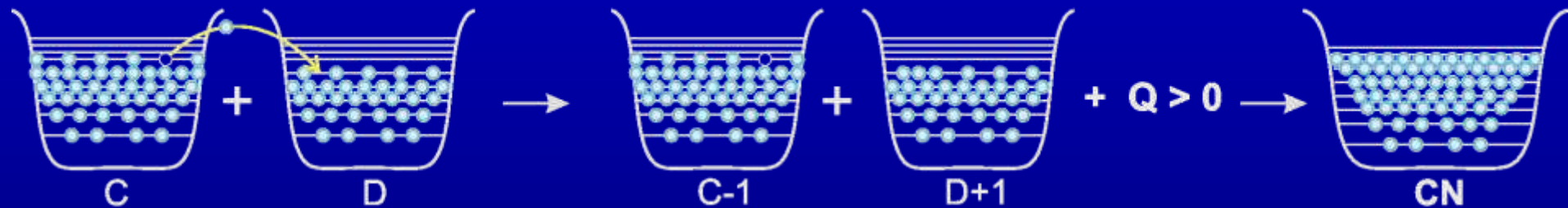
Wave functions of valence neutrons follow the two-center quasi-molecular states and **spread over the volumes of both nuclei** rather fast, before nuclei come in contact and even **before** they overcome the Coulomb barrier !

# Gain due to positive Q-value of neutron rearrangement

In most combinations neutrons may be transferred only with negative Q-values



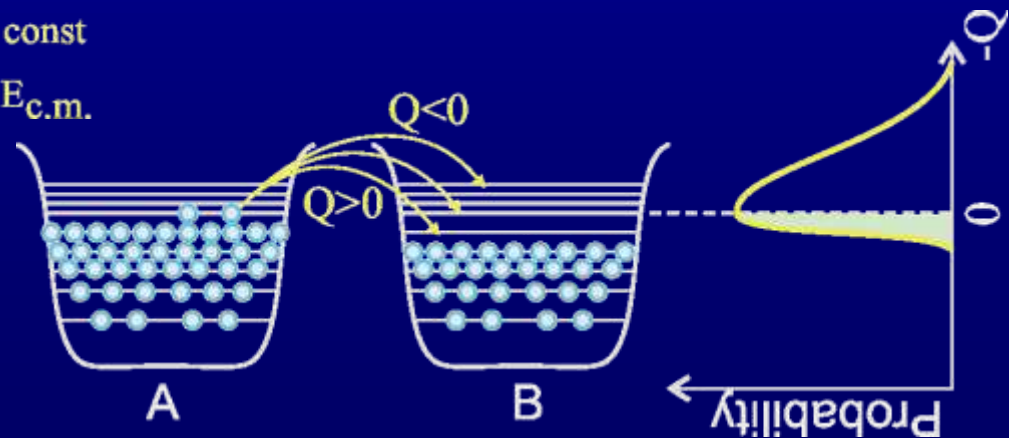
Nevertheless there are combinations with a chance for positive Q-value neutron transfer !



This gain in energy may go into the relative motion energy !

$$E_{c.m.} + M(A) + M(B) = E'_{c.m.} + M(A-1) + M(B+1) = \text{const}$$

$$\text{if } [M(A-1)+M(B+1)] < [M(A)+M(B)] \text{ then } E'_{c.m.} > E_{c.m.}$$





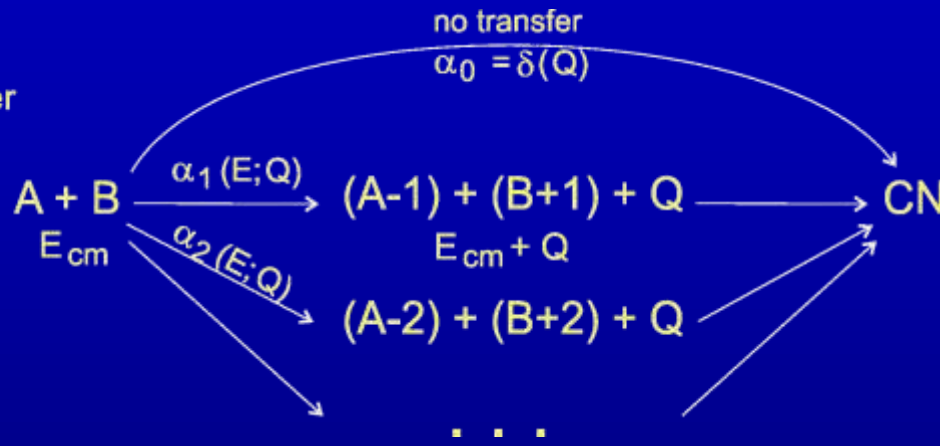
# Sequential fusion process

$$\sigma_{\text{fusion}}(E) = \frac{\pi}{k^2} \sum (2\ell+1) \cdot T(\ell, E)$$

$$T(\ell, E) = \int_0^{\infty} f(B) P_0(B; \ell, E) dB$$

barrier distribution function

subsequent neutron transfer and "sequential fusion":



$$T(\ell, E) = \int_0^{\infty} f(B) \int_{-E}^{Q_0(k)} \frac{1}{N} \left[ \delta(Q) + \sum_{k \geq 1} \alpha_k(E; \ell, Q) \right] P_0(B; \ell, E+Q) dQ dB$$

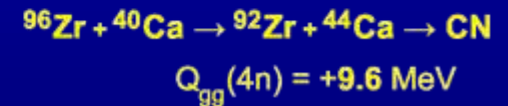
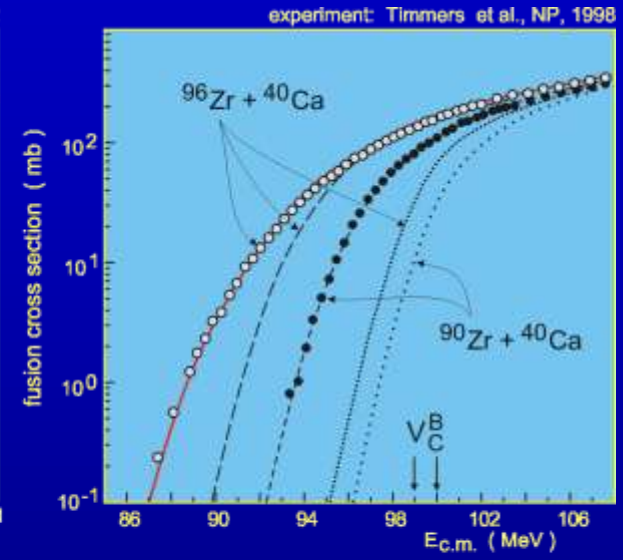
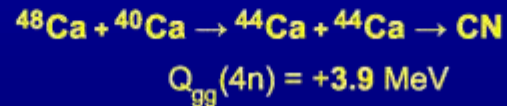
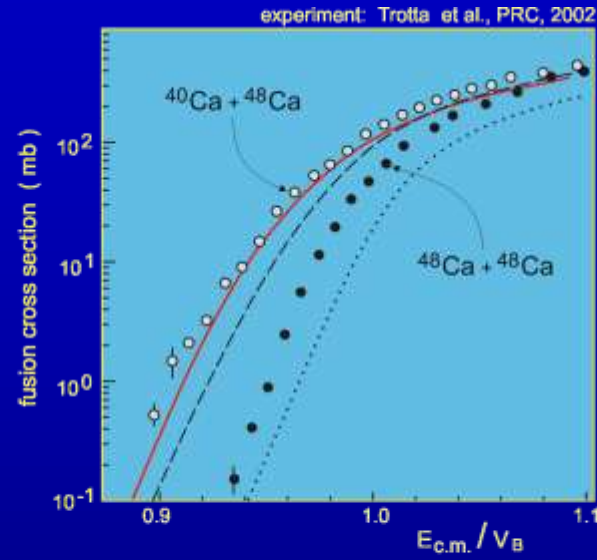
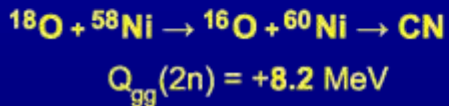
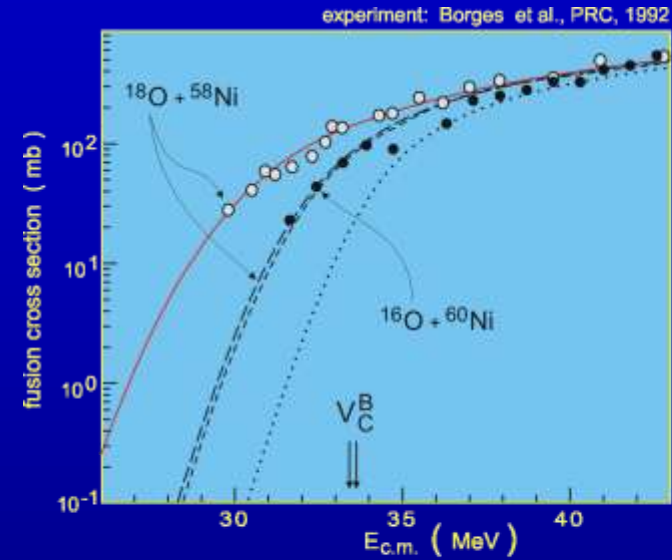
$Q_0(k)$  is the Q-value of g.s. transfer

probability for transfer of k neutrons:  $\alpha_k(E; \ell, Q) = N_k e^{-C[Q-Q_{\text{opt}}]^2} e^{-2\gamma[D(E, \ell) - D_0]}$

$D(E, \ell)$  is the distance of closest approach, and  $D_0 = d_0(A^{1/3} + B^{1/3})$ ,  $d_0 \approx 1.4$  fm

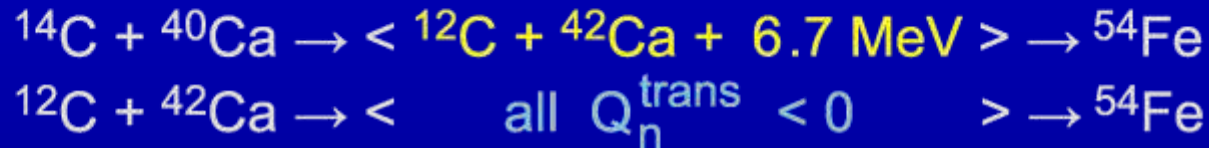
$\gamma = \gamma(\epsilon_1) + \gamma(\epsilon_2) + \dots + \gamma(\epsilon_k)$ ,  $\gamma(\epsilon) = \sqrt{2\mu\epsilon/\hbar^2}$

# Fusion enhancement for stable neutron rich nuclei

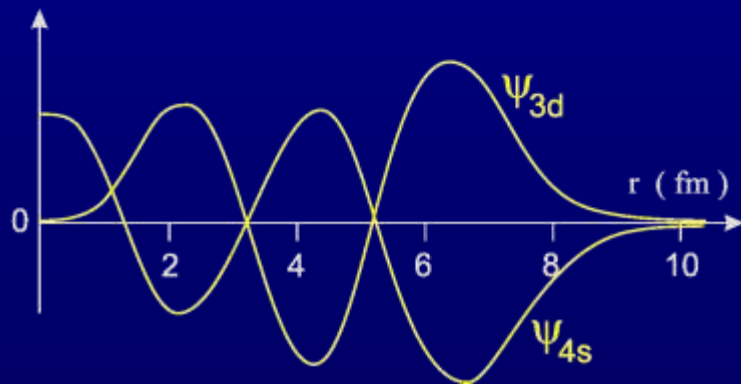
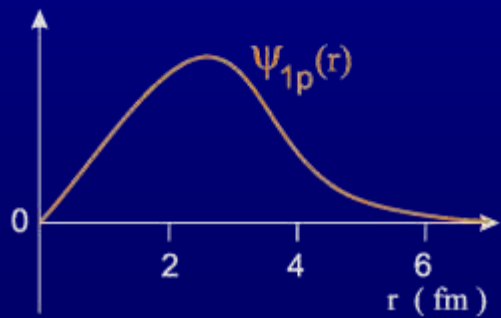
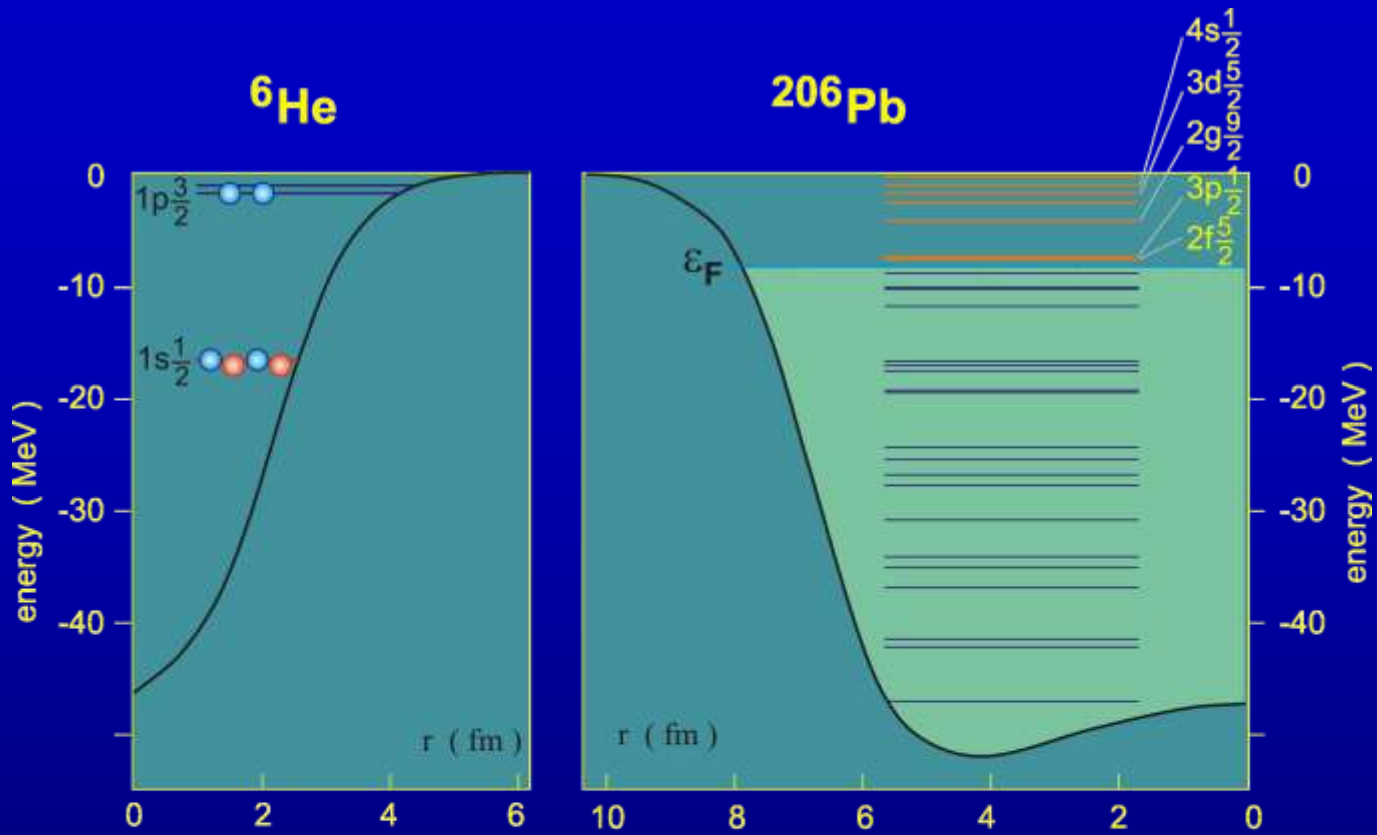


Idea of “sequential fusion” mechanism with intermediate neutron rearrangement seems to work !

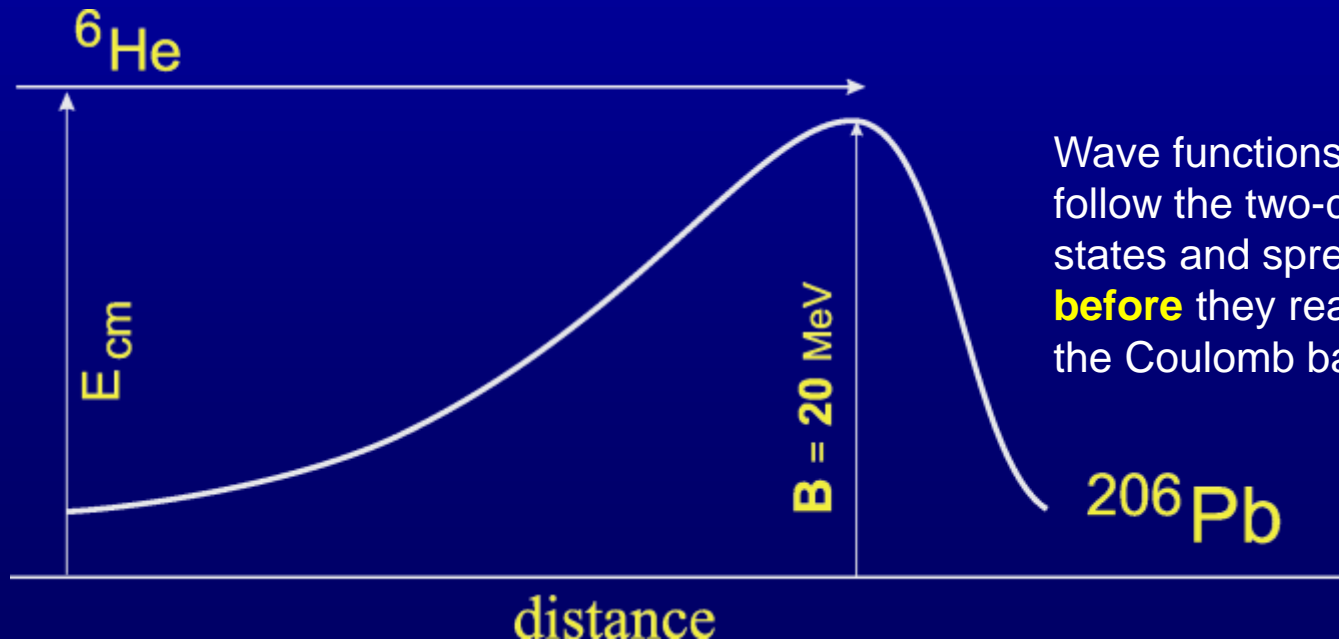
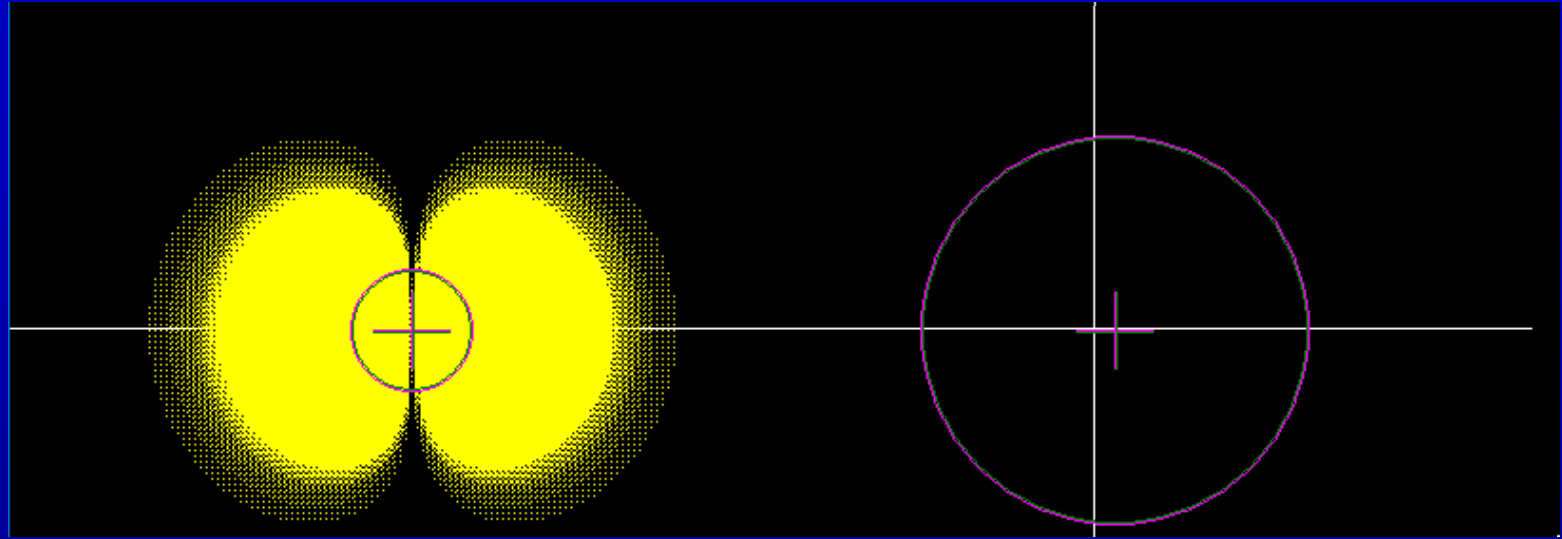
# Test for the “neutron rearrangement” mechanism in sub-barrier fusion reactions



# ${}^6\text{He} + {}^{206}\text{Pb}$

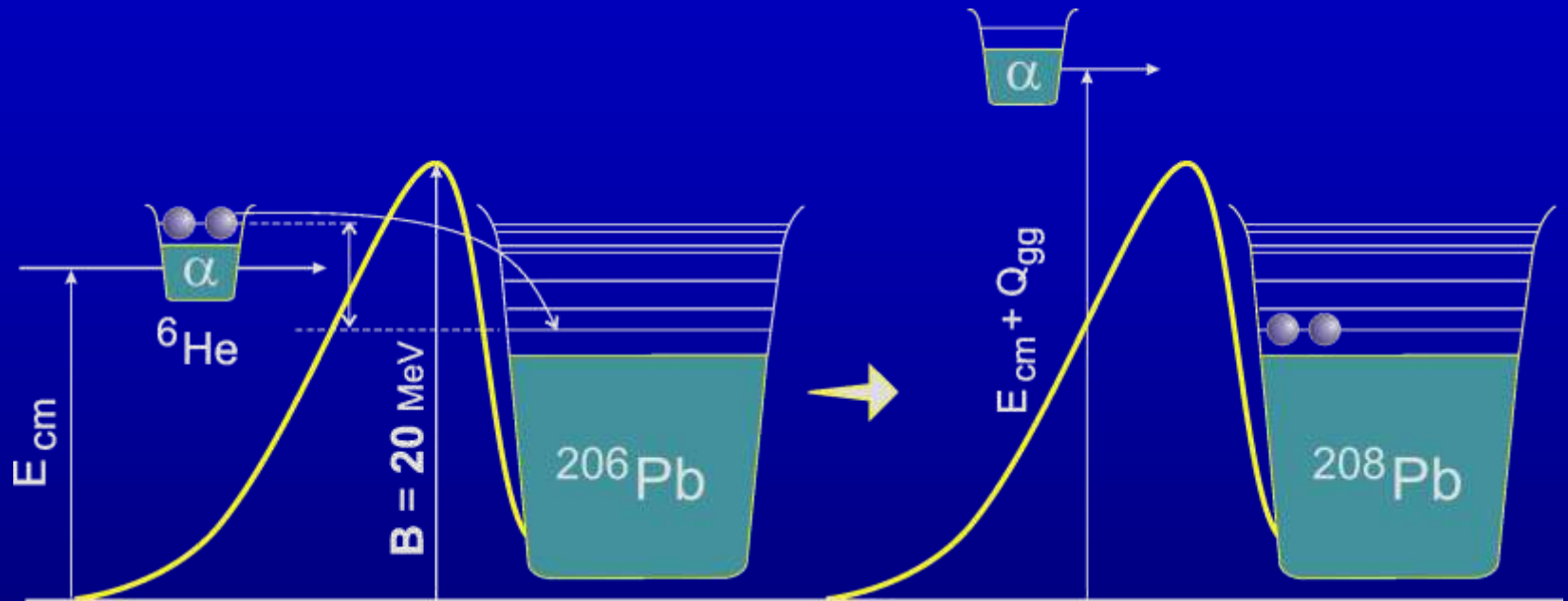


# Time-dependent analysis of ${}^6\text{He} + {}^{206}\text{Pb}$ collision



Wave functions of valence neutrons follow the two-center molecular states and spread over both nuclei **before** they reach and overcome the Coulomb barrier !

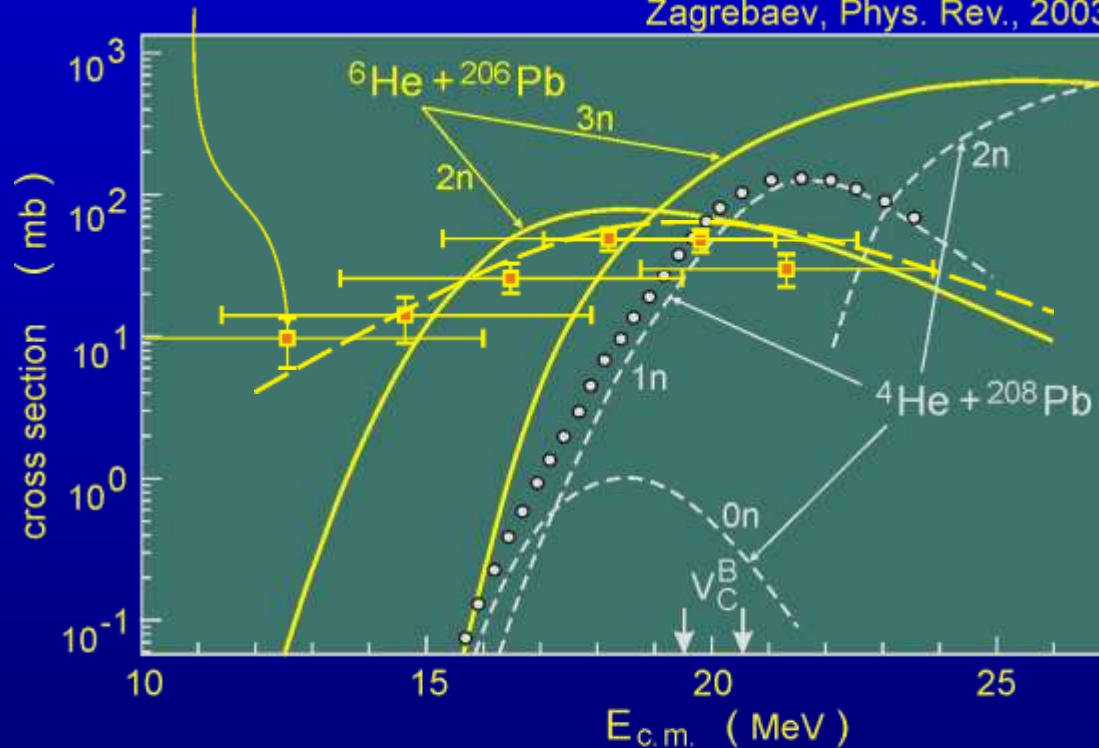
# Schematic picture for sequential fusion of ${}^6\text{He}$



# Huge enhancement in deep sub-barrier fusion of weakly bound nuclei

Penionzhkevich, Zagrebaev, Lukyanov and Kalpakchieva,  
Phys. Rev. Lett, **96**,2006

Zagrebaev, Phys. Rev., 2003



Raabe et al., Nature, 2004

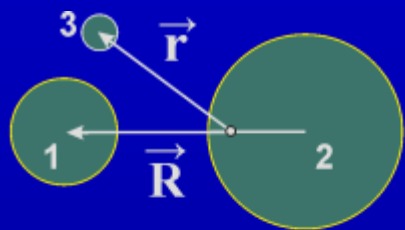
${}^6\text{He} + {}^{238}\text{U}$ :

**No fusion enhancement !**

$$\left. \frac{\sigma({}^6\text{He} + {}^{206}\text{Pb} \rightarrow {}^{212}\text{Po})}{\sigma({}^4\text{He} + {}^{208}\text{Pb} \rightarrow {}^{212}\text{Po})} \right|_{E_{c.m.} = 15 \text{ MeV}} = 10^3$$

( same "temperature" )

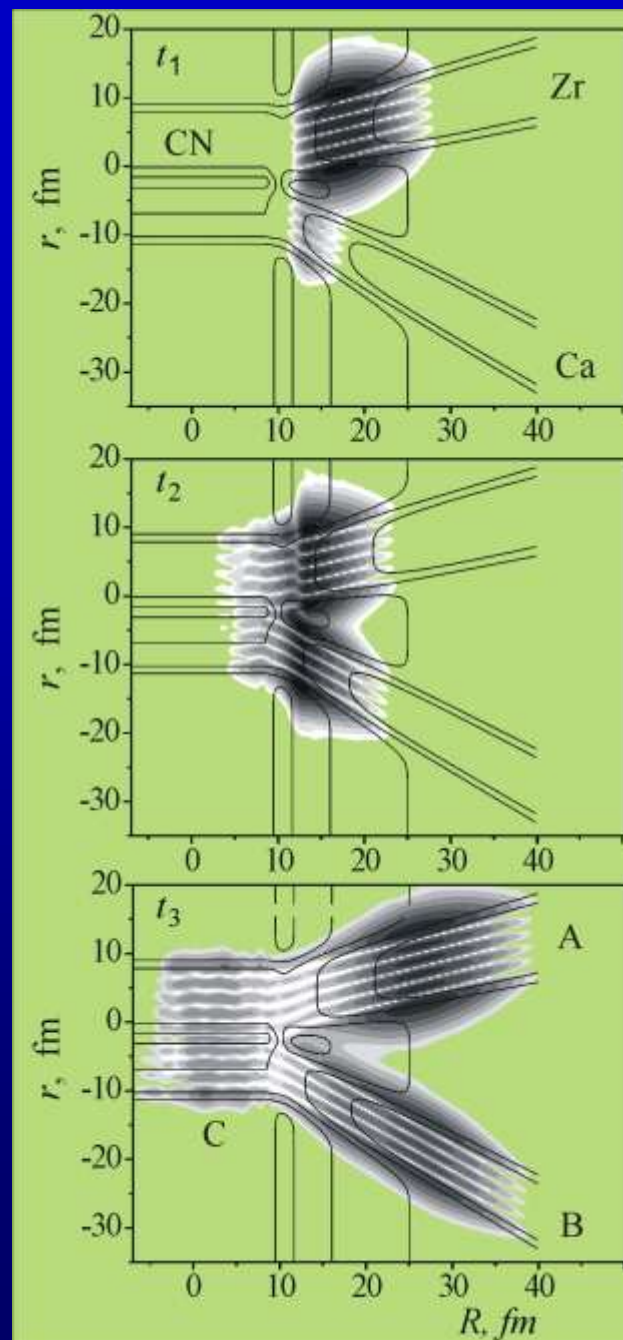
# Time-dependent Scrodinger equation



$$i\hbar \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2m_3} \Delta_{\mathbf{r}} + V(\mathbf{r}, \mathbf{R}(t)) \right] \Psi(\mathbf{r}, t, \mathbf{R}(t))$$

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[ -\frac{\hbar^2}{2m_3} \Delta_{\mathbf{r}} - \frac{\hbar^2}{2M} \frac{\partial^2}{\partial \mathbf{R}^2} + V_{12}(\mathbf{R}) + V_n(\mathbf{r}, \mathbf{R}) \right] \Psi(\mathbf{r}, \mathbf{R}, t)$$

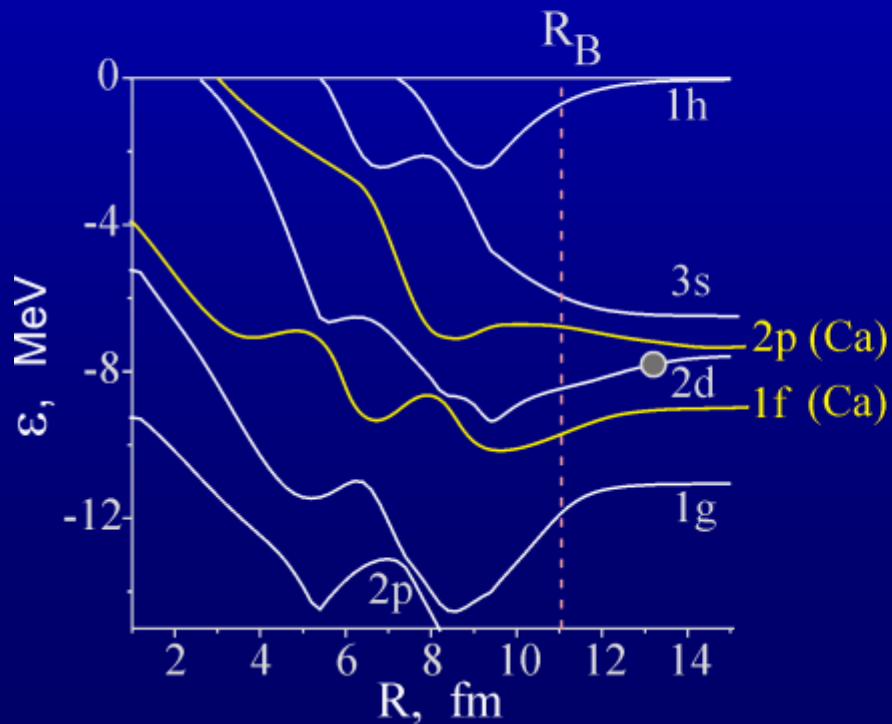
$$T(t, E) = \int_{-\infty}^{R_B} dR \int d^3\mathbf{r} |\Psi(\mathbf{r}, \mathbf{R}, t)|^2$$





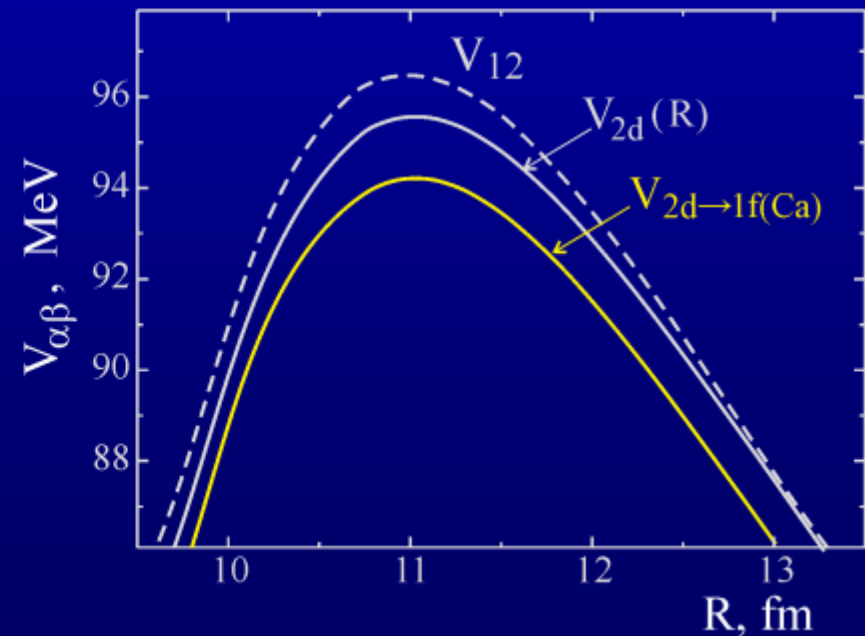
# Two mechanisms of fusion enhancement due to neutron exchange

single particle levels  
in the  $^{96}\text{Zr} + ^{40}\text{Ca}$  system

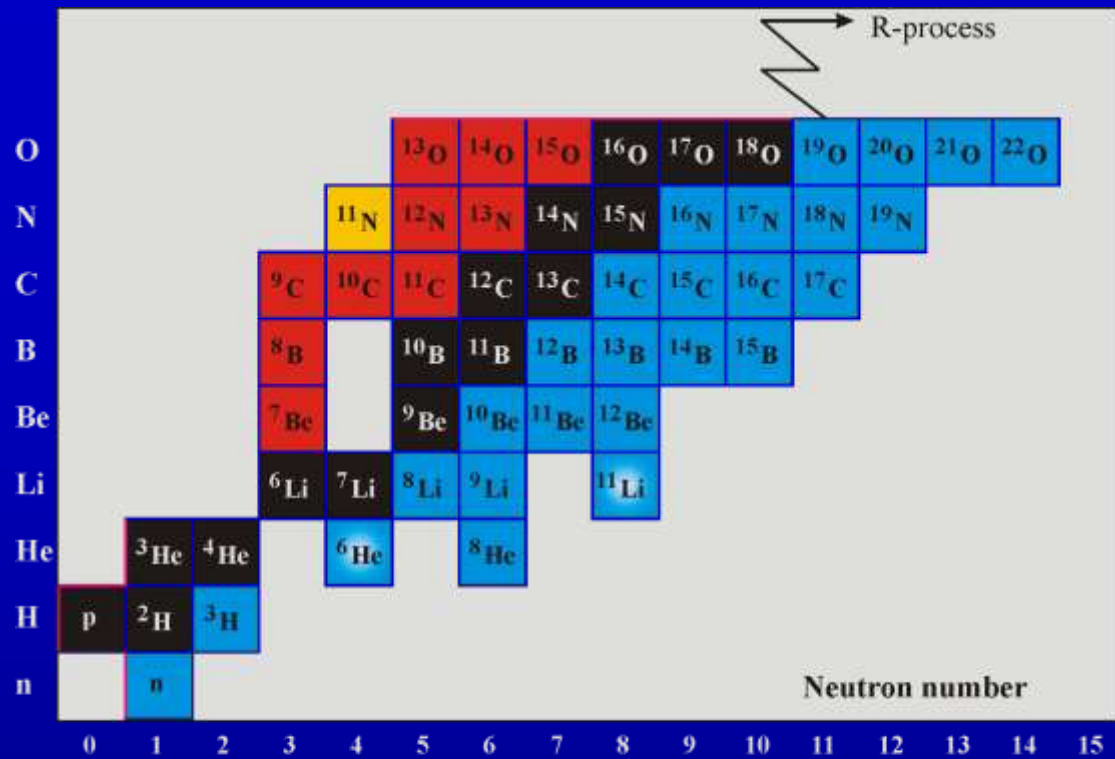


$$V_{\alpha\beta}(R) = V_{12}(R) + \epsilon_{\beta}(R) - \epsilon_{\alpha}(\infty)$$

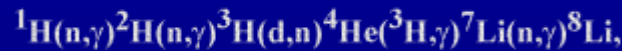
nucleus-nucleus potential energy  
of  $^{40}\text{Ca} + ^{96}\text{Zr}$



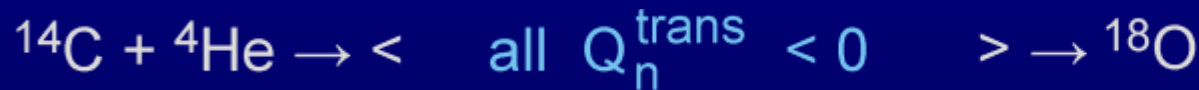
# Nucleosynthesis



some chains :



$^{14}\text{C}(^4\text{He},\gamma)^{18}\text{O}$  plays the major role in heavy element production (Malanay and Fowler, *Astroph. Journal*, 1988)



# Conclusion

- “Sequential fusion” mechanism plays an important role
- Rearrangement of neutrons with positive Q-value significantly increases sub-barrier fusion probability
- Huge enhancement in fusion of weakly bound nuclei is revealed
- New experiments on sub-barrier fusion of light weakly bound nuclei - like  $^{12}\text{C}(^6\text{He},n\gamma)^{17}\text{O}$  - are to be performed
- Scenario of primordial nucleosynthesis may be revised