Superheavies:

1. Theoretical models of formation dynamics
(the problems to be solved)

- **Fusion reactions** **(new SH elements and isotopes)**
- **Transfer reactions** **(new neutron-rich SH nuclei)**
- **Neutron capture** **(SHE in nature)**

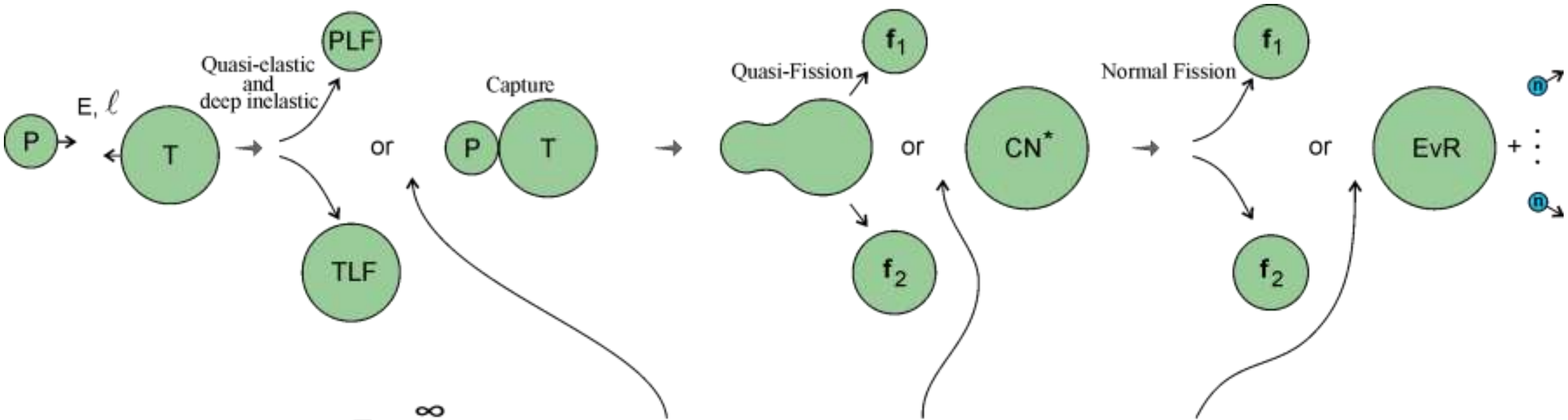
2. SHE experiments
(what could be really done within the next few years)

Valeriy Zagrebaev

Flerov Laboratory of Nuclear Reactions, JINR, Dubna

for “Future of Super Heavy Elements”, *May 14, 2012*, Weiltal, Germany

Synthesis of SHE in fusion reactions (conventional view)



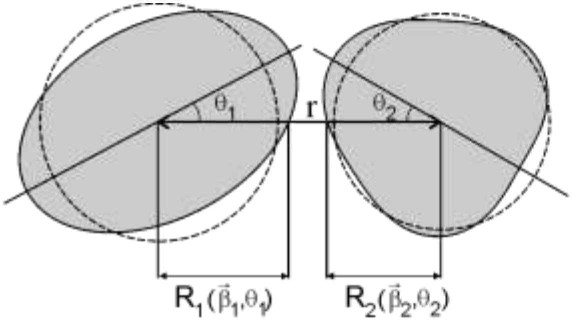
$$\sigma_{ER}^{xn}(E) = \underbrace{\underbrace{\frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\cdot P_{cont}(\ell,E)}_{\sigma_{capture}(E)\overset{exp}{=}\sum(f_{1,2}^{all}+EvR),\ \text{if}\ f_1f_2\neq PLF, TLF}\cdot P_{CN}(\ell,E^*)}_{\sigma_{CN}\equiv\sigma_{fusion}(E)\overset{exp}{=}\sum(f_{1,2}^{NF}+EvR),\ \text{if}\ f_{1,2}^{NF}\neq f_{1,2}^{QF}}\cdot P_{xn}(\ell,E^*)$$

P_{cont} : Penetration probability of the multi-dimensional Coulomb barrier $V_C^B(r,\beta_1,\beta_2,\varphi_1,\varphi_2)$ - ?

P_{CN} : Probability of CN formation (evolution in the space of shape parameters)

P_{xn} : Survival probability of excited CN (Statistical Model: Γ_n, Γ_f, E_n^{sep}, B_{fis})

Capture cross section (Channel Coupling approach)



$$R(\vec{\beta},\theta)=\tilde{R}\cdot\left(1+\sum_{\lambda\geq 2}\beta_{\lambda}Y_{\lambda 0}(\theta,0)\right)$$

$$\widetilde{R}=Ro/[1+\frac{3}{4\pi}\sum_{\lambda}\beta_{\lambda}^{2}+...]^{1/3}$$

$$V_{12}(r;\vec{\beta}_1,\theta_1,\vec{\beta}_2,\theta_2)=V_C(r;\vec{\beta}_1,\theta_1,\vec{\beta}_2,\theta_2)+V_N(r;\vec{\beta}_1,\theta_1,\vec{\beta}_2,\theta_2)+\frac{1}{2}\sum_{i=1}^{2}\sum_{\lambda}C_{i\lambda}\cdot\beta_{i\lambda}^{2}$$

$$H=-\frac{\hbar^2\nabla_r^2}{2\mu}+V_C(r;\vec{\beta}_1,\theta_1,\vec{\beta}_2,\theta_2)+V_N(r;\vec{\beta}_1,\theta_1,\vec{\beta}_2,\theta_2)+\underbrace{\sum_{i=1,2}\frac{\hbar^2\hat{I}_i^{\,2}}{2\,\mathrm{J}_i}+\sum_{i=1,2}\sum_{\lambda\geq 2}\left(-\frac{1}{2d_{i\lambda}}\frac{\partial^2}{\partial s_{i\lambda}^2}+\frac{1}{2}c_{i\lambda}s_{i\lambda}^2\right)}$$

$$H\Psi=E\Psi$$

$$H_{\mathrm{int}}\varphi_{\nu}(\vec{\alpha})=\varepsilon_{\nu}\varphi_{\nu}(\vec{\alpha})$$

$$\Psi_{\vec{\mathbf{k}}}^{(+)}(r,\vartheta,\vec{\alpha})=\frac{1}{kr}\sum_{l=0}^{\infty}i^l e^{i\sigma_l}(2l+1)\chi_l(r,\vec{\alpha})P_l(\cos\vartheta)\,,$$

$$\chi_l(r,\vec{\alpha})=\sum_{\nu}y_{l,\nu}(r)\cdot\varphi_{\nu}(\vec{\alpha})$$

$$y_{l,\nu}''-\frac{l(l+1)}{r^2}+\frac{2\mu}{\hbar^2}\left[E-\varepsilon_{\nu}-V_{\nu\nu}(r)\right]y_{l,\nu}-\sum_{\mu\neq\nu}\frac{2\mu}{\hbar^2}V_{\nu\mu}(r)y_{l,\mu}=0$$

boundary conditions:

$$y_{l,\nu}(r\rightarrow\infty)=\frac{i}{2}\left[h_l^{(-)}(\eta_{\nu},k_{\nu}r)\cdot\delta_{\nu 0}-\left(\frac{k_0}{k_{\nu}}\right)^{1/2}S_{\nu 0}^{l}\cdot h_l^{(+)}(\eta_{\nu},k_{\nu}r)\right]$$

$$y_{l,\nu}'(r<R_{\mathrm{fus}})\sim -ik_{l,\nu}y_{l,\nu}(r)$$

incoming flux in channel ν:

$$j_{l,\nu}=-i\frac{\hbar}{2\mu}\left(y_{l,\nu}\frac{dy_{l,\nu}^{*}}{dr}-y_{l,\nu}^{*}\frac{dy_{l,\nu}}{dr}\right)\Big|_{r\leq R_{\mathrm{fus}}}$$

$$P_l(E)=\sum_{\nu}{}^{j_{l,\nu}}\!/_{j_0}$$

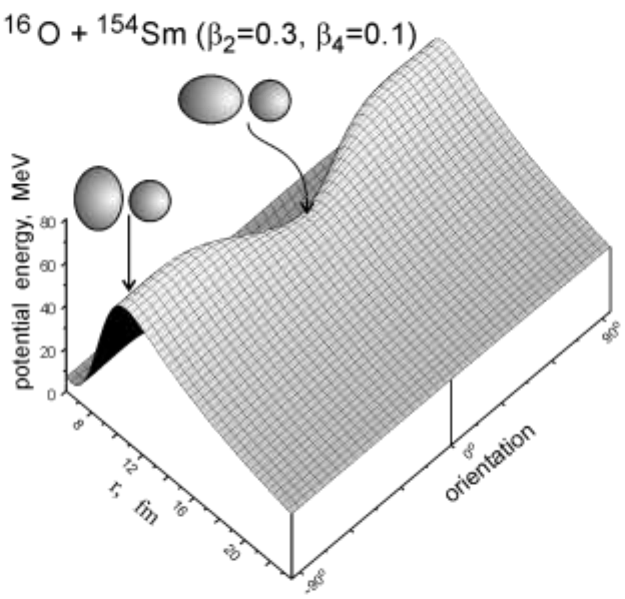
$$\sigma_{\mathrm{fus}}(E)=\frac{\pi}{k_0^2}\sum_{l=0}^{\infty}(2l+1)\cdot P_l(E)$$

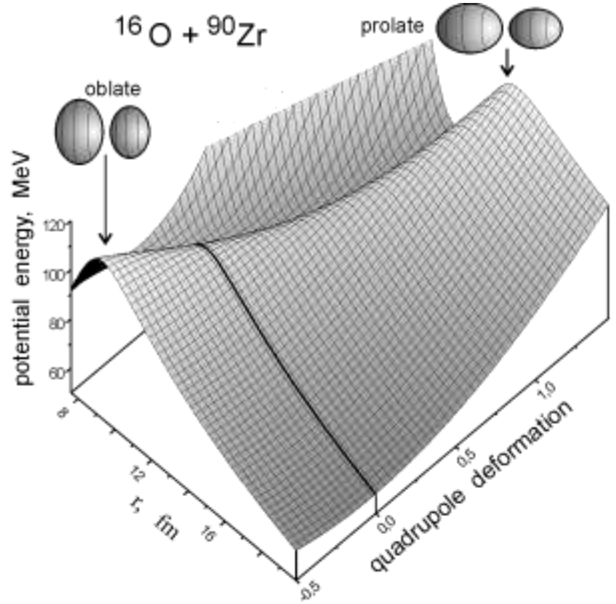
CCFULL code (Hagino, Rowley, Kruppa)

NRV-codes: Fusion-CC (Zagrebaev, Samarin)

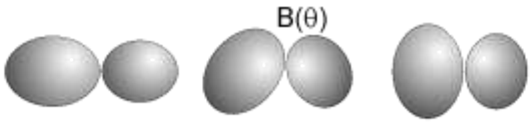
Web: http://nrv.jinr.ru/nrv

Capture cross section (Empirical Channel Coupling approach)





deformed nuclei: B(θ) depends on orientation



spherical nuclei: B depends on dynamical deformation

$$\sigma_{fus}(E) = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\cdot P(\ell, E)$$

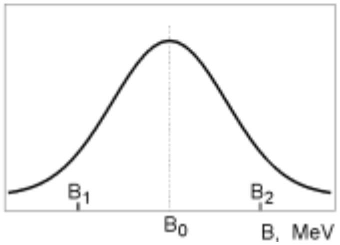
$$P(\ell,E) = \int_0^{\pi} P^{HW}(B(\theta);\ell,E)\sin\theta\, d\theta$$

$$P(\ell,E) = \int_0^{\infty} P^{HW}(B;\ell,E)\ f(B)\ dB$$

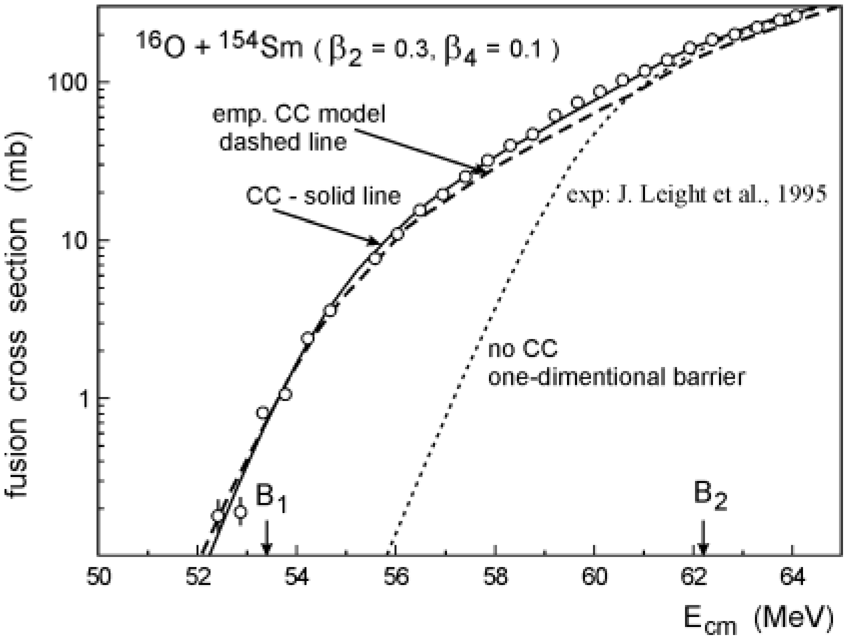
Hill - Wheeler approximation:

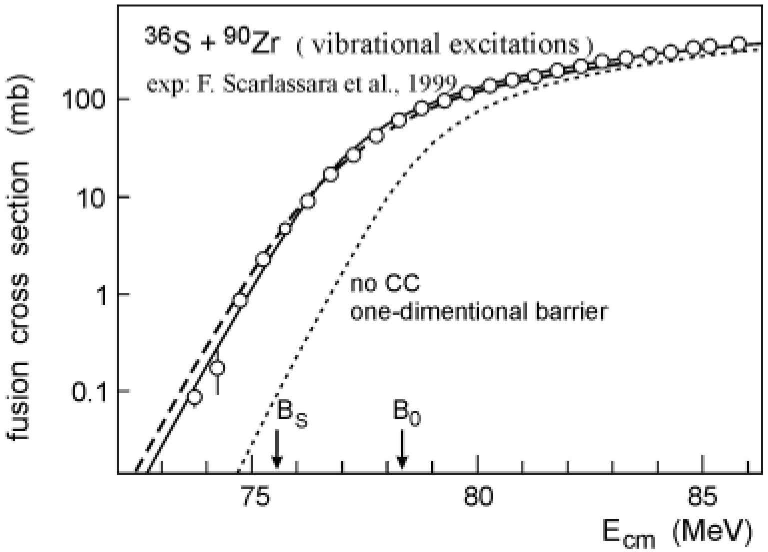
$$P^{HW} = \left[1 + \exp\left(\frac{2\pi}{\hbar\omega(\ell)}\left[B + \frac{\hbar^2\ell(\ell+1)}{2\mu R_B^2(\ell)} - E\right]\right)\right]^{-1}$$

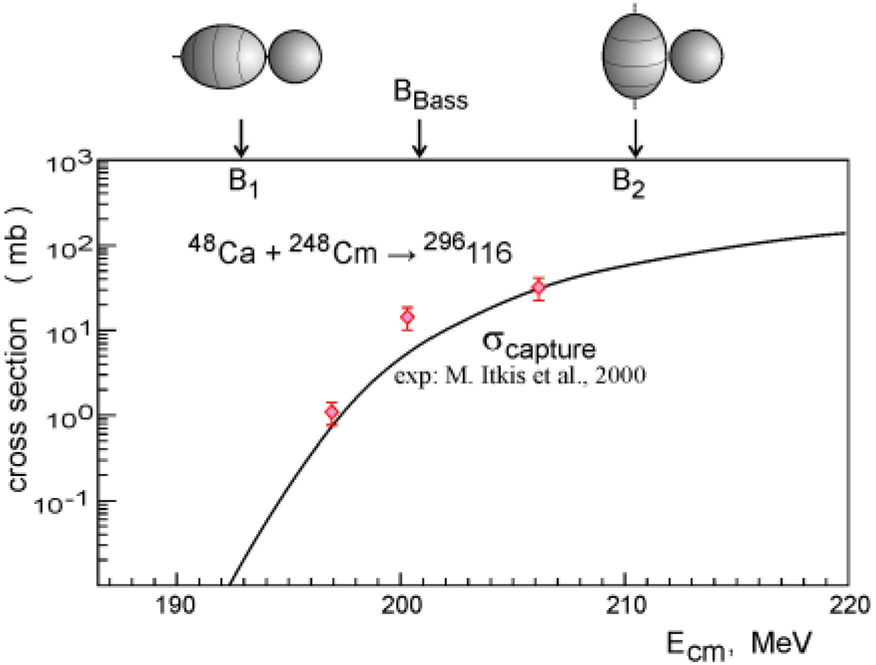
$$f(B) \sim \exp\left[-\left(\frac{B-B_0}{\Delta}\right)^2\right],\quad B_0 = (B_1+B_2)/2$$

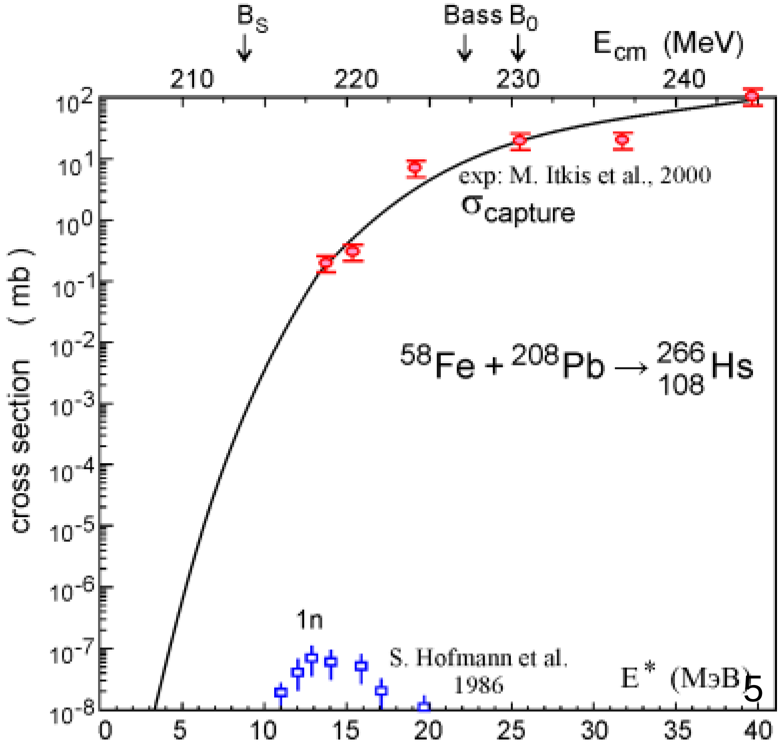


Triumph of Theory

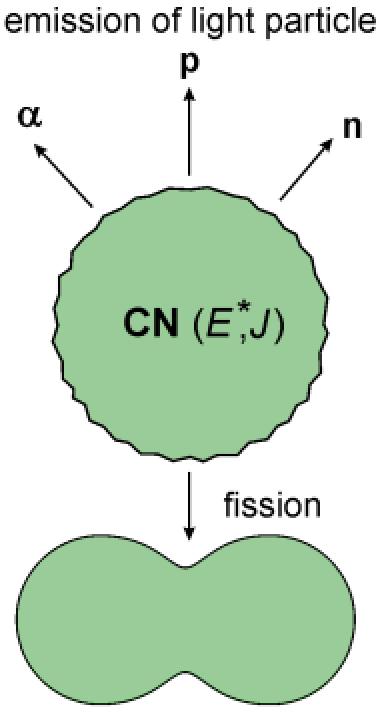








Cooling (survival) of excited compound nucleus (Statistical Model)



$$\Gamma_{A\to B+a}(E^*J) = \frac{1}{2\pi\rho_A(E^*,J)} \int_0^{E^*-E_a^{sep}} \sum_{l,j} T_{lj}(e_a)\cdot\sum_{I=|J-j|}^{I=J+j} \rho_B(E^*-E_a^{sep}-e_a, I;\beta_{g.s.})\,de_a$$

$$\Gamma_{fission}(E^*,J) = \frac{K_{Kramers}(\eta,T)}{2\pi\rho_A(E^*,J)} \int_0^{E^*} T_{fis}(e)\cdot\rho_A(E^*-e,J;\beta_2^{saddle})\,de$$

$$\Gamma_\gamma^L(E^*,J) = \frac{1}{2\pi\rho_A(E^*,J)} \int_0^{E} \sum_{I=|J-L|}^{I=J+L} f_L(e_\gamma)\cdot e_\gamma^{2L+1}\cdot\rho_A(E^*-e_\gamma, I)\,de_\gamma$$

Survival probability: **CN** $(E_0^*,J_0) \rightarrow$ **EvR**(g.s.) + xn + Nγ

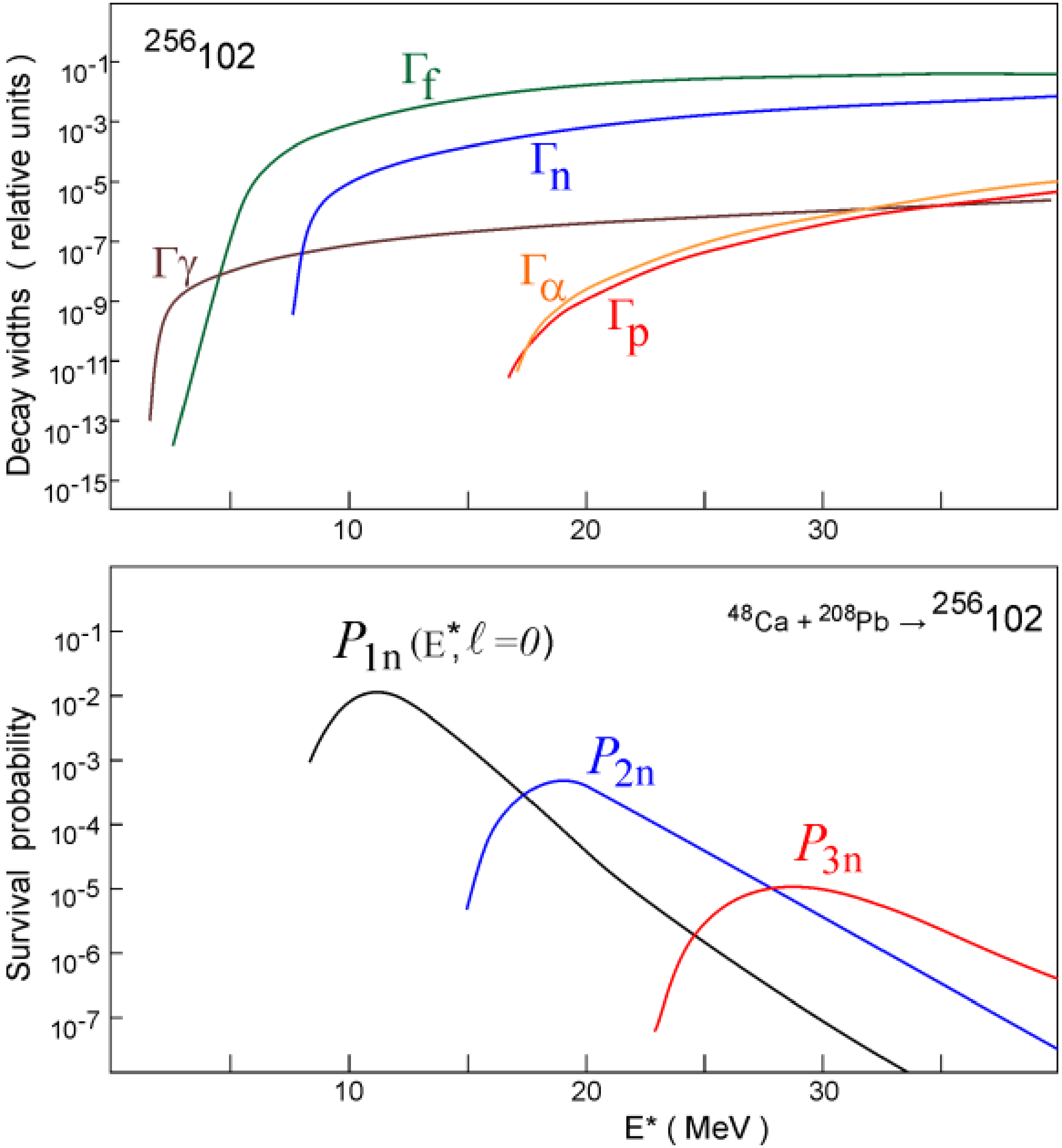
$$P_{xn} = \int_0^{E_0^*-E_n^{sep}(1)} \frac{\Gamma_n}{\Gamma_{tot}}(E_0^*,J_0)\ P_n(E_0^*,e_1)\,de_1 \int_0^{E_1^*-E_n^{sep}(2)} \frac{\Gamma_n}{\Gamma_{tot}}(E_1^*,J_1)\ P_n(E_1^*,e_2)\,de_2 \cdots \int_0^{E_1^*-E_n^{sep}(x)} \frac{\Gamma_n}{\Gamma_{tot}}(E_{x-1}^*,J_{x-1})\ P_n(E_{x-1}^*,e_x)\ G_{N\gamma}(E_x^*,J_x\to g.s.)\,de_x$$

Cross section for formation of evaporation residues:

$$\sigma_{EvR}^{xn}(E) = \frac{\pi}{k^2}\sum_{\ell}(2\ell+1)P(E,\ell)\cdot P_{CN}(E^*,\ell)\cdot P_{xn}(E^*,\ell)$$

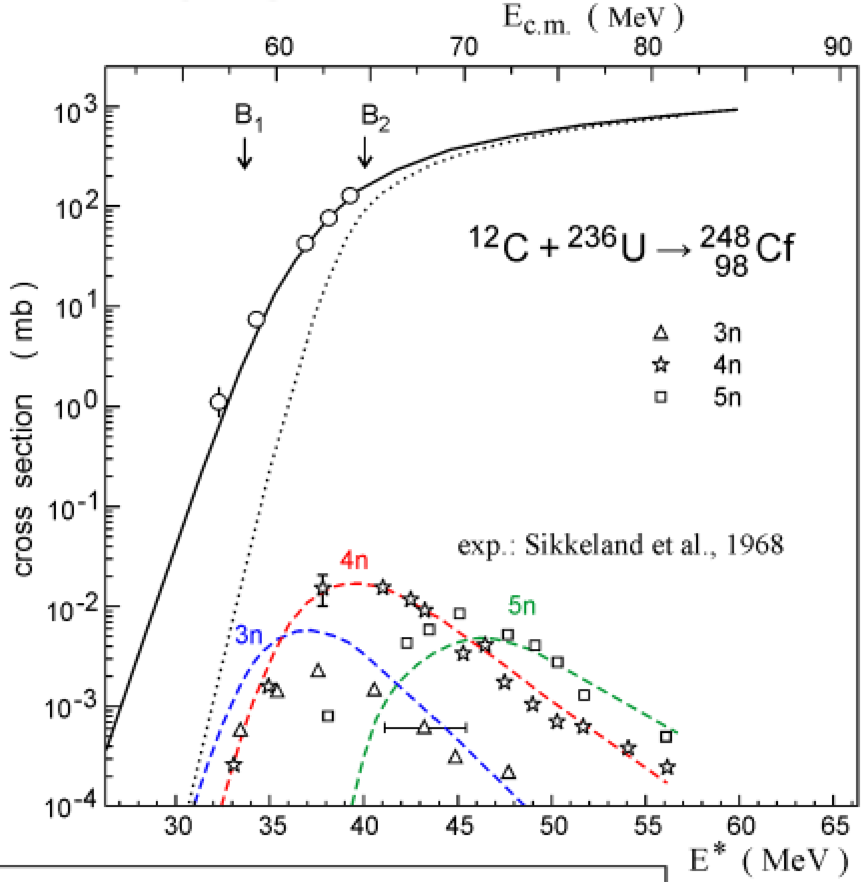
http://nrv.jinr.ru/nrv/Statistical Model

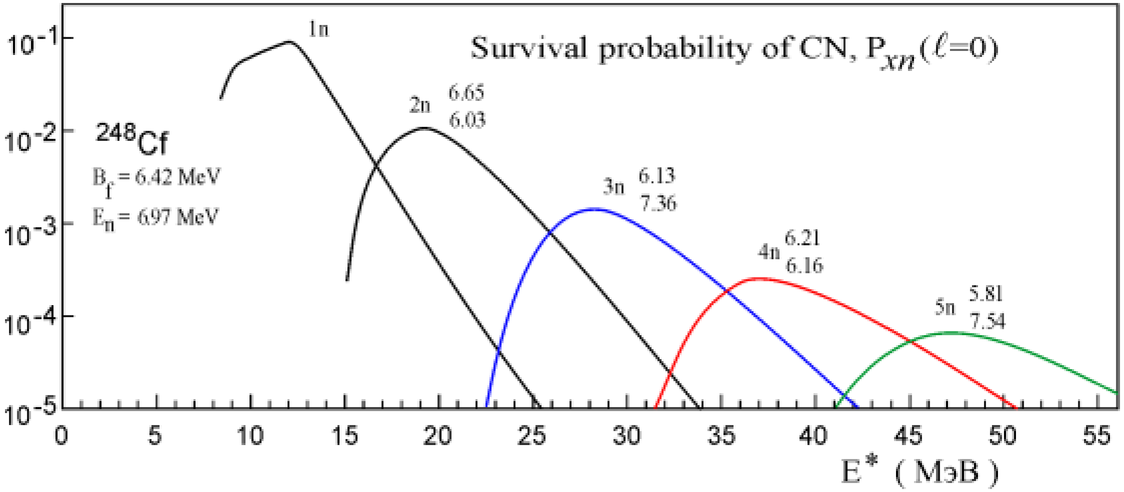
Decay widths and survival probability



Triumph of Theory for SHE formation in very asymmetric fusion reactions

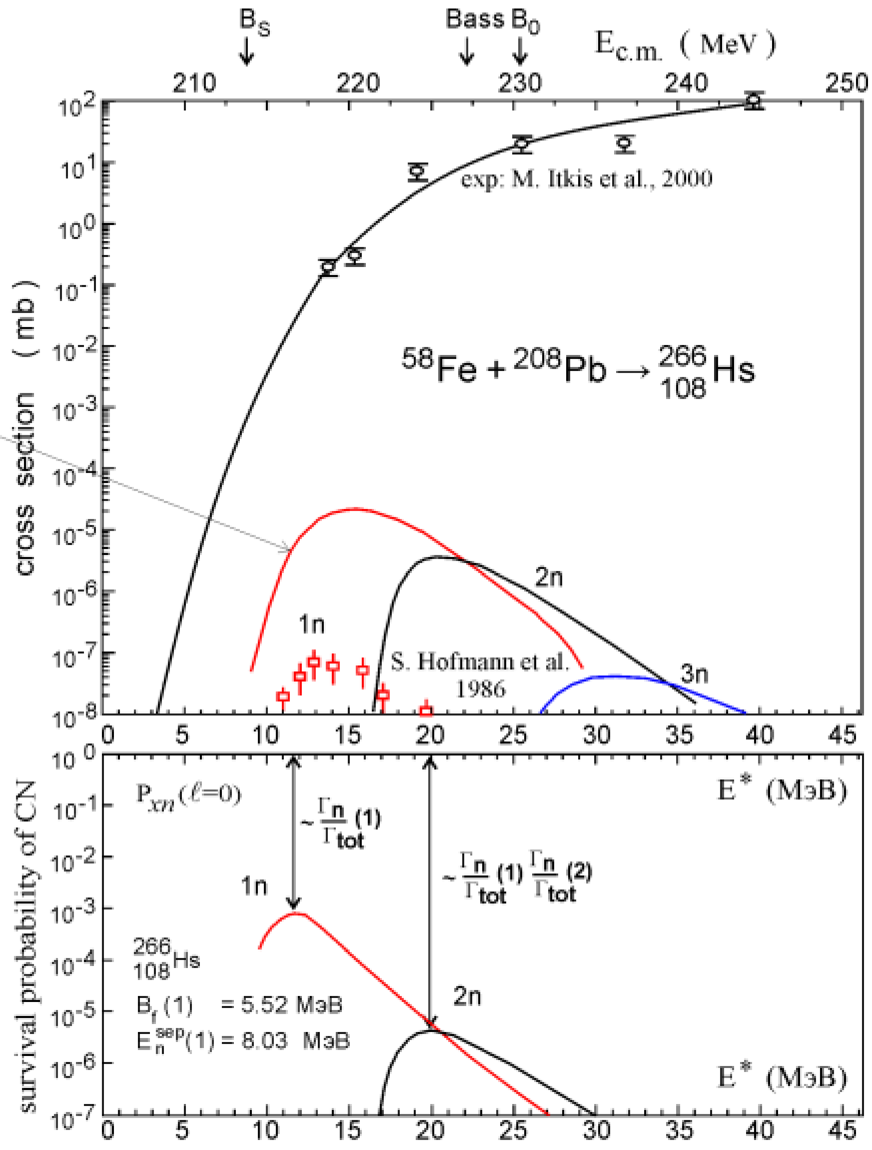
$$\sigma^{xn}_{\mathrm{EvR}}(E) = \frac{\pi\hbar^2}{2\mu E}\sum_{\ell}(2\ell+1)\underbrace{P(\ell,E)}_{\text{standard CC model}}\cdot\underbrace{P_{\mathrm{CN}}(\ell,E^*)}_{\doteq 1}\cdot\underbrace{P_{xn}(\ell,E^*)}_{\text{statistical model}}$$





Lack of Theory for SHE formation in more symmetric fusion reactions

$$\sigma^{xn}_{\mathrm{EvR}}(E) = \frac{\pi\hbar^2}{2\mu E}\sum_{\ell}(2\ell+1)\underbrace{P(\ell,E)}_{\text{standard CC model}}\cdot\underbrace{P_{\mathrm{CN}}(\ell,E^*)}_{=1}\cdot\underbrace{P_{xn}(\ell,E^*)}_{\text{statistical model}}$$

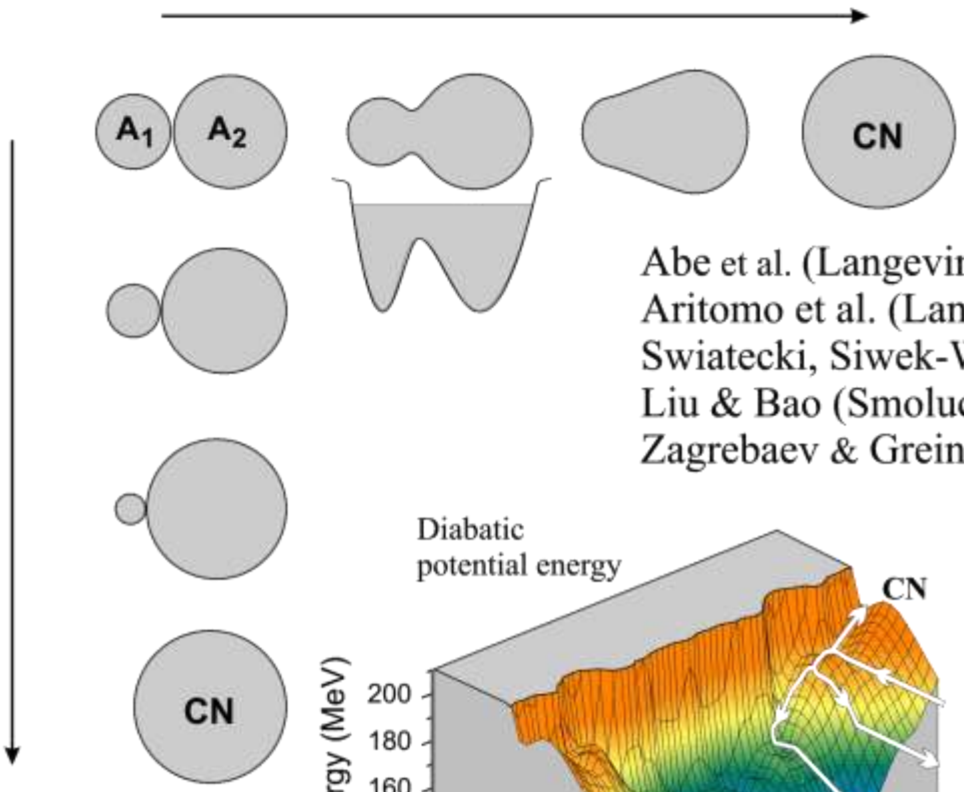


Two (quite opposite) theoretical approaches for calculation of CN formation

- overlapped mean fields
- two-center shell model
- adiabatic potential energy

Microscopic basis:

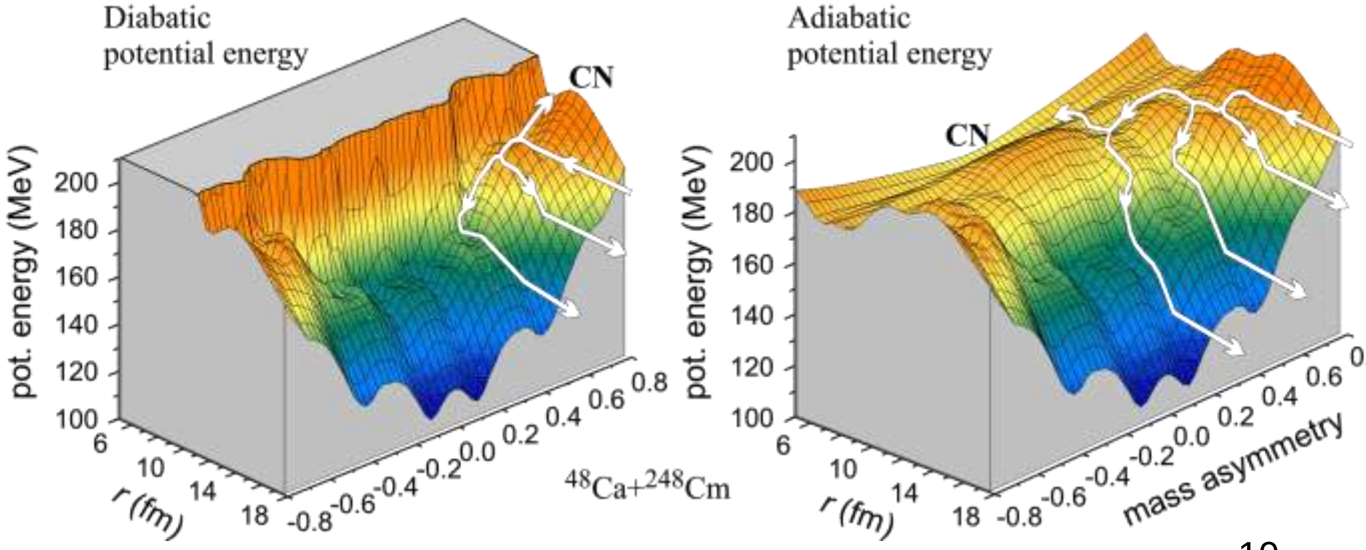
- time-tependent Schrödinger equation for single particle wave functions *(Zagrebav, Samarin, Greiner, 2007);*
- TDHF calculations *(Umar, Oberacker, Maruhn, 2008)*



- two individual (frozen) nuclei
- isolated mean fields
- nucleons jump from A_1 to A_2

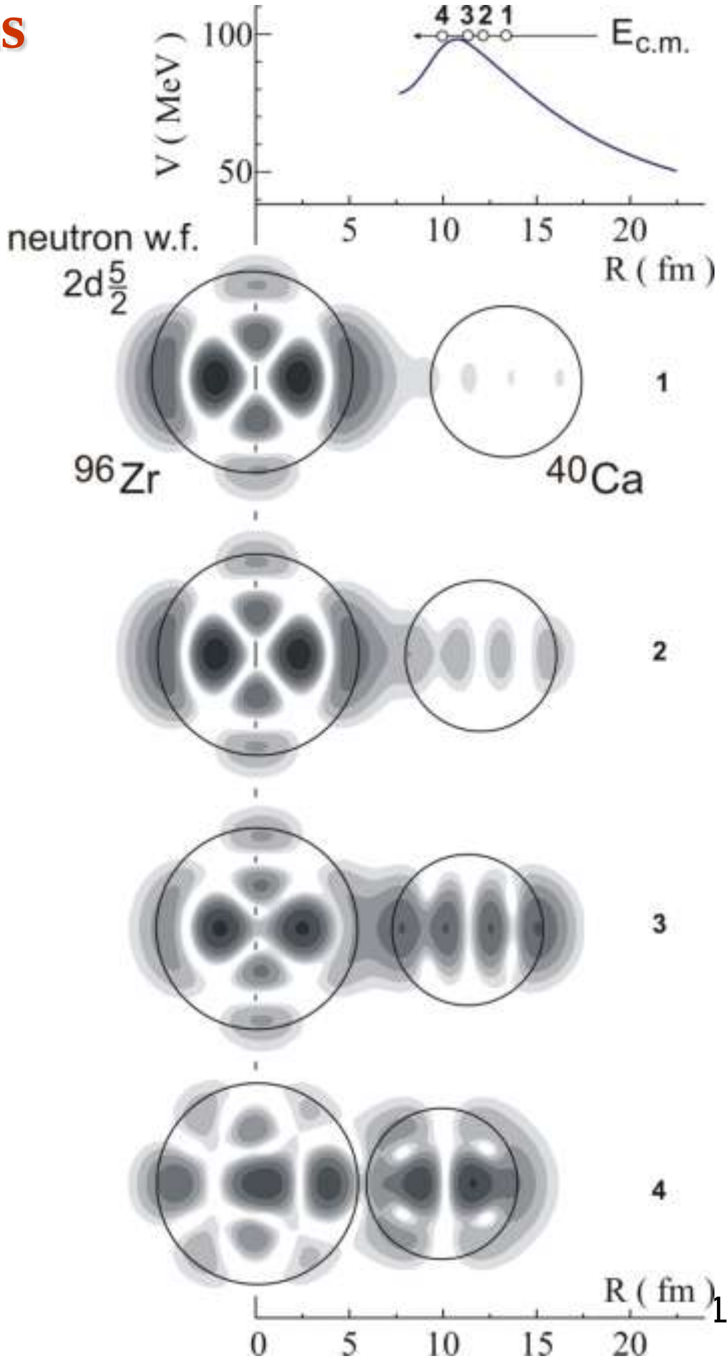
Adamian et al.
Nasirov et al.
Z-Q. Feng et al.

Abe et al. (Langevin eqs.)
Aritomo et al. (Langevin eqs.)
Swiatecki, Siwek-Wilczynska et al. (Fusion by Difusion)
Liu & Bao (Smoluchowski eqs.)
Zagrebaev & Greiner (Langevin type eqs.)



What is behavior of valence nucleons at near-barrier collisions of HI ?

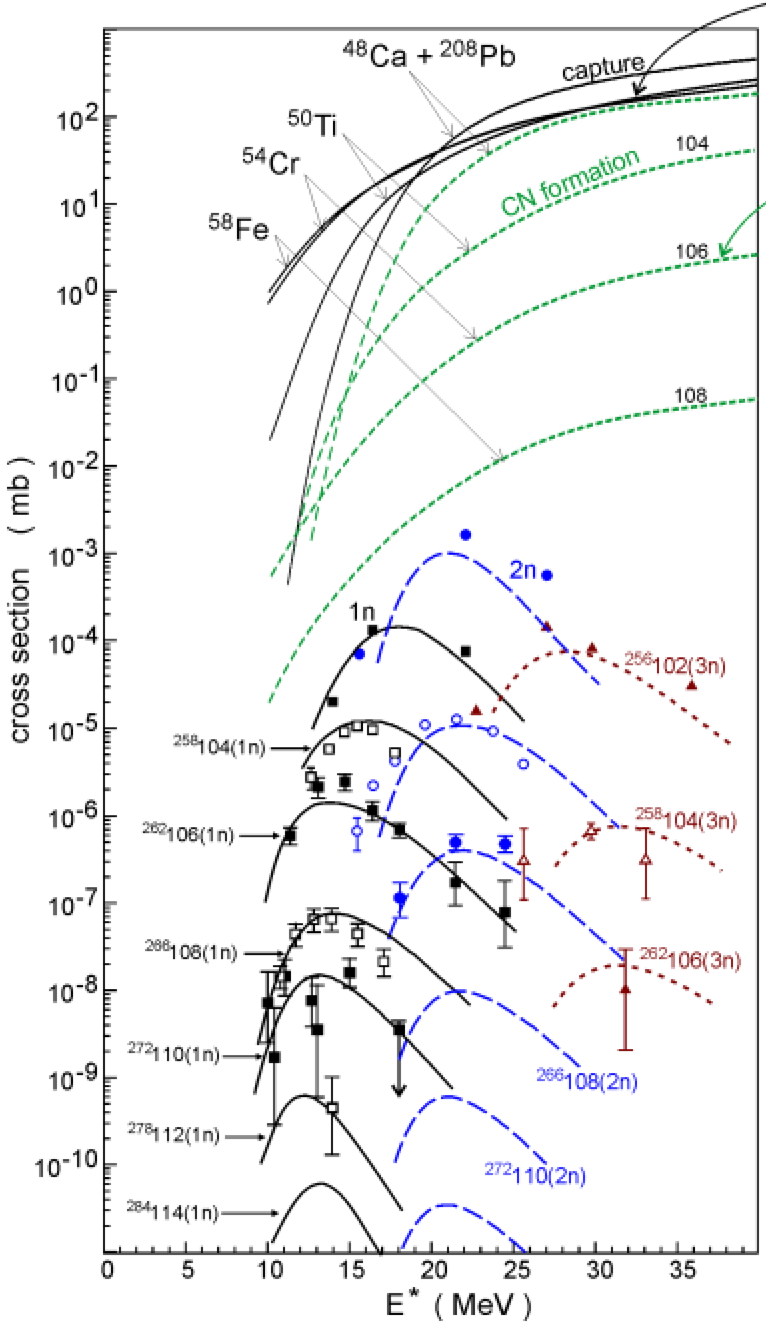
(Zagrebaev, Samarin and Greiner, PRC 2007)



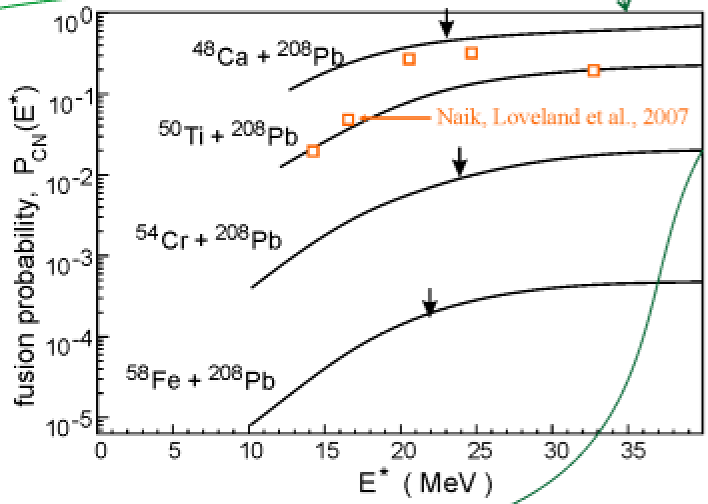
Time-dependent Schrödinger equation shows that at low-energy collisions nucleons do not “jump” from one nucleus to another.
Wave functions of valence nucleons follow the ***two-center molecular states*** spreading over both nuclei.

Two-Center Shell Model +
Adiabatic Potential Energy Surface +
→ **Transport (Langevin type) Equations of Motion**
are appropriate for description
of low-energy nucleon rearrangement

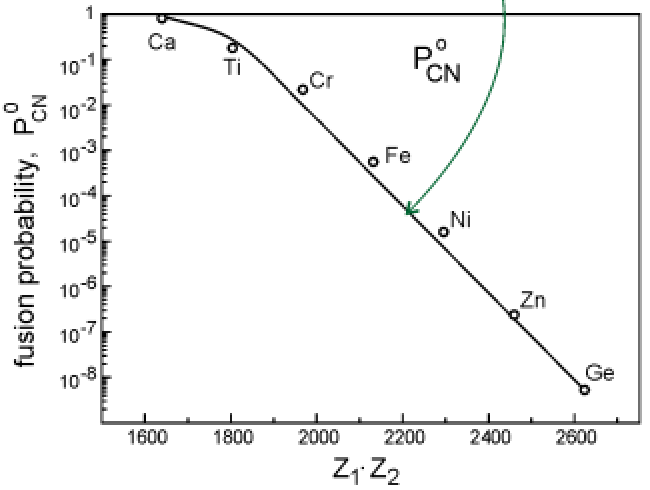
CN formation probability in cold fusion reactions



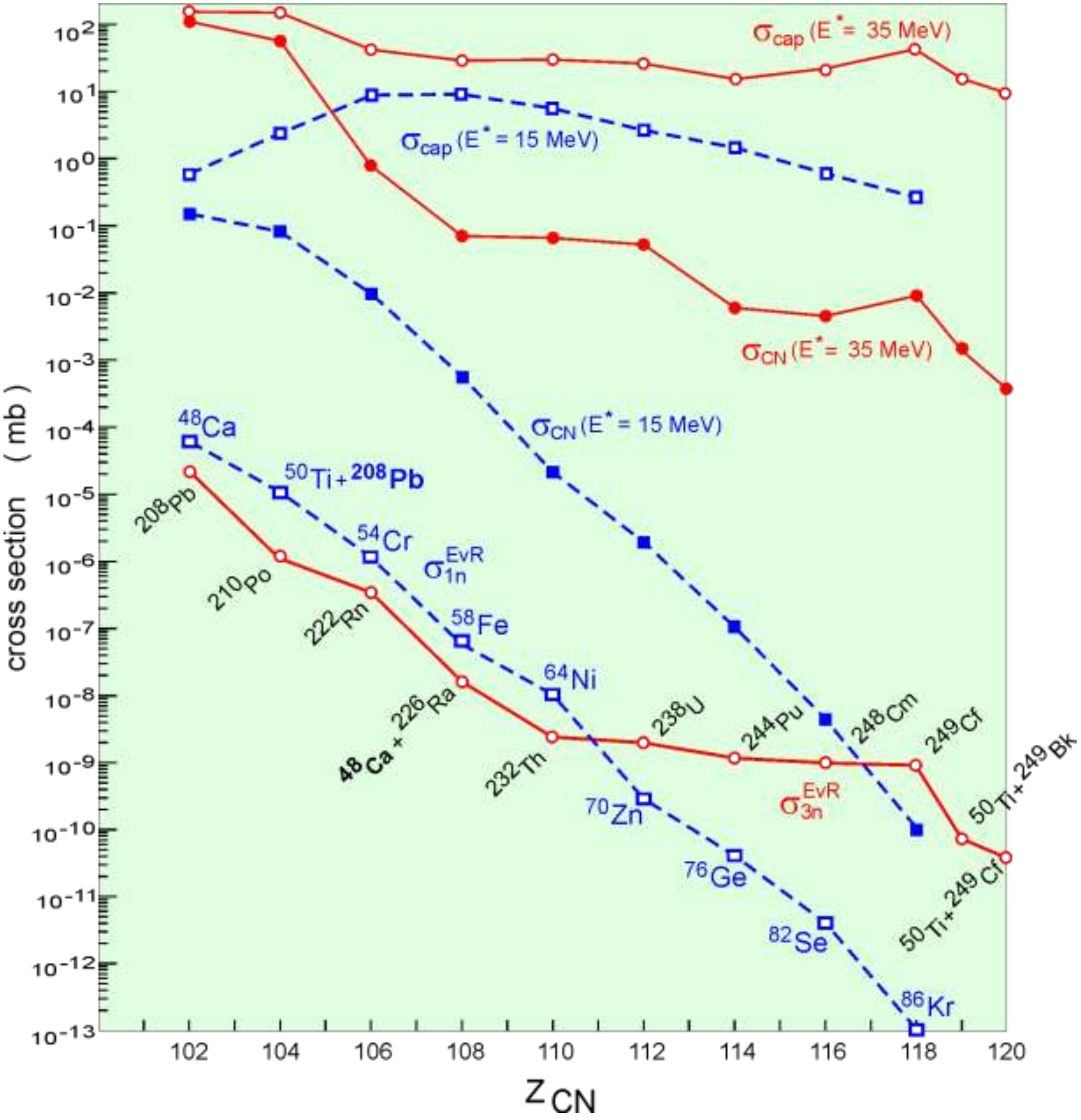
$$\sigma_{ER}^{xn}(E) = \underbrace{\overbrace{\frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\cdot P_{cont}(E,\ell)}\cdot P_{CN}(E^*,\ell)}\cdot P_{xn}(E^*,\ell)$$



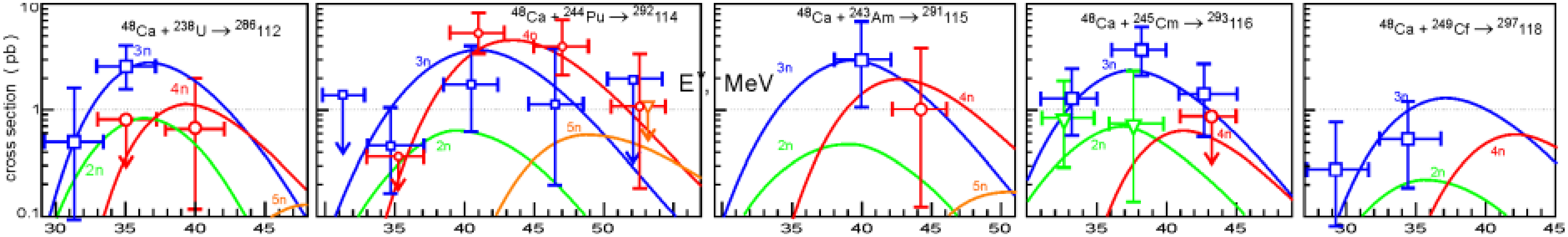
$$P_{CN}(E^*,\ell) = \frac{P_{CN}^{0}}{1+\exp\left[\frac{E_B^* - E_{int}^*(\ell)}{\Delta}\right]}, \quad P_{CN}^{0}\ (Z_1 \cdot Z_2) \approx \frac{1}{1+\exp\left[\frac{Z_1 Z_2 - 1760}{45}\right]}$$

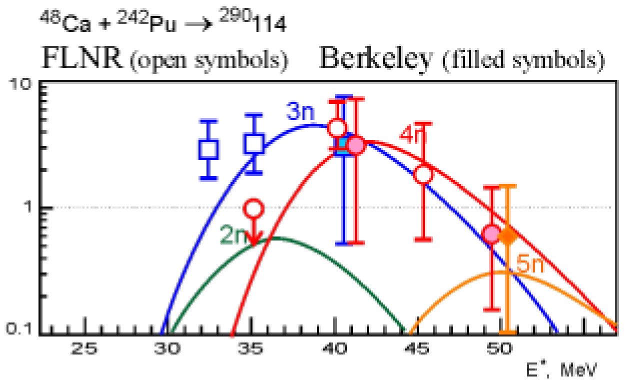


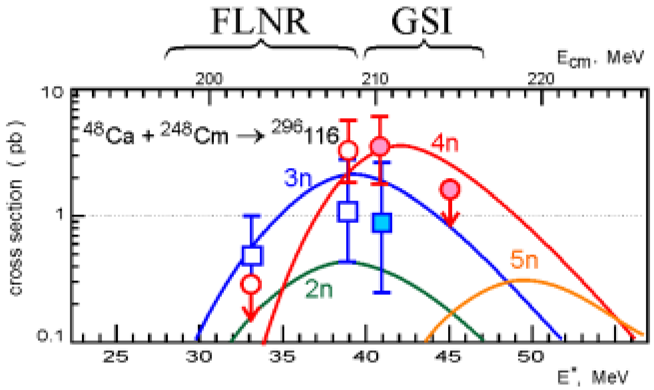
“Cold” and “Hot” synthesis of SHE



Predictive power of the theory for the hot fusion reactions







looks quite impressive, but…

System of coupled Langevin type Equations of Motion

Variables: $\{R, \theta, \varphi_1, \varphi_2, \beta_1, \beta_2, \eta_Z, \eta_N\}$

$$\frac{dR}{dt} = \frac{p_R}{\mu_R}$$

$$\frac{d\vartheta}{dt} = \frac{\ell}{\mu_R R^2}$$

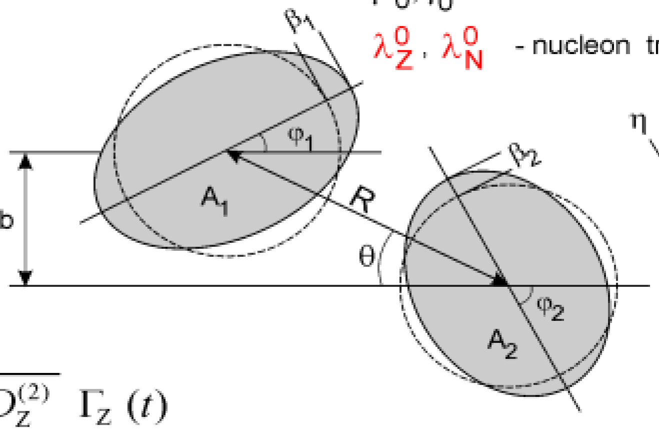
$$\frac{d\varphi_1}{dt} = \frac{L_1}{\Im_1}, \quad \frac{d\varphi_2}{dt} = \frac{L_2}{\Im_2}$$

$$\frac{d\beta_1}{dt} = \frac{p_{\beta 1}}{\mu_{\beta 1}}$$

$$\frac{d\beta_2}{dt} = \frac{p_{\beta 2}}{\mu_{\beta 2}}$$

$$\frac{d\eta_Z}{dt} = \frac{2}{Z_{CN}} D_Z^{(1)} + \frac{2}{Z_{CN}} \sqrt{D_Z^{(2)}} \; \Gamma_Z(t)$$

$$\frac{d\eta_N}{dt} = \frac{2}{N_{CN}} D_N^{(1)} + \frac{2}{N_{CN}} \sqrt{D_N^{(2)}} \; \Gamma_N(t)$$



Most uncertain parameters:

μ_0, γ_0 - nuclear viscosity and friction,

λ_Z^0, λ_N^0 - nucleon transfer rate

$$\eta = \frac{A_1 - A_2}{A_1 + A_2}$$

$$\eta_Z = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$\eta_N = \frac{N_1 - N_2}{N_1 + N_2}$$

$$\lambda_Z^0 = \lambda_N^0 = \frac{\lambda^0}{2}$$

$$\frac{dp_R}{dt} = -\frac{\partial V}{\partial R} + \frac{\ell^2}{\mu_R R^3} + \left(\frac{\ell^2}{2\mu_R^2 R^2} + \frac{p_R^2}{2\mu_R^2}\right)\frac{\partial \mu_R}{\partial R} + \frac{p_{\beta 1}^2}{2\mu_{\beta 1}^2}\frac{\partial \mu_{\beta 1}}{\partial R} + \frac{p_{\beta 2}^2}{2\mu_{\beta 2}^2}\frac{\partial \mu_{\beta 2}}{\partial R} - \gamma_R \frac{p_R}{\mu_R} + \sqrt{\gamma_R T}\,\Gamma_R(t)$$

$$\frac{d\ell}{dt} = -\frac{\partial V}{\partial \vartheta} - \gamma_{\rm tang}\left(\frac{\ell}{\mu_R R} - \frac{L_1}{\Im_1} a_1 - \frac{L_2}{\Im_2} a_2\right) R + \sqrt{\gamma_{\rm tang}\, T}\,\Gamma_{\rm tang}(t)$$

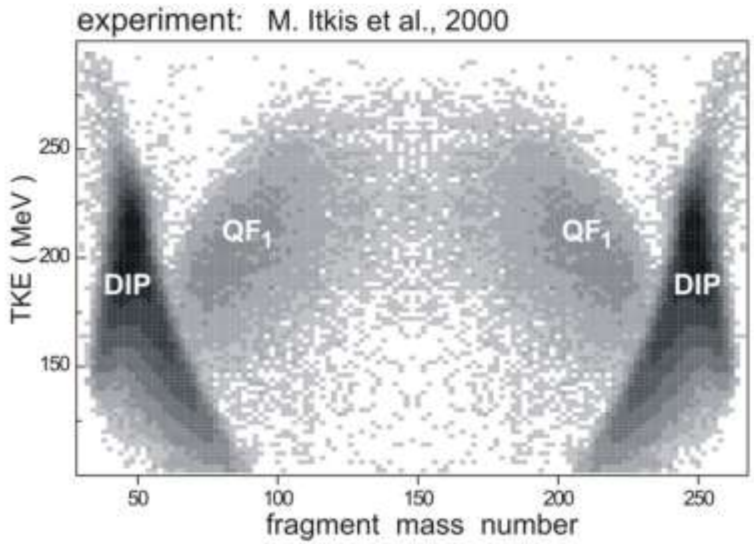
$$\frac{dL_1}{dt} = -\frac{\partial V}{\partial \varphi_1} + \gamma_{\rm tang}\left(\frac{\ell}{\mu_R R} - \frac{L_1}{\Im_1} a_1 - \frac{L_2}{\Im_2} a_2\right) a_1 - \frac{a_1}{R}\sqrt{\gamma_{\rm tang}\, T}\,\Gamma_{\rm tang}(t)$$

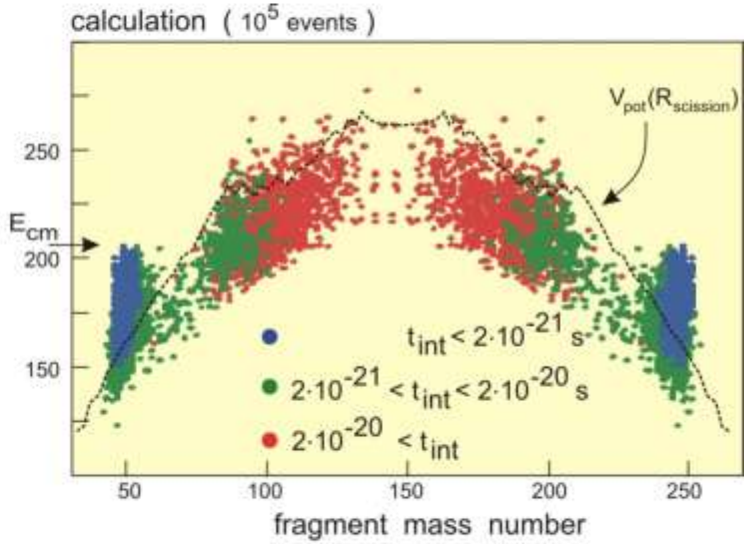
$$\frac{dL_2}{dt} = -\frac{\partial V}{\partial \varphi_2} + \gamma_{\rm tan}\left(\frac{\ell}{\mu_R R} - \frac{L_1}{\Im_1} a_1 - \frac{L_2}{\Im_2} a_2\right) a_2 - \frac{a_2}{R}\sqrt{\gamma_{\rm tang}\, T}\,\Gamma_{\rm tang}(t)$$

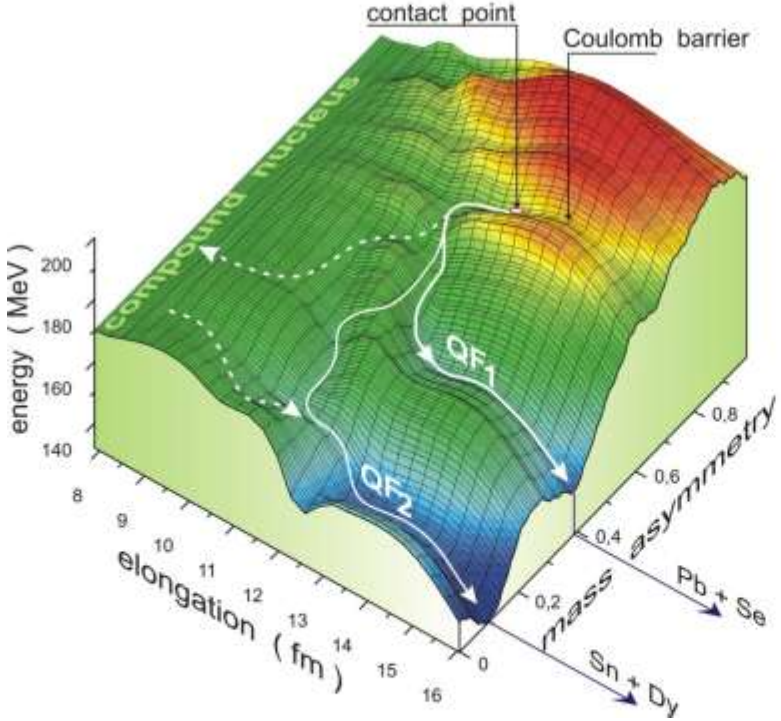
$$\frac{dp_{\beta 1}}{dt} = -\frac{\partial V}{\partial \beta_1} + \frac{p_{\beta 1}^2}{2\mu_{\beta 1}^2}\frac{\partial \mu_{\beta 1}}{\partial \beta_1} + \frac{p_{\beta 2}^2}{2\mu_{\beta 2}^2}\frac{\partial \mu_{\beta 2}}{\partial \beta_1} + \left(\frac{\ell^2}{2\mu_R^2 R^2} + \frac{p_R^2}{2\mu_R^2}\right)\frac{\partial \mu_R}{\partial \beta_1} - \gamma_\beta \frac{p_{\beta 1}}{\mu_{\beta 1}} + \sqrt{\gamma_{\beta 1}\, T}\,\Gamma_{\beta 1}(t)$$

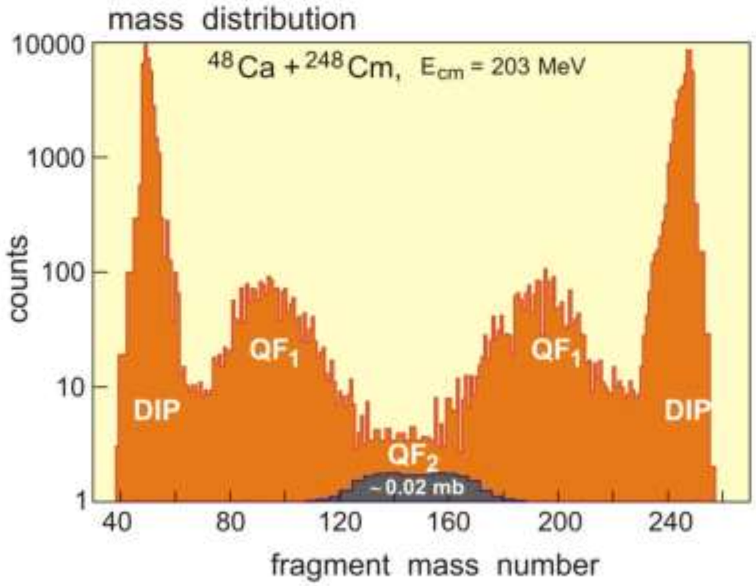
$$\frac{dp_{\beta 2}}{dt} = -\frac{\partial V}{\partial \beta_2} + \frac{p_{\beta 1}^2}{2\mu_{\beta 1}^2}\frac{\partial \mu_{\beta 1}}{\partial \beta_2} + \frac{p_{\beta 2}^2}{2\mu_{\beta 2}^2}\frac{\partial \mu_{\beta 2}}{\partial \beta_2} + \left(\frac{\ell^2}{2\mu_R^2 R^2} + \frac{p_R^2}{2\mu_R^2}\right)\frac{\partial \mu_R}{\partial \beta_2} - \gamma_\beta \frac{p_{\beta 2}}{\mu_{\beta 2}} + \sqrt{\gamma_{\beta 2}\, T}\,\Gamma_{\beta 2}(t)$$

^{48}Ca + ^{248}Cm collisions at E_{cm} = 203 MeV (quasi-fission)



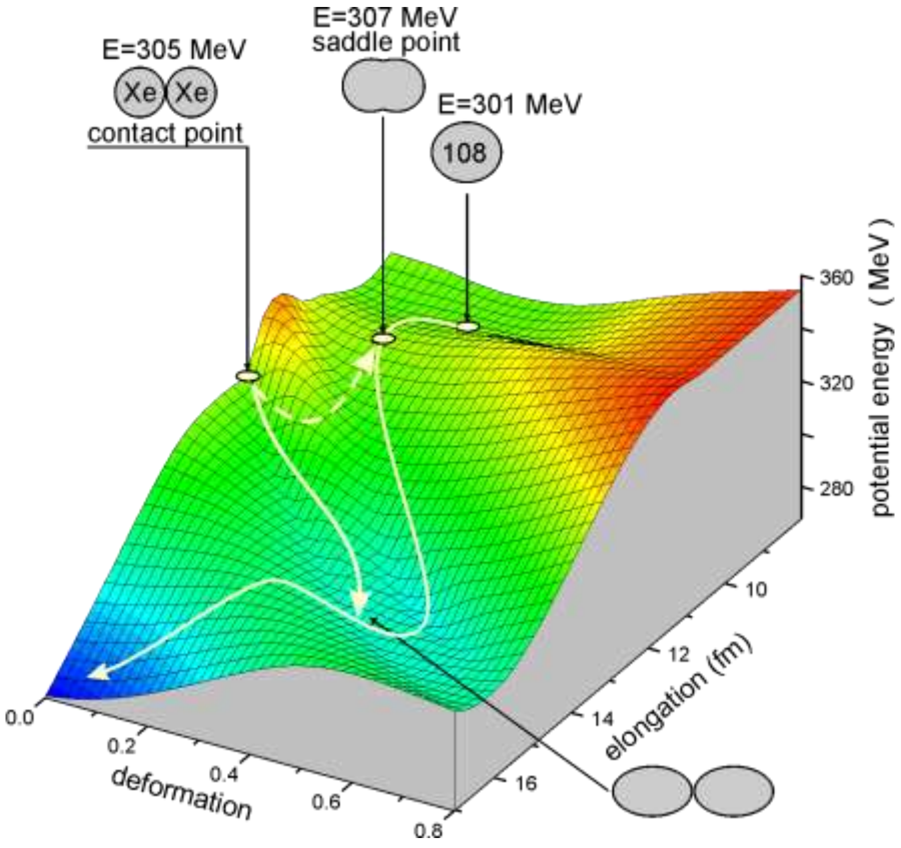


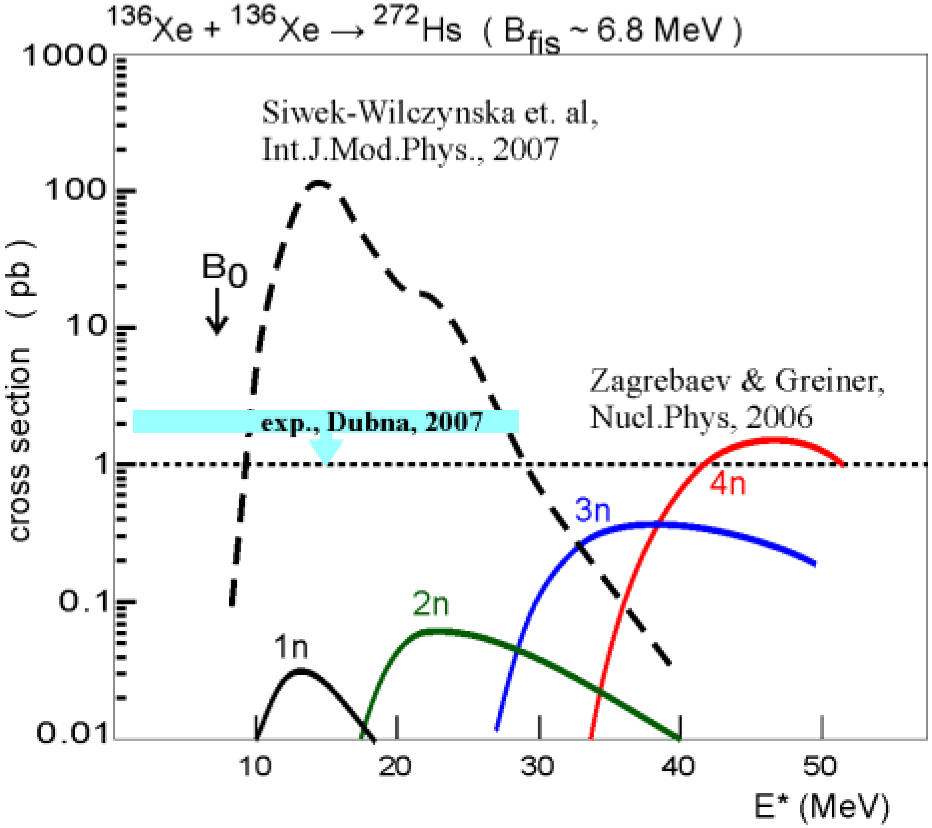




Fusion of “fission fragments”: ^{136}Xe + ^{136}Xe → ^{272}108

(theoretical troubles)





Synthesis of SHE in **fusion reactions**
(theoretical problems to be solved)

1. **Capture (contact) reaction stage**
   standard CC calculation:
   → no problems with predictions of capture cross sections
   (within factor 2 or 3)

2. **CN formation stage**
   two-center shell model and transport equations:
   - explicit potential energy surface?
   - appropriate degrees of freedom and equations of motion?
   - nuclear viscosity?
   - nucleon transfer rate?
   → uncertainty factor may vary from 10 to 1000

3. **Cooling stage**
   standard Statistical Model calculation:
   - collective enhancement factor in level density?
   - damping of shell corrections and fission barrier?
   - unknown fission barriers for SH nuclei?
   → uncertainty factor is about 10

Synthesis of SHE in **transfer reactions:**
Which models are on the market?

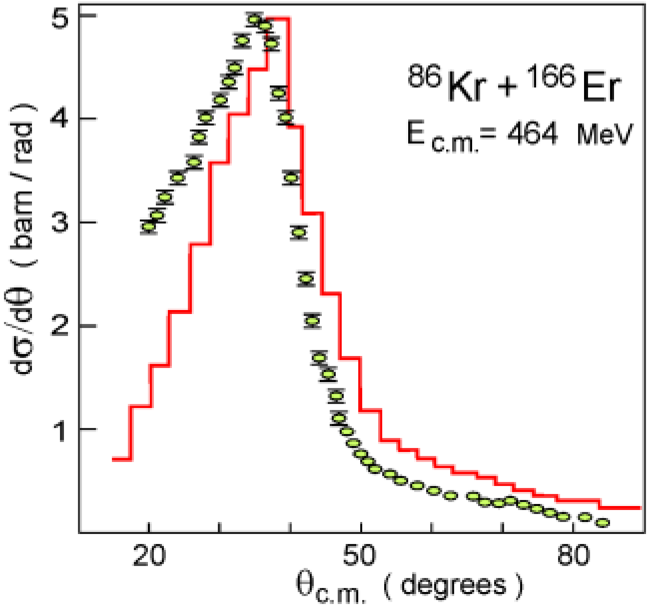
1. **Semiclassical Model: code GRAZING**
   A. Winther, 2005
   Good agreement with experiment for few-nucleon transfers
   and quasi-elastic excitations (grazing collisions).
   Does not describe properly deep inelastic scattering and multi-nucleon transfers.

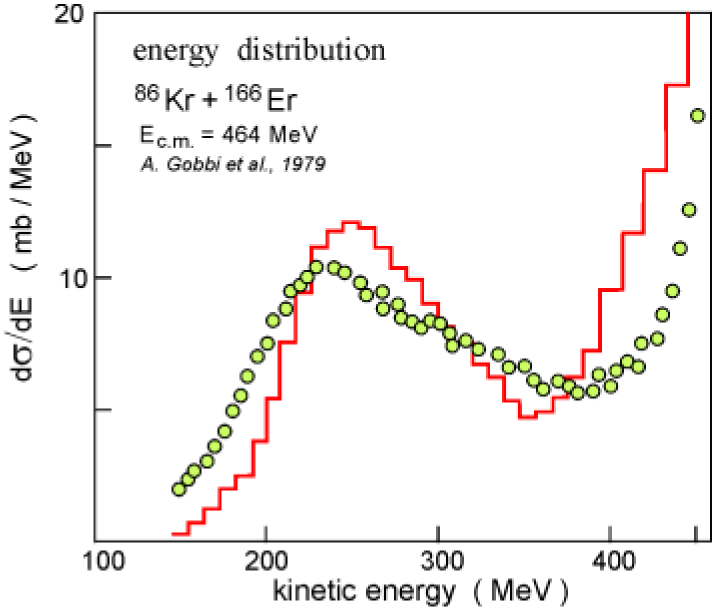
2. **Quantum Molecular Dynamics**
   J. Tian et al., 2008, Z.Q.Feng et al., 2009
   Only 2 or 3 papers on SHE formation have been published so far.

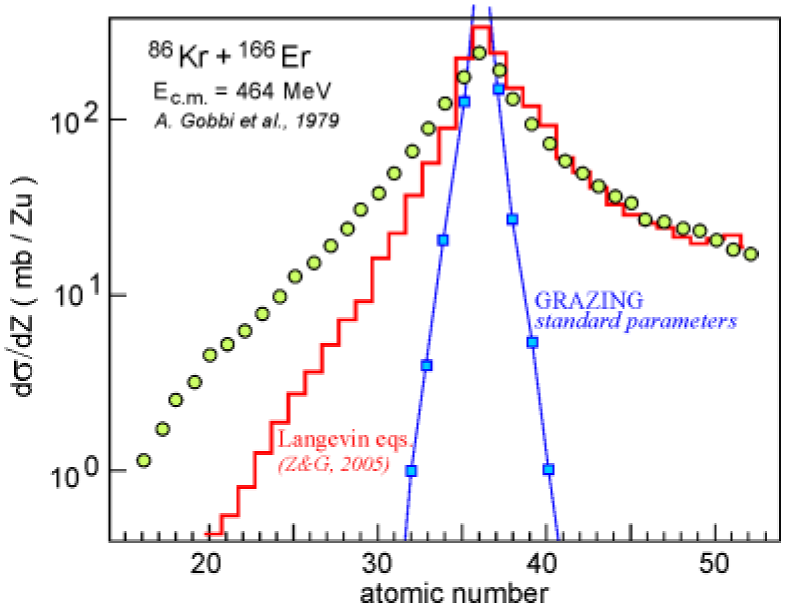
3. **TDHF calculations**
   C. Simenel et al., 2010
   Mostly qualitative results.
   Cross sections for SHE production were not obtained yet.

4. **Macroscopic transport equations**
   Zagrebaev & Greiner, 2005
   Poor description of quasi-elastic scattering and few-nucleon transfers.
   Appropriate description of deep inelastic scattering and multi-nucleon transfers.

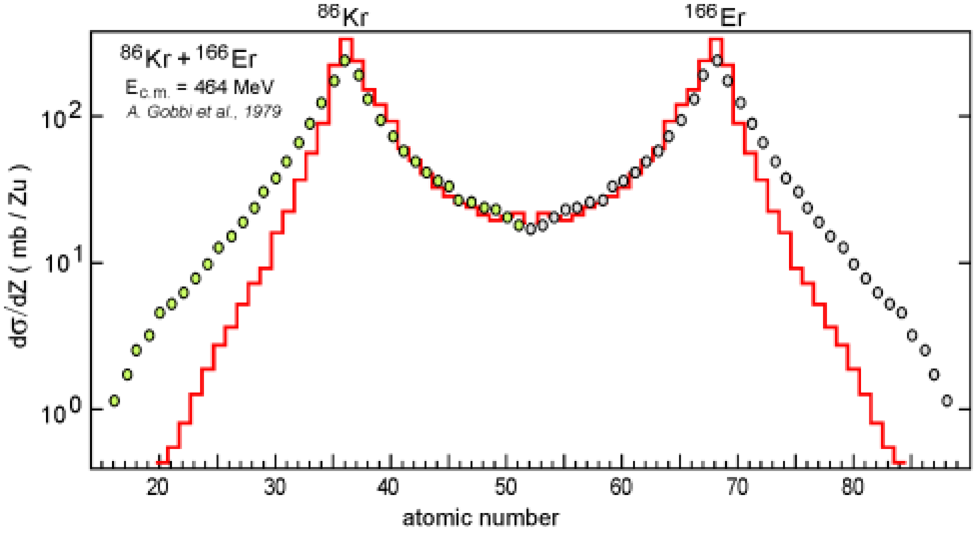
Satisfactory agreement with experiment

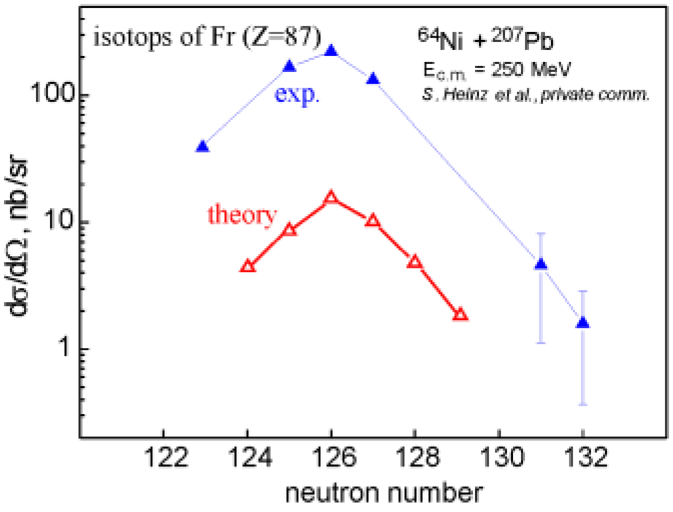


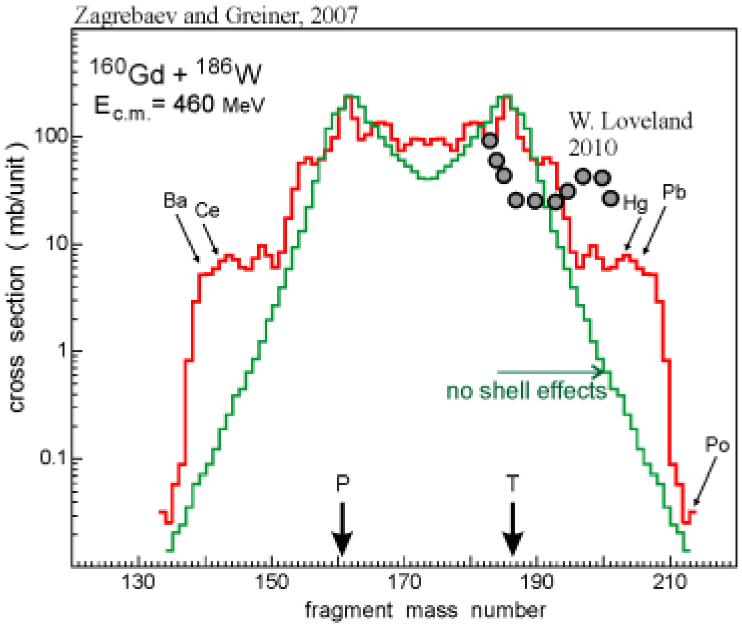


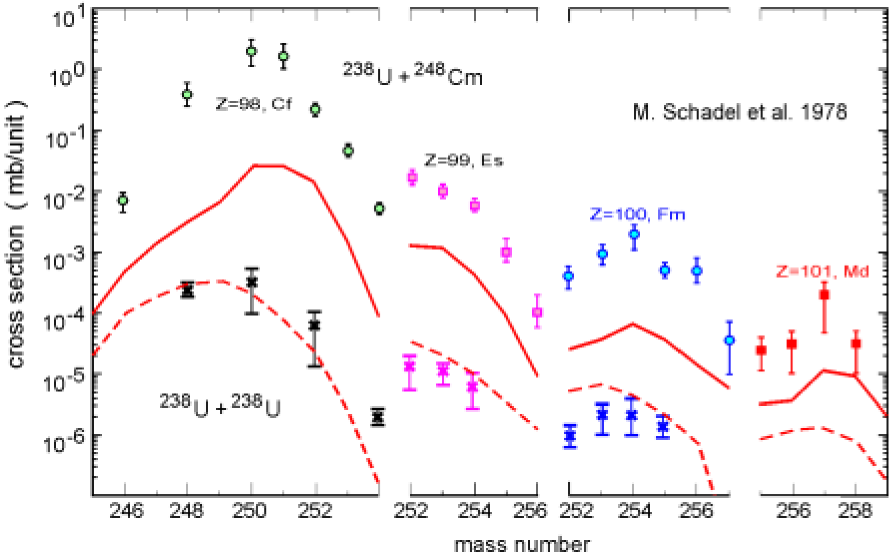


Underestimation of “anti-symmetrizing” dynamics

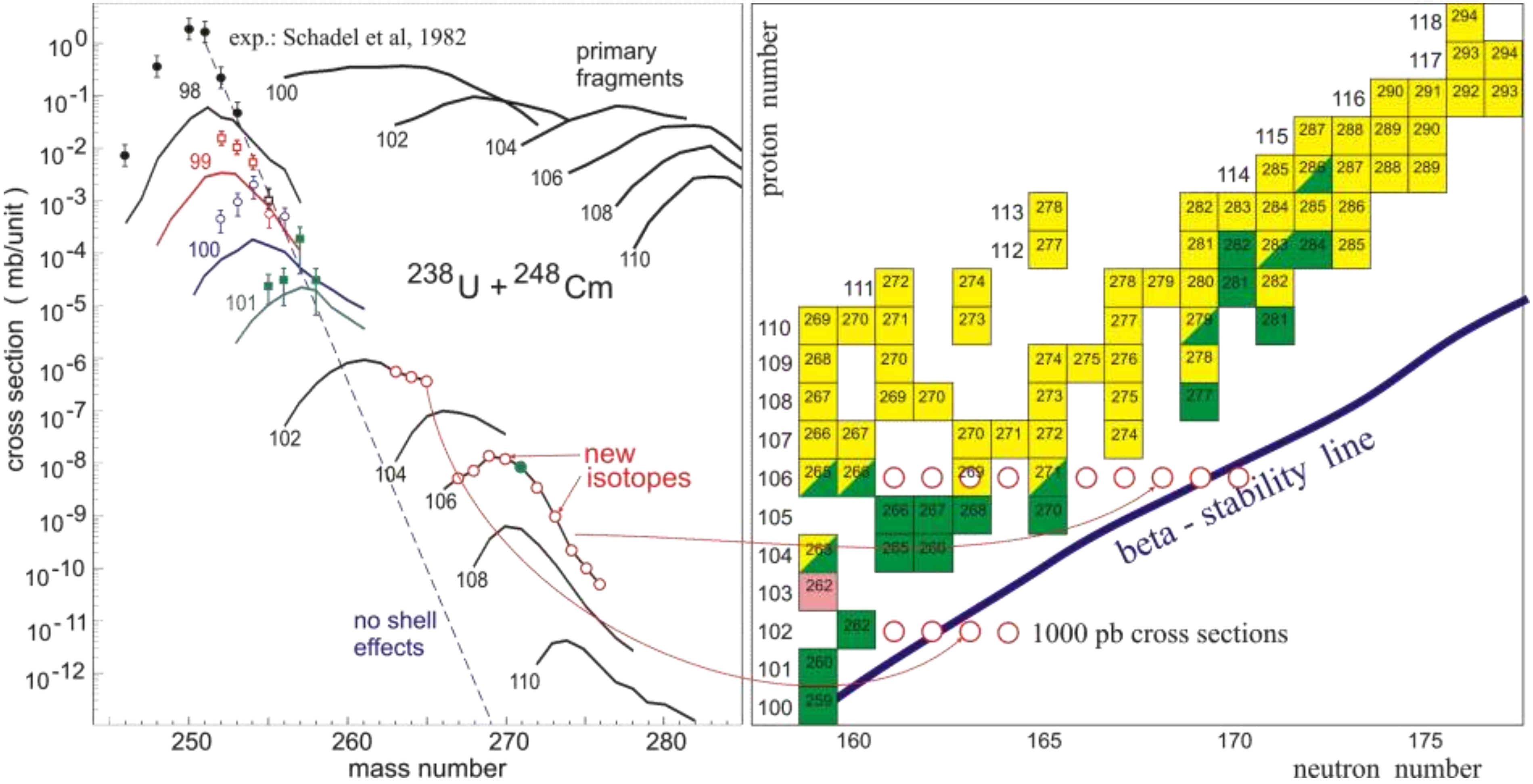








Production of transfermium nuclei along the line of stability looks quite possible



118																		294	
117																		293	294
116																290	291	292	293
115														287	288	289	290		
114													285	286	287	288	289		
113							278				282	283	284	285	286				
112							277				281	282	283	284	285				
111			272		274				278	279	280	281	282						
110	269	270	271		273				277		279		281						
109	268		270				274	275	276		278								
108	267		269	270			273		275		277								
107	266	267			270	271	272		274										
106	265	266	○	○	269	○	271	○	○	○	○	○							
105			266	267	268		270												
104	263		265	266															
103	262																		
102		262	○	○	○	○													
101	260																		
100	259																		

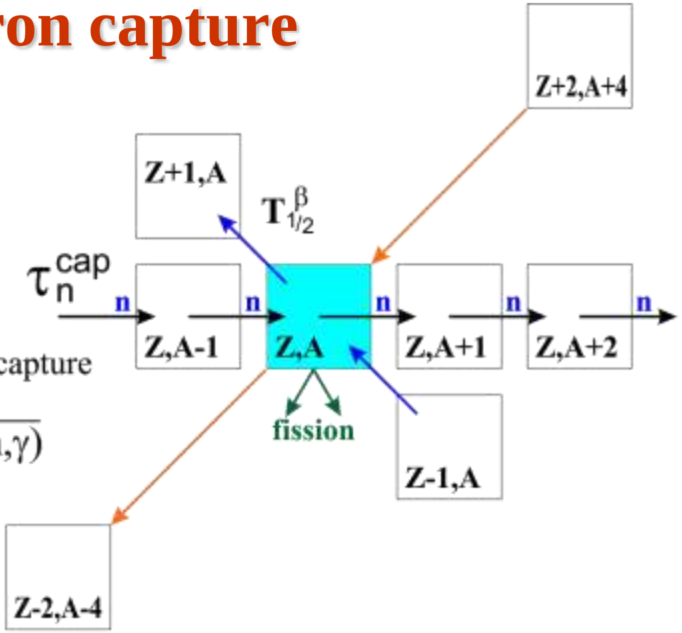
Synthesis of SHE in **transfer reactions**
(theoretical problems to be solved)

1. **Microscopic (and semi-microscopic) models need further development:**
   The models should be applied first to description of numerous experimental data on deep inelastic scattering and multi-nucleon transfers in low energy HI collisions

2. **Macroscopic (classical) approaches:**
   There are several uncertain parameters and quantities:
   - too many important degrees of freedom,
   - explicit adiabatic potential energy surface?
   - appropriate equations of motion?
   - nuclear viscosity?
   - nucleon transfer rate?

3. **Decay of excited heavy (and superheavy) primary fragments:**
   standard Statistical Model calculation:
   - collective enhancement factor in level density?
   - damping of shell corrections and fission barrier?
   - unknown fission barriers for SH nuclei?
   → uncertainty factor is about 10

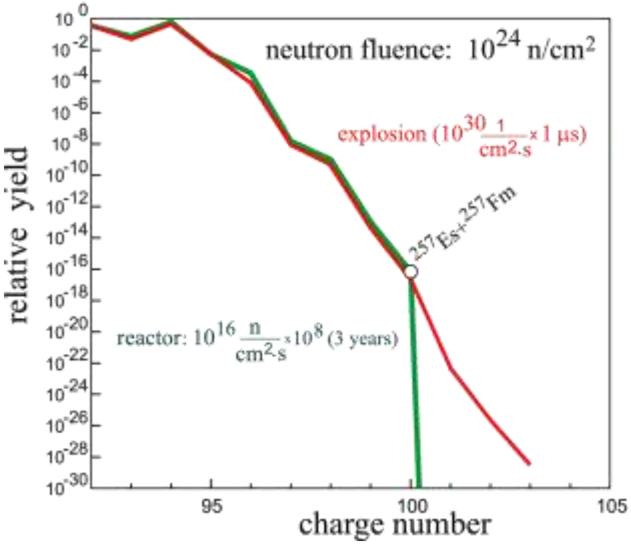
Nucleosynthesis by neutron capture

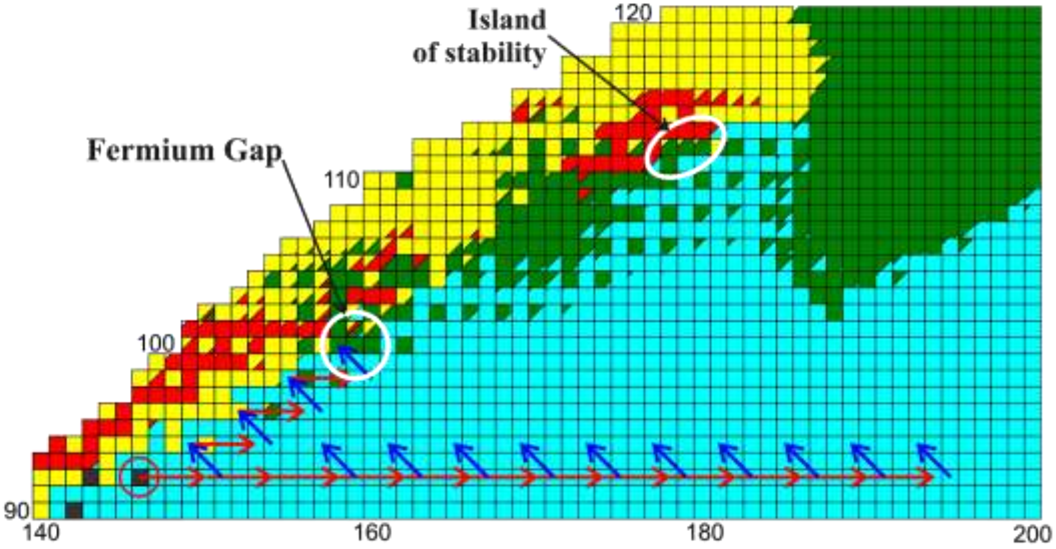


time of neutron capture

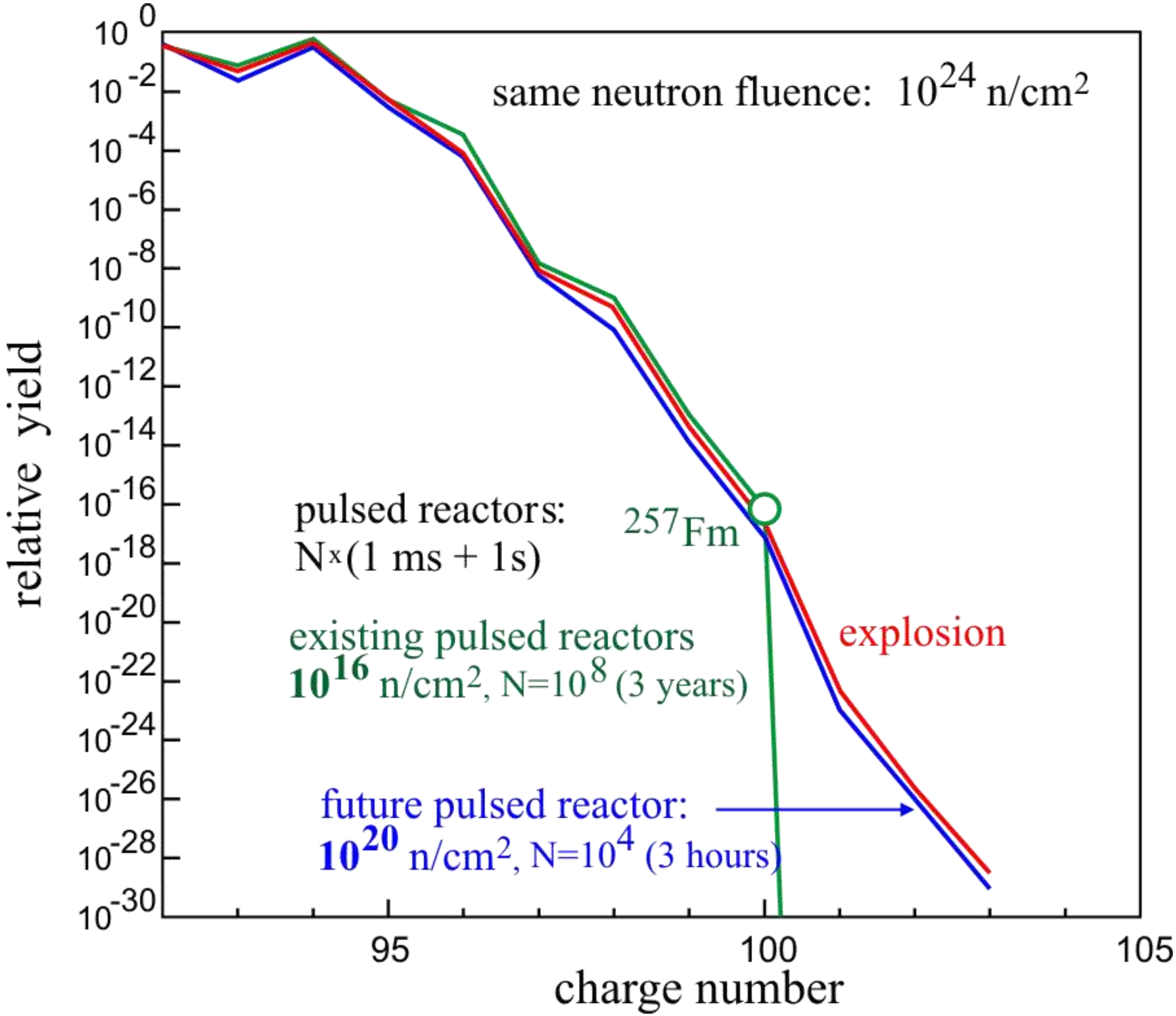
$$\tau^{cap}_{n} = \frac{1}{n_0 \times \sigma(n,\gamma)}$$

$$\frac{dN_{ZA}}{dt} = N_{ZA-1}\, n_0\, \sigma^{n\gamma}_{ZA-1} - N_{ZA}\, n_0\, \sigma^{n\gamma}_{ZA} - N_{ZA}\frac{\ln 2}{T^{\beta}_{ZA}} - N_{ZA}\frac{\ln 2}{T^{\alpha}_{ZA}} - N_{ZA}\frac{\ln 2}{T^{fis}_{ZA}} + N_{Z-1A}\frac{\ln 2}{T^{\beta}_{Z-1A}} + N_{Z+2A+4}\frac{\ln 2}{T^{\alpha}_{Z+2A+4}}$$





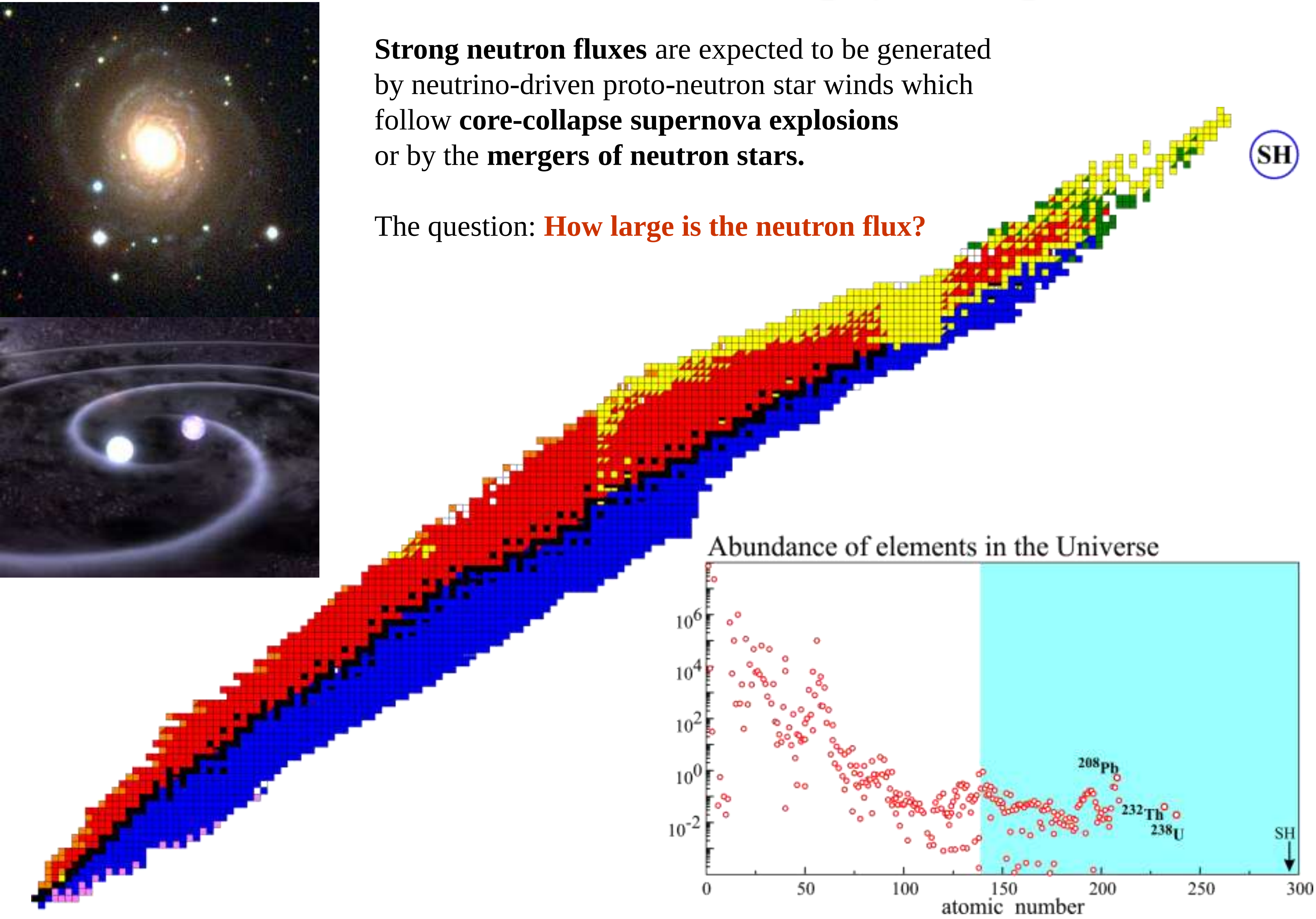
Next generation of pulsed reactors: We need factor 1000 only !



Formation of SH elements in astrophysical r-process

Strong neutron fluxes are expected to be generated by neutrino-driven proto-neutron star winds which follow **core-collapse supernova explosions** or by the **mergers of neutron stars.**

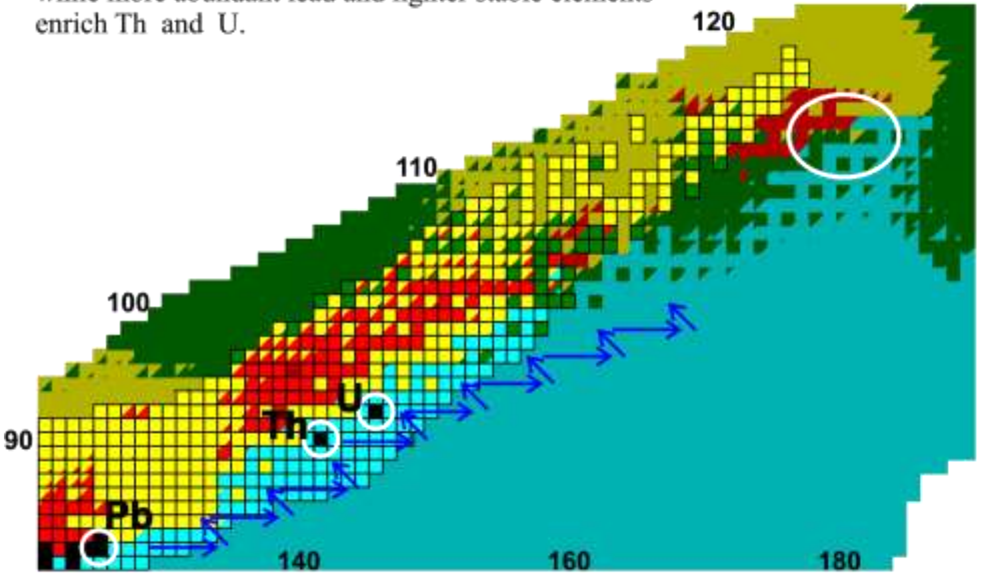
The question: **How large is the neutron flux?**



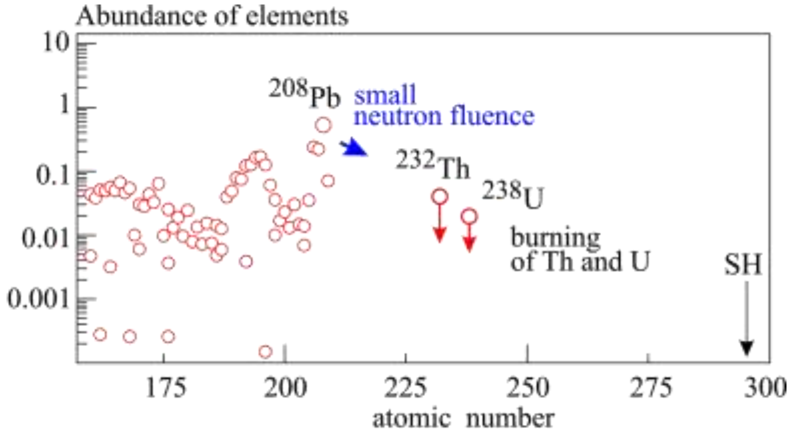
Abundance of elements in the Universe

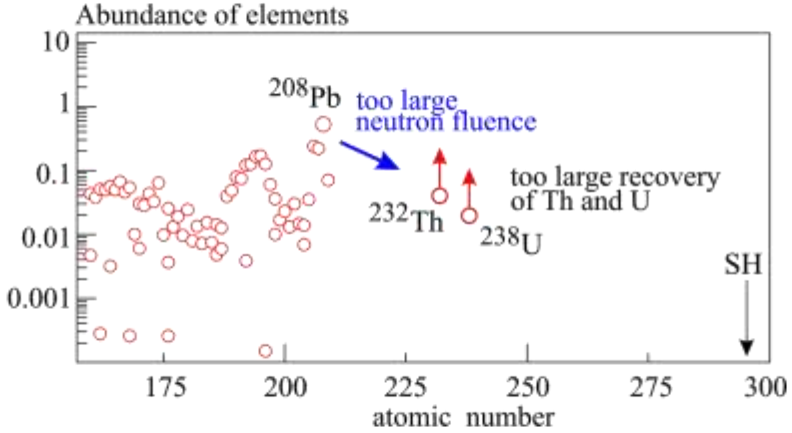
Formation of SH elements in astrophysical r-process: fit of unknown neutron fluence

During neutron irradiation initial Th and U material are depleted transforming to heavier elements and going to fission, while more abundant lead and lighter stable elements enrich Th and U.



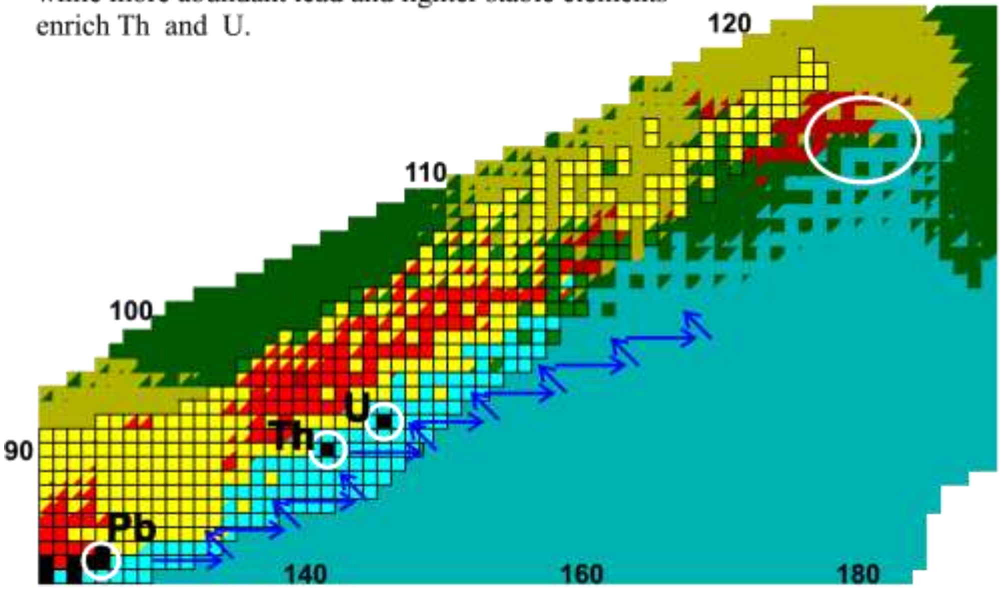
Unknown total neutron fluence is adjusted in such a way that the ratios Th/Pb and U/Pb keep its experimental values.

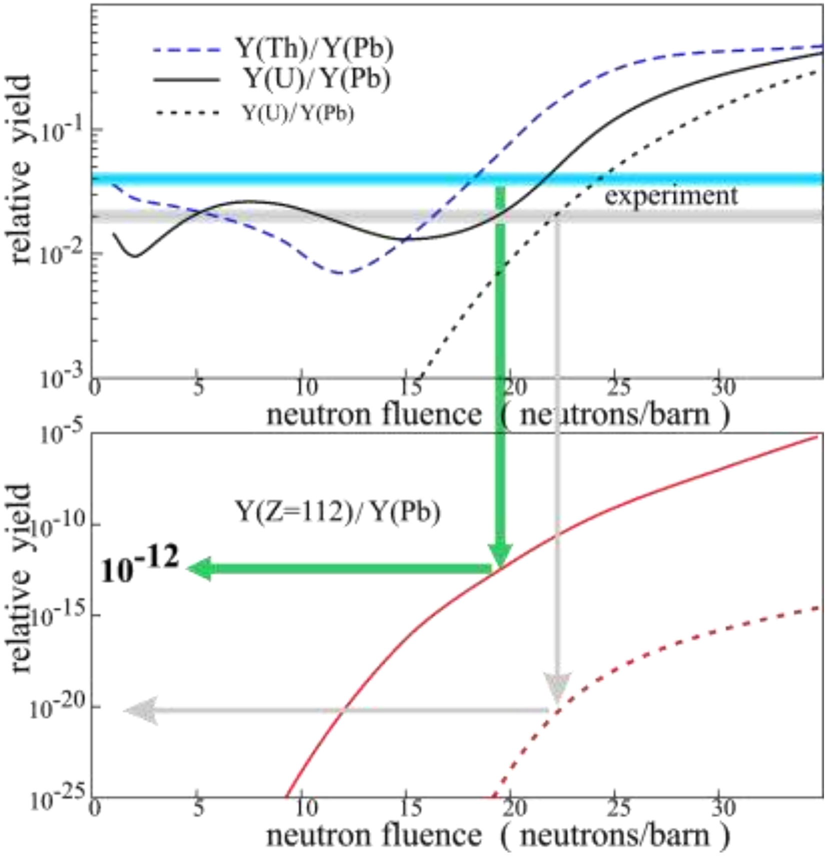




Formation of SH elements in astrophysical r-process

During intensive neutron irradiation initial Th and U material are depleted transforming to heavier elements and going to fission, while more abundant lead and lighter stable elements enrich Th and U.





Unknown total neutron fluence is adjusted in such a way that the ratios Th/Pb and U/Pb keep its experimental values.

For a given neutron fluence one gets the relative yield of SH elements, SH/Pb.

Synthesis of SHE by **neutron capture** in r process (“experimental” problems to be solved)

1. **Equations are well defined.**

2. **Neutron capture cross sections and decay properties of heavy neutron rich nuclei are unknown:**
   - only theoretical estimations,
   - most uncertain are the fission half-lives,
   - beta(-) decay half-lives are also unknown.

3. **Neutron fluence ?**
   - adjusted to reproduce experimental abundances?

SHE experiments

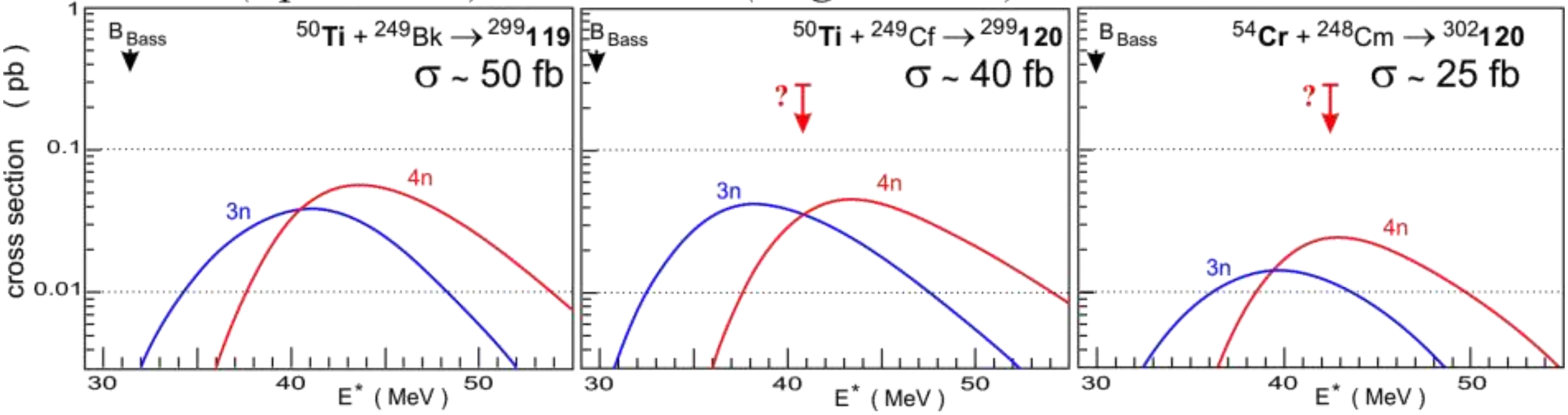
What new could be done within the next few years?

Valeriy Zagrebaev

Flerov Laboratory of Nuclear Reactions, JINR, Dubna

for “Future of Super Heavy Elements”, *May 14, 2012*, Weiltal, Germany

Beyond ^{48}Ca: ^{50}Ti and ^{54}Cr induced fusion reactions


Ti beam:

TASCA (April, 2012)

TASCA (August, 2011)

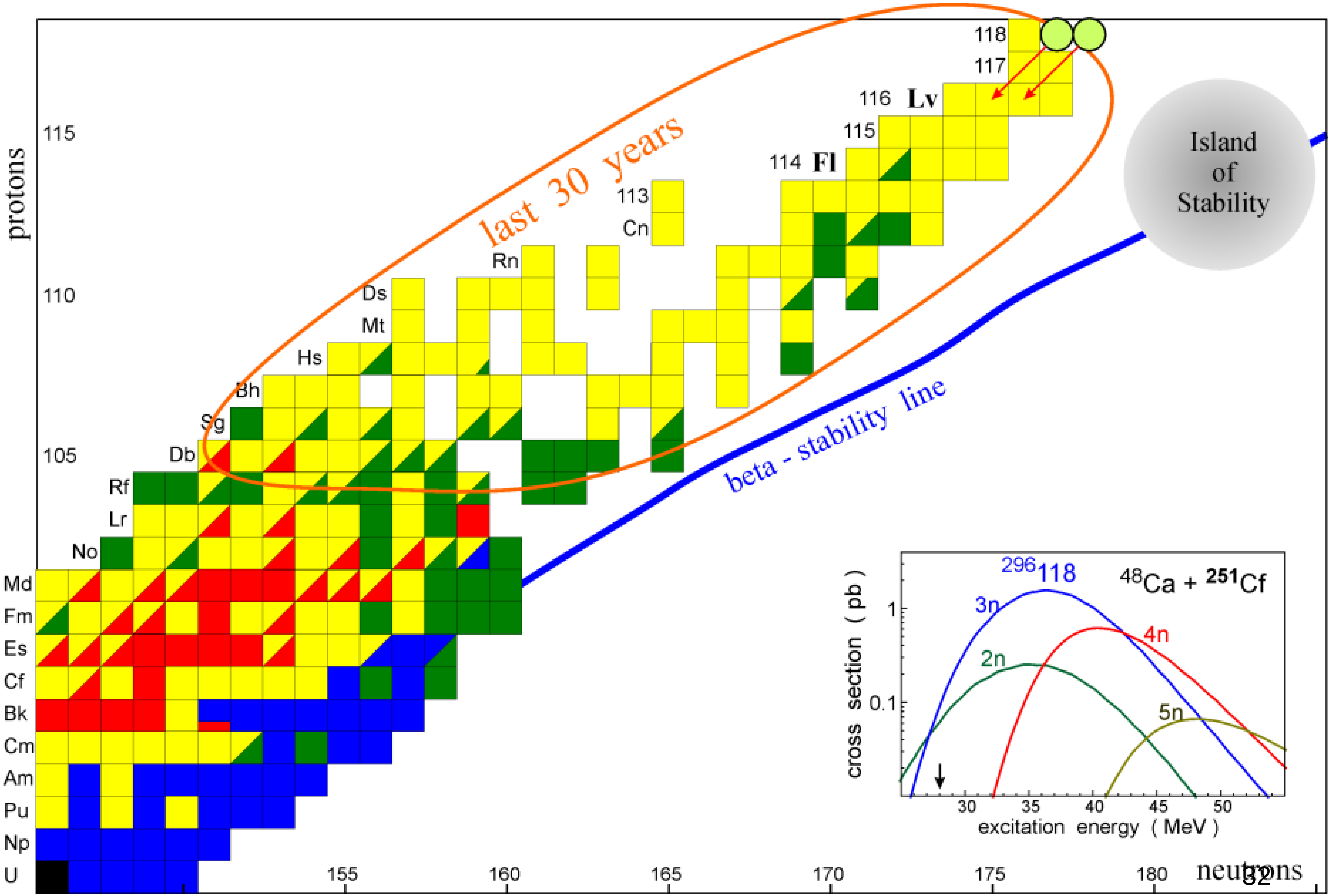
Cr beam:

SHIP (May, 2011)

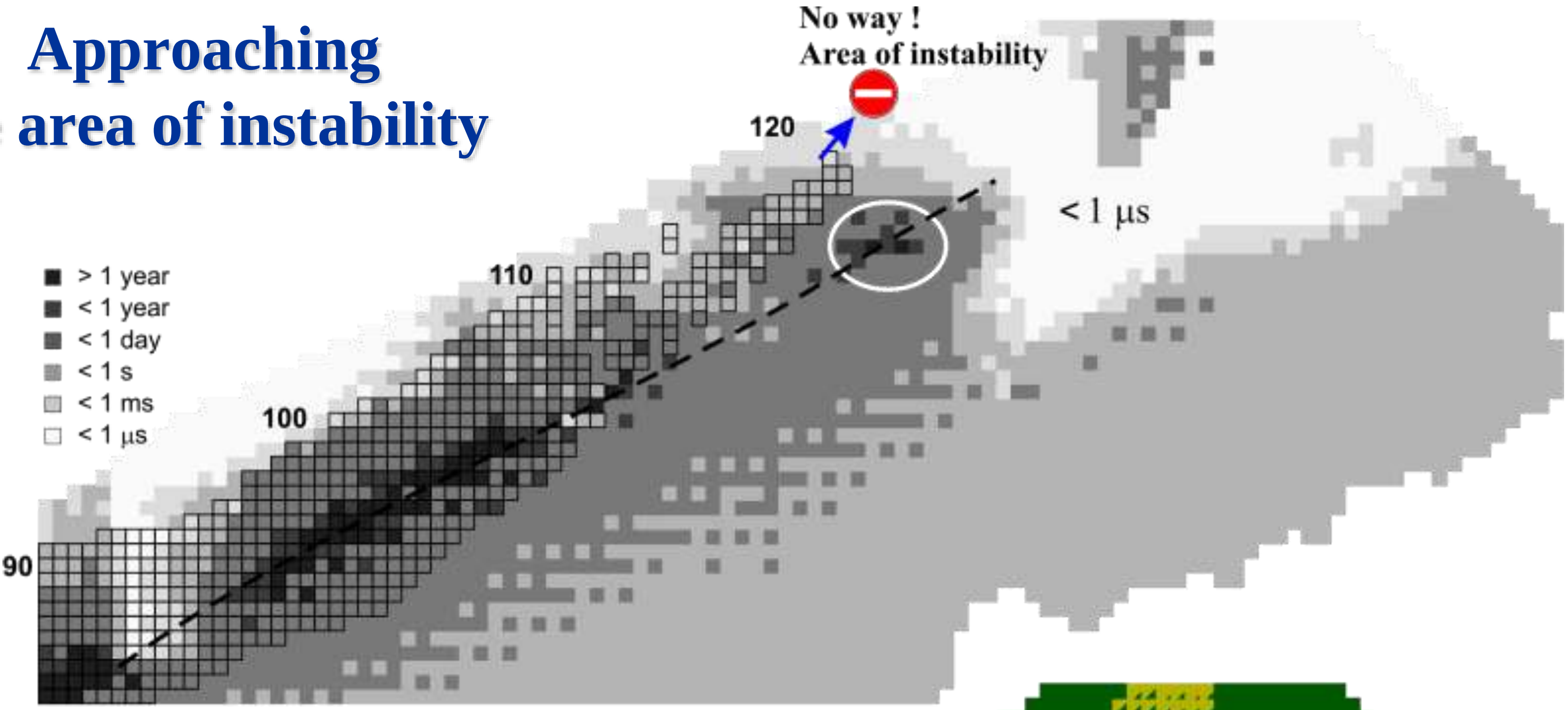
factor $\frac{1}{20}$ as compared to ^{48}Ca

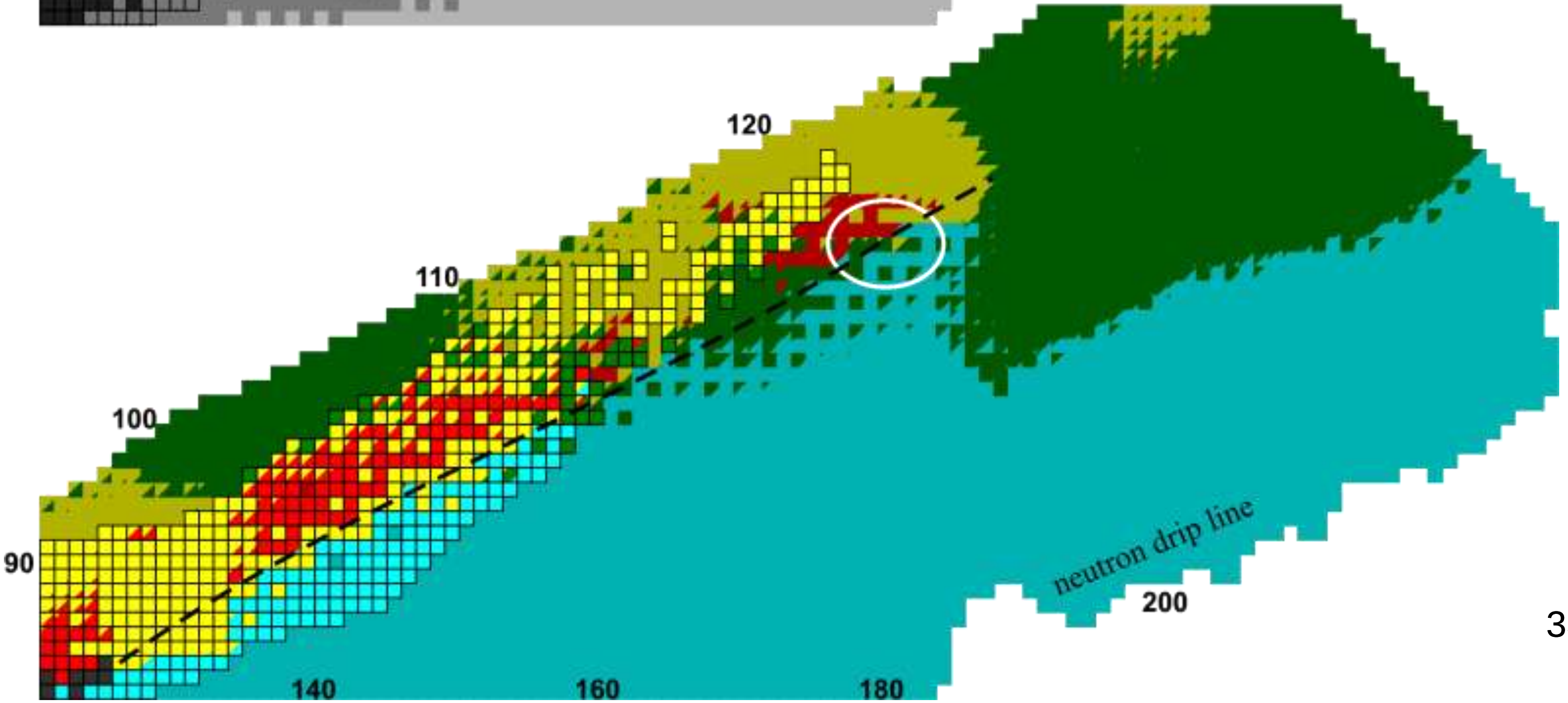
Probably these elements are the last ones
which will be synthesized in the nearest future

We are still far from the island of stabilty

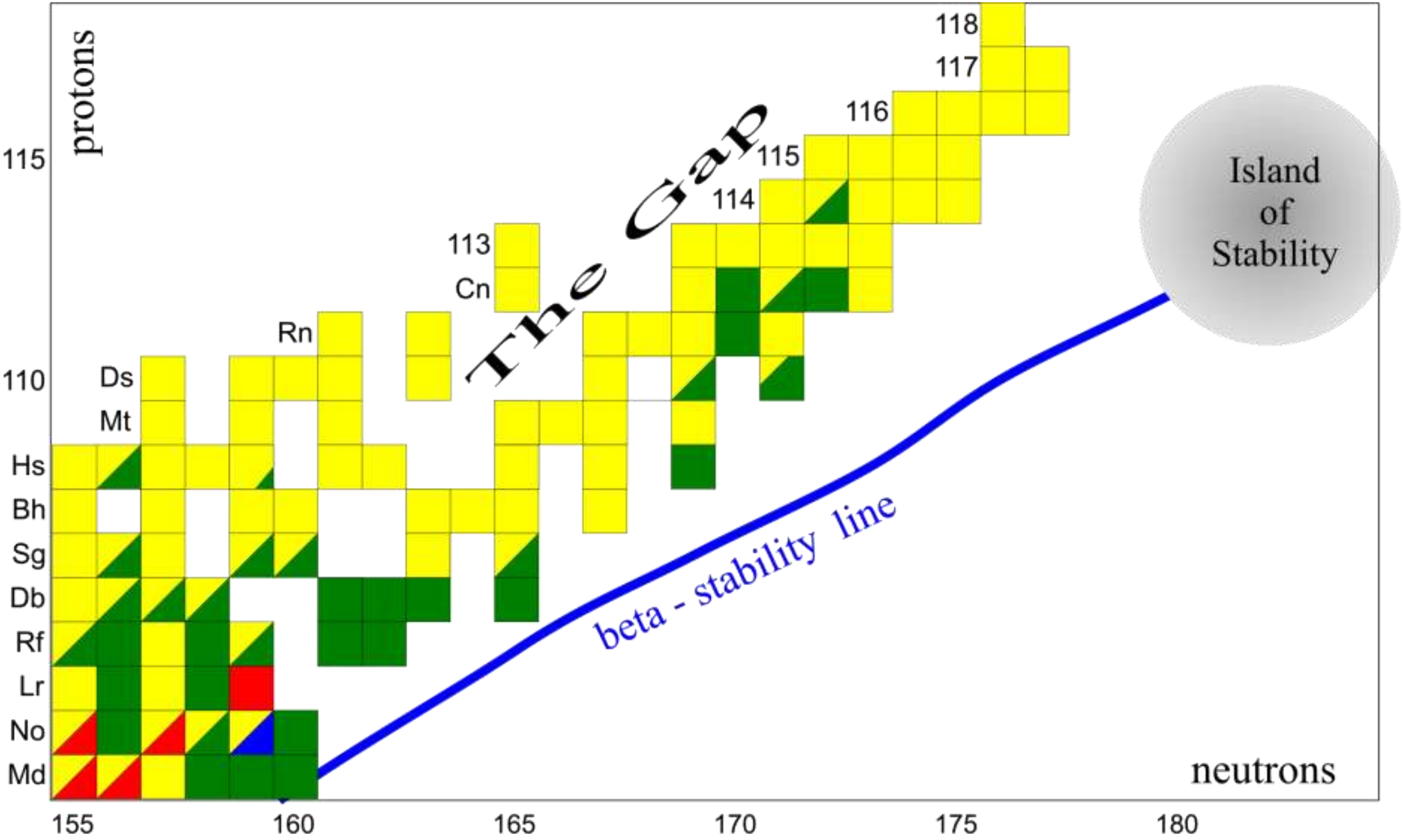


Approaching the area of instability

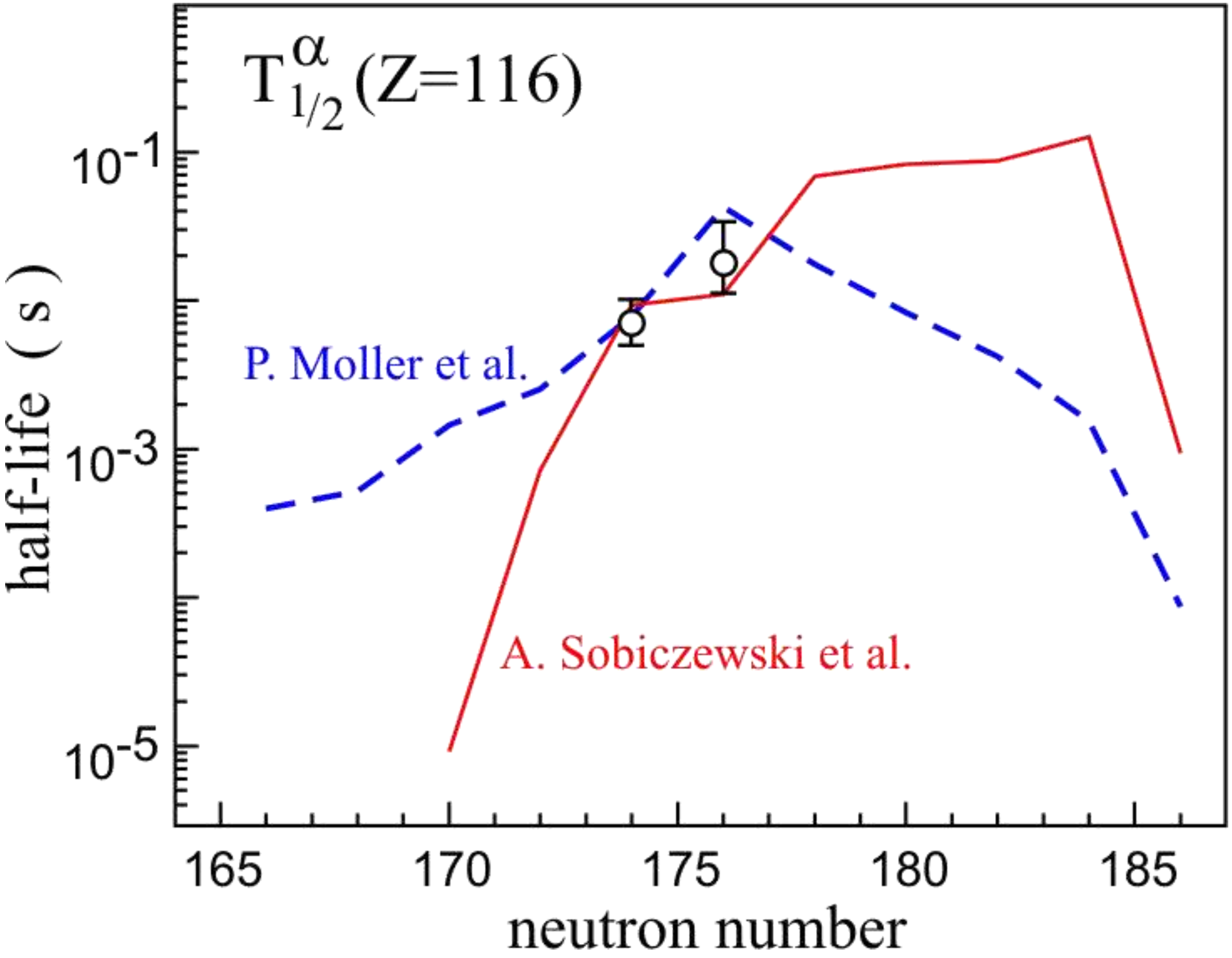




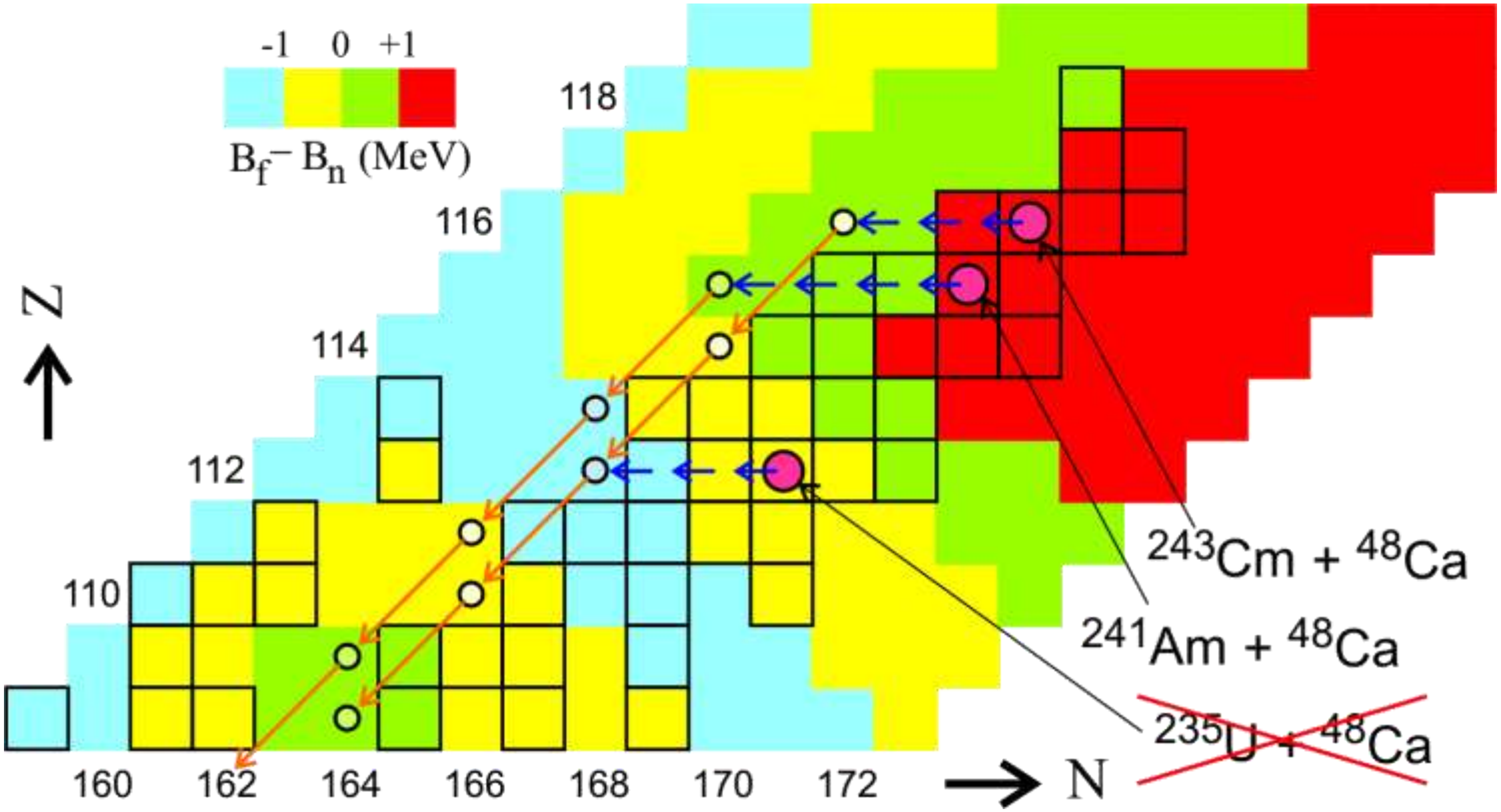
The gap in SH mass area must be filled somehow



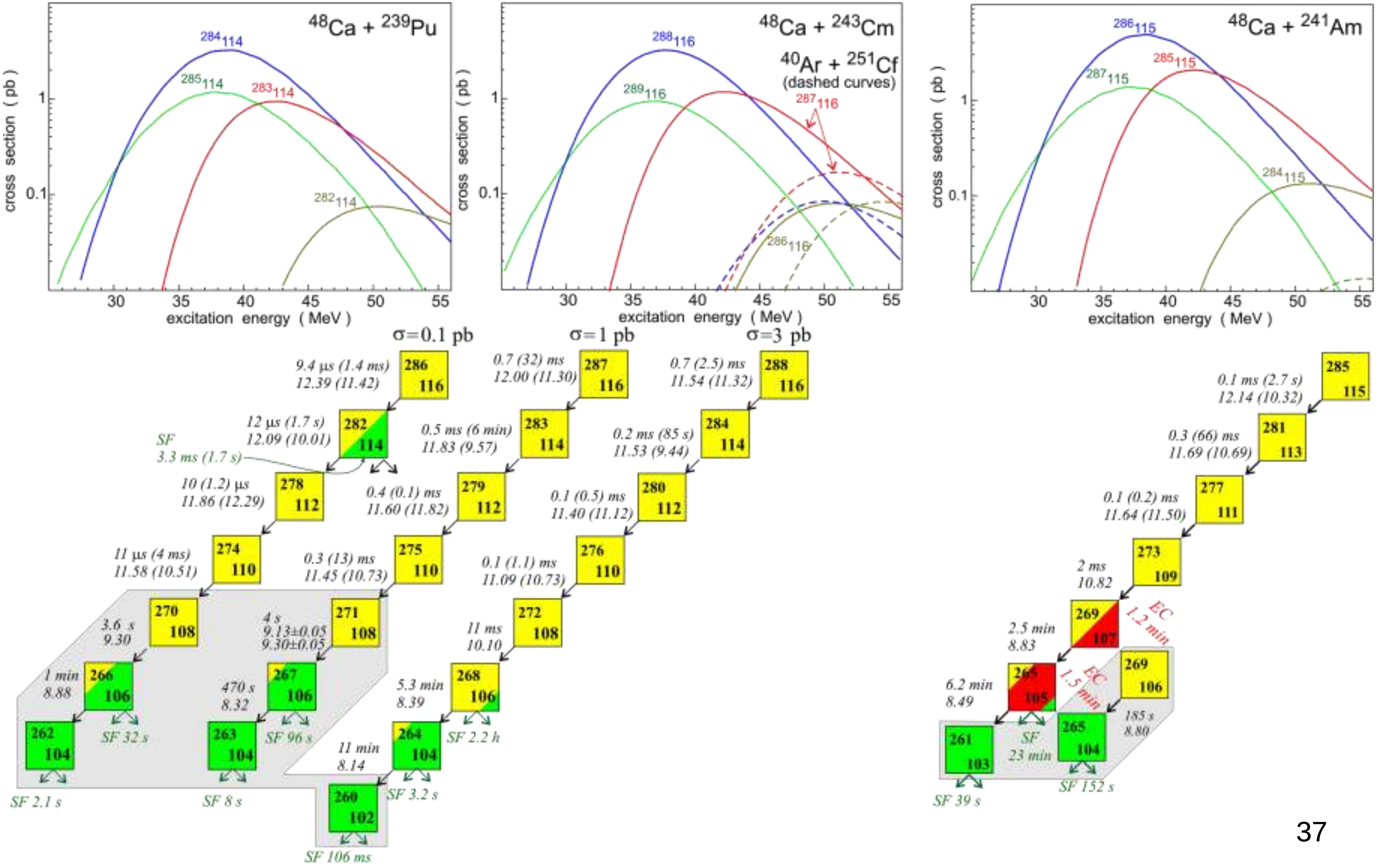
Our ability of predictions in superheavy mass area



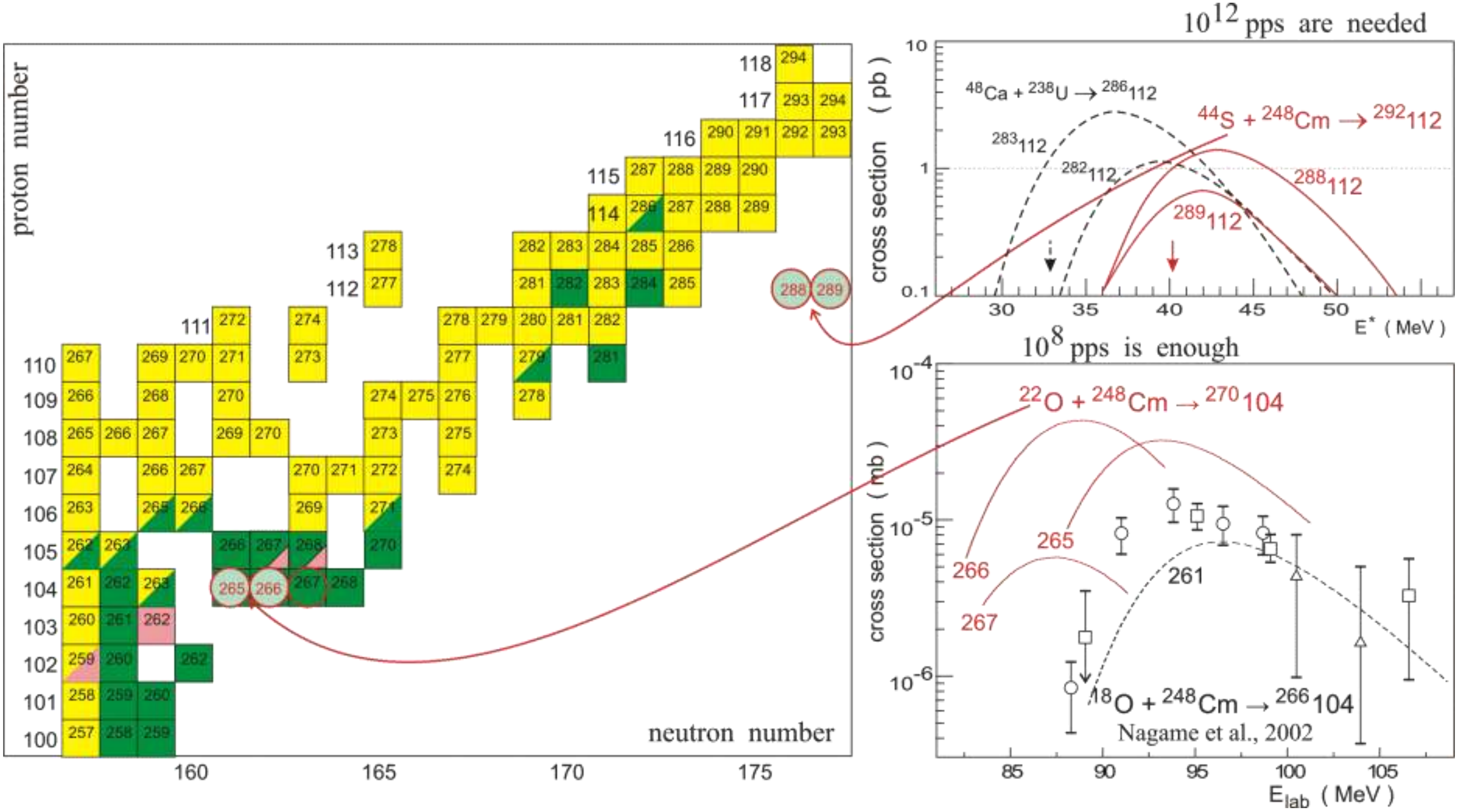
It is easier to fill the gap from above



Cross sections are high enough to perform experiments at available facilities just now

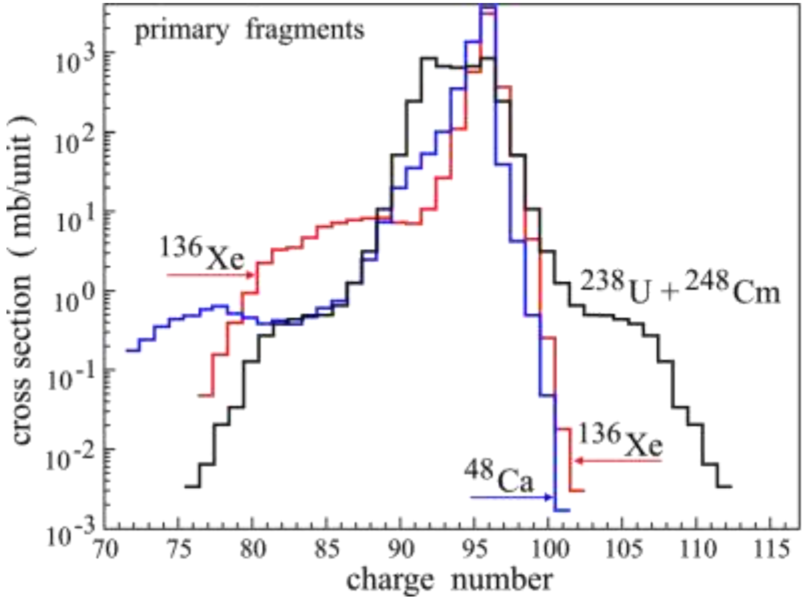


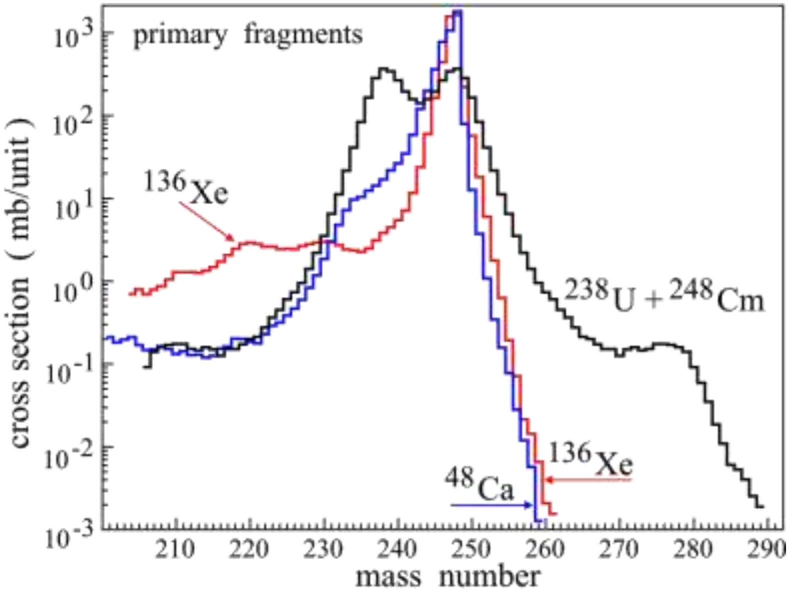
Use of low-energy Radioactive Ion Beams for production of neutron rich superheavy nuclei ?



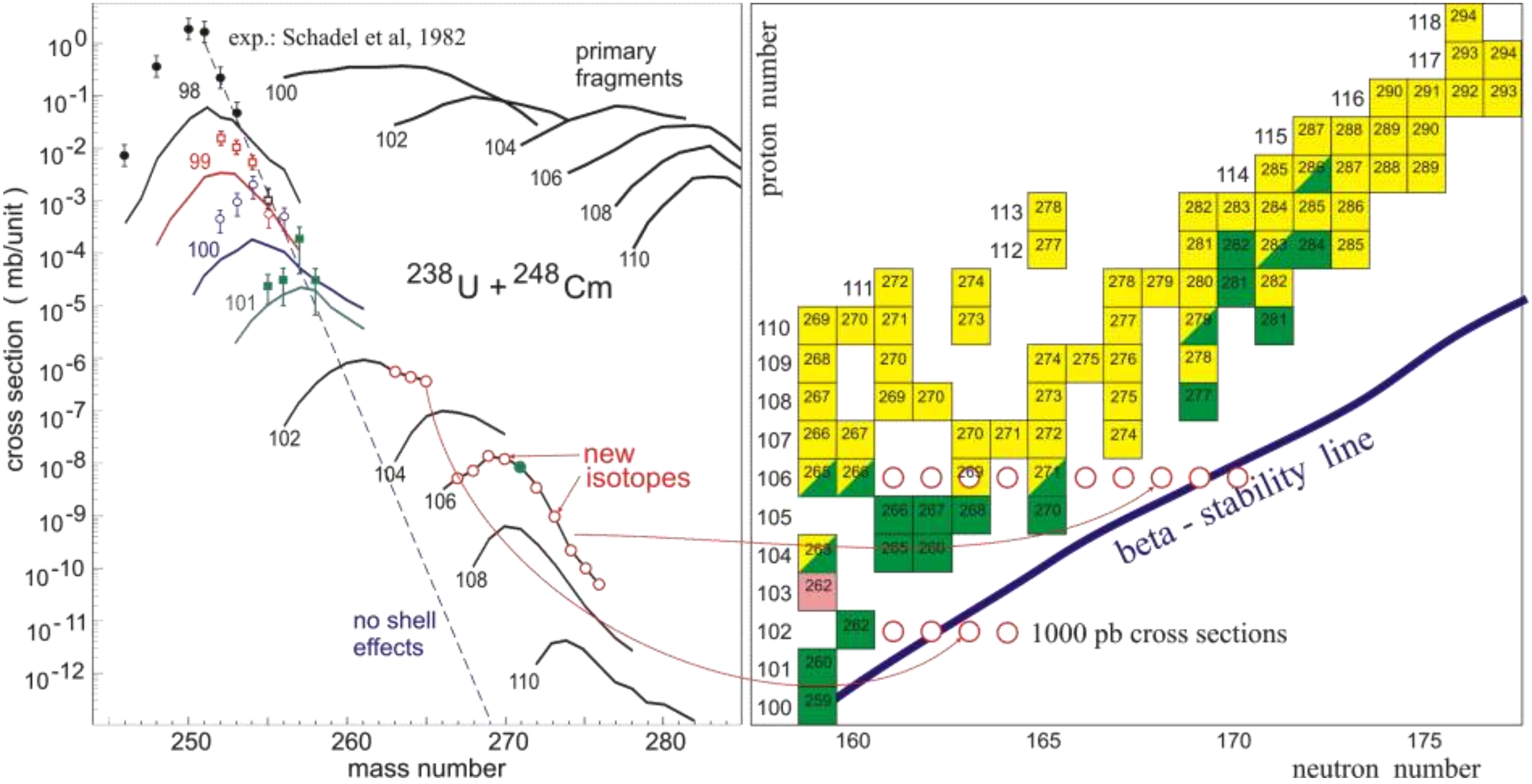
No chances today and in the nearest future

Multi-nucleon transfer for production of superheavies:
U-like beams give us more chances to produce neutron rich SH nuclei in transfer reactions

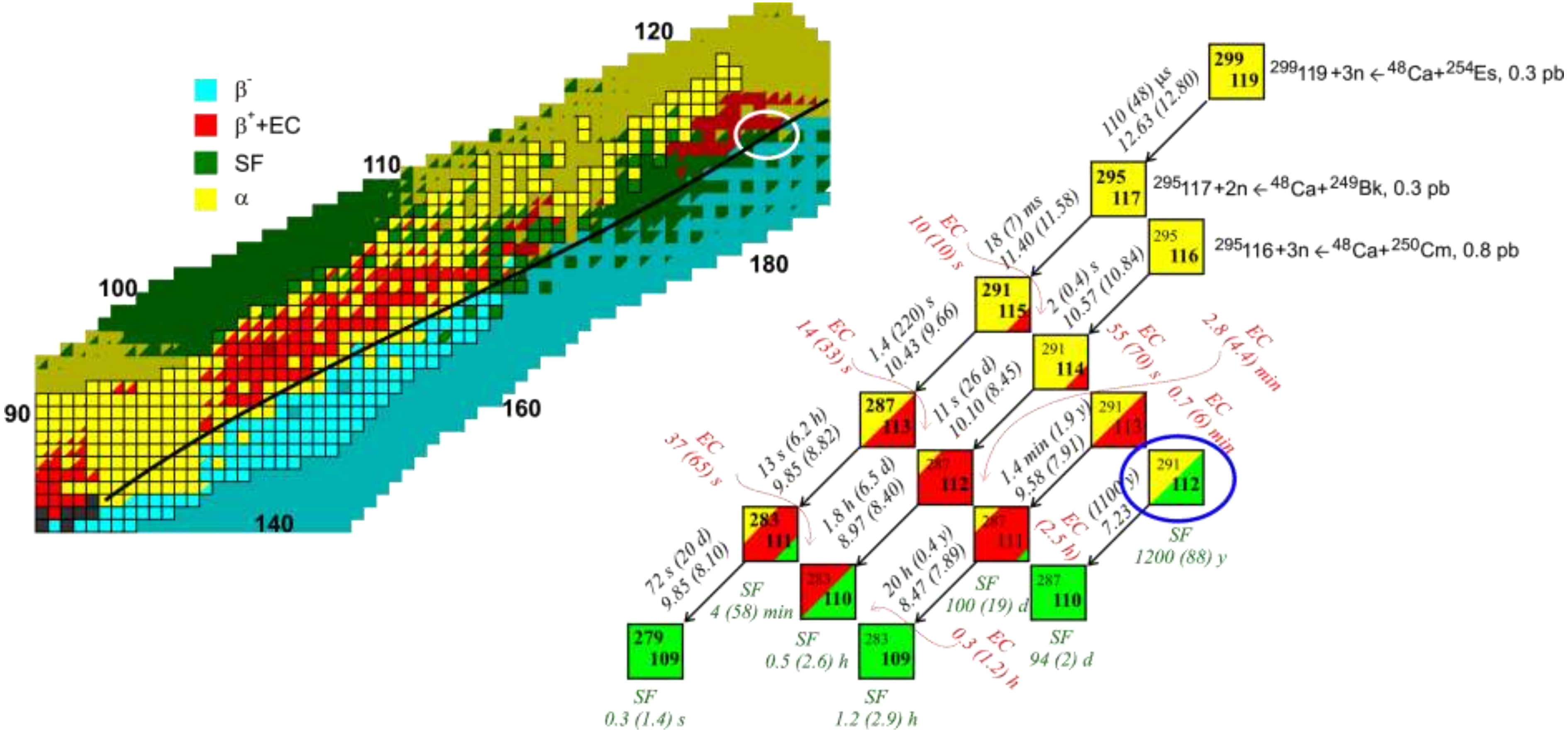




Production of transfermium nuclei along the line of stability looks quite possible



Narrow pathway to the island of stability just by fusion reactions !



Experiments for the next several years:

- **Elements 119 and 120 may be really synthesized in the Ti and/or Cr fusion reactions with cross sections of about 0.05 - 0.02 pb.**

  Perhaps they are the heaviest SH elements with $T_{1/2} > 1\ \mu s$?

  (beam time: 0.5 year + 0.5 year)

- **The gap in SH mass area (Z=106 – 116) can be easily filled in fusion reactions of 48Ca with lighter isotopes of actinides (239Pu, 241Am, 243Cm, …).**

  (beam time: one weak for one decay chain of a new SH isotope)

- **The narrow pathway to the island of stability is found at last !**

  (beam time: 20 days to check the idea)

- **Multi-nucleon transfer reactions have to be used for synthesis of neutron enriched long-living SH nuclei located along the beta-stability line. 48Ca and 136Xe beams are insufficient. Uranium-like beam is needed !**

  (beam time: one day for one new neutron-rich isotope of Fm, Md, No…)